Traveling small-scale Josephson vortices

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(Submitted 28 July 1994; resubmitted 6 August 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 6, 442–445 (25 September 1994)

An analytic description is offered for a traveling periodic structure of small-scale vortices in the strong-dissipation limit of a nonlocal Josephson electrodynamics. © *1994 American Institute of Physics.*

Theoretical interest has recently been focused on the study of moving Josephson vortex structures with a length scale shorter than the London depth, in which case it becomes necessary to use a nonlocal Josephson electrodynamics[1–3] (see also Refs. 4–7 for the case of superconducting thin films). In contrast with a local Josephson electrodynamics based on the sine-Gordon equation, in a nonlocal electrodynamics there is no common path for seeking possible nonlinear solutions. Accordingly, only a few solutions describing moving small-scale vortex structures have been derived so far. A first exact solution in the dissipation-free limit, describing a traveling 4π kink, was derived in Refs. 3 and 8. An exact solution for a traveling kink in the strong-dissipation limit was derived in Ref. 9. Finally, solutions describing spatially periodic, dissipationless, traveling vortex structures were found in Ref. 10. In the limit of a large distance between the vortices making up these structures, those solutions transform into a traveling 4π kink.

In this letter we describe a new traveling periodic structure of Josephson vortices in the strong-dissipation case. In the limit of a large distance between individual vortices, this structure becomes the kink of Ref. 9. At the same time, the solution given below makes it possible to examine the transition to a periodic, static, small-scale structure with a magnetic field as in Ref. 10.

The small-scale vortex structure of interest in this letter is described by the solution of the equation

$$\sin\varphi+\frac{\beta}{\omega_j^2}\frac{\partial\varphi}{\partial t}=\frac{l}{\pi}\int_{-\infty}^{+\infty}\frac{dz'}{z'-z}\frac{\partial\varphi(z',t)}{\partial z'}+i, \tag{1}$$

where φ is the phase difference between the Cooper pairs on the different sides of the Josephson junction, ω_j is the Josephson frequency, β corresponds to a description of ordinary dissipation, $l=\lambda_0^3(\lambda_+^2+\lambda_-^2)^{-1}$, λ_+ and λ_- are the London depths on the two sides of the junction, $\lambda_0^3=\lambda_j^2(\lambda_++\lambda_-+2d)$, λ_j is the Josephson length, $2d$ is the width of the junction, and $i=j/j_c$ is the dimensionless current density, which is assumed to be distributed uniformly over the contact (cf. Refs. 9 and 11). This equation corresponds to the strong-dissipation limit of a nonlocal Josephson electrodynamics in the asymptotic limit of a small-scale vortex structure with a spatial scale smaller than the London depths.

We should refine the terminology used in this letter. A "small-scale structure" is one whose length scale is smaller than the London depth. In particular, the state discussed below, in which the magnetic field vector lies in the plane of the Josephson junction, is a traveling mixed state with a periodic structure of period (cf. Ref. 10)

$$L=\frac{\phi_0}{2\pi(\lambda_+ + \lambda_- + 2d)\bar{H}},$$

where $\bar{H}$ is the magnetic field averaged along the contact. Such a period is evidently smaller than the London depth if the magnetic field strength is below the lower critical field but close to it (cf. Ref. 10).

According to Refs. 2, 3, and 6–9, Eq. (1) arises specifically when the length scale of the vortex structures is short in comparison with the London depths. This circumstance is the sole distinction between the model of this paper and the customary model. The transition to ordinary local Josephson electrodynamics is made with the help of not asymptotic equation (1) but a nonlocal equation for the phase difference between the Cooper pairs. That equation was found in Ref. 1 (see also Refs. 2 and 3). Two asymptotic limits of this equation are Eq. (1) (the Peierls equation) and the sine-Gordon equation, which is the foundation of local Josephson electrodynamics.

The periodic structure we are assuming here is described by

$$\varphi(z,t)=\theta+\pi+2\arctan\frac{\tan[(z+vt)/2L]}{\tanh(\alpha/2)}. \tag{2}$$

To explain the onset of this solution we note that we have

$$\frac{l}{\pi}\int_{-\infty}^{+\infty}\frac{dz'}{z'-z}\frac{\partial\varphi(z',t)}{\partial z'}=\frac{l}{L}\frac{\sin[(z+vt)/L]}{\cos[(z+vt)/L]-\cosh\alpha},$$

$$\frac{\beta}{\omega_j^2}\frac{\partial\varphi}{\partial t}=\frac{\beta v}{\omega_j^2 L}\frac{\sinh\alpha}{\cosh\alpha-\cos[(z+vt)/L]},$$

$$\sin\varphi=-\frac{\cos\theta\sinh\alpha\sin[(z+vt)/L]}{\cosh\alpha-\cos[(z+vt)/L]}+\sin\theta\left[\cosh\alpha-\frac{\sinh^2\alpha}{\cosh\alpha-\cos[(z+vt)/L]}\right].$$

It is easy to see that by substituting these expressions into Eq. (1) we find the following relations, which determine θ and α:

$$\sin\theta\cosh\alpha=i,\quad \cos\theta\sinh\alpha=l/L. \tag{3}$$

We also find that the velocity v of the moving vortex structure is given by

$$v=(\omega_j^2 L i/\beta)\tanh\alpha. \tag{4}$$

It is not difficult to see that we have

$$\cos^2\theta=\left[\frac{1}{4}\left(i^2+\frac{l^2}{L^2}-1\right)^2+\frac{l^2}{L^2}\right]^{1/2}-\frac{1}{2}\left[i^2+\frac{l^2}{L^2}-1\right], \tag{5}$$

$$\sinh^2\alpha=\left[\frac{1}{4}\left(i^2+\frac{l^2}{L^2}-1\right)^2+\frac{l^2}{L^2}\right]^{1/2}+\frac{1}{2}\left[i^2+\frac{l^2}{L^2}-1\right], \tag{6}$$

$$v=\frac{\omega_j^2 L}{\beta}\left\{\left[\frac{1}{4}\left(i^2+\frac{l^2}{L^2}-1\right)^2+\frac{l^2}{L^2}\right]^{1/2}+\frac{1}{2}\left[i^2-\frac{l^2}{L^2}-1\right]\right\}^{1/2}. \tag{7}$$

In the limit $i=0$, in which we have $v=0$ and $\theta=0$ according to (5) and (7), solution (2) becomes the static periodic solution of Ref. 10. In the other limit $L\rightarrow\infty$, the periodic structure of Josephson vortices in (2) degenerates into the traveling kink of Ref. 9.

The magnetic field inside the junction is described by (cf. Ref. 9)

$$H_y(z,t)=-\bar{H}+\delta H_y(z+vt), \tag{8}$$

where the average field is given by the usual expression

$$\bar{H}=\frac{\phi_0}{2\pi L(\lambda_+ +\lambda_- +2d)}, \tag{9}$$

and the oscillatory part is

$$\delta H_y(z+vt)=-\frac{\phi_0}{2\pi(\lambda_+^2+\lambda_-^2)}\left\{\alpha-\ln\left[2\left(\cosh\alpha-\cos\frac{z+vt}{L}\right)\right]\right\}$$

$$=-\frac{\phi_0}{\pi(\lambda_+^2+\lambda_-^2)}\sum_{n=1}^{\infty}\frac{1}{n}e^{-n\alpha}\cos\frac{n(z+vt)}{L}. \tag{10}$$

Here $\phi_0=\pi\hbar c/|e|=2.05\times 10^{-7}$ Oe·cm^2 is the quantum of magnetic flux. Relation (9) shows that the traveling periodic Josephson structure is a small-scale structure for magnetic fields below the lower critical field. The magnetic field in superconductors is described by Eq. (3.5) of Ref. 10, in which z should be replaced by $z+vt$, and in which expression (6) of the present paper should be used for α.

Finally, the electric potential at the junction is

$$V=-\frac{\hbar}{2|e|}\frac{\partial\varphi}{\partial t}=\frac{\hbar v}{2|e|L}\frac{\sinh\alpha}{\cos[(z+vt)/L]-\cosh\alpha}$$

$$=-\frac{\hbar v}{2|e|L}\left[1+\sum_{k=1}^{\infty}2e^{-k\alpha}\cos\left(k\frac{z+vt}{L}\right)\right]. \tag{11}$$

In particular, the average electric potential over a period is related to the dimensionless current density i by

$$\bar{V}=-\frac{\hbar\omega_j^2}{2|e|\beta}\left\{\left[\frac{1}{4}\left(i^2+\frac{l^2}{L^2}-1\right)^2+\frac{l^2}{L^2}\right]^{1/2}+\frac{1}{2}\left[i^2-\frac{l^2}{L^2}-1\right]\right\}^{1/2}. \tag{12}$$

In the limits of low and high values of i, the average potential is a linear function of the current density. At low currents, the resistance depends on the length scale L, i.e., on the average magnetic field. At high currents it does not depend on the magnetic field. All these circumstances distinguish the small-scale structure under consideration here in a qualitative way from the usual structure (cf. Ref. 9).

We wish to stress that in the case $i=1$ Eq. (12) of the present letter yields a finite value for the potential, in contrast with the case in Ref. 9. The singular current–voltage characteristic of Ref. 9 [see Eq. (31) of that paper] follows from expression (12) of the present letter in the limit $L \gg l$ and under the condition

$$0 > 1 - \epsilon^2 \gg (l/L)^2.$$

The right side of this inequality corresponds to the elimination of the singularity.

In summary, an analytic solution has been derived for the fundamental equation of a nonlocal Josephson electrodynamics in the strong-dissipation limit. This solution describes a traveling chain of periodic small-scale vortices.

I am indebted to A. Gurevich, who furnished me a reprint of his article.[9] This study was supported by the Scientific Council on High Temperature Superconductivity. It was carried out as part of Project 93015 of the State Program on High Temperature Superconductivity.

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Translated by D. Parsons