Observation of nonequilibrium Bose condensation of quasiphonons excited by a noisy microwave pump

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The parametric excitation of quasiphonons in the antiferromagnet $FeBO_3$ by a noisy microwave field has been studied. There are two critical pump amplitudes: one corresponding to the onset of nonlinear absorption of microwave power and one above which strong phase correlations, i.e., a nonequilibrium Bose condensate, arise in the system of excited quasiphonons. © *1994 American Institute of Physics.*

Spin waves and magnetoelastic waves in magnetic materials are exceptionally useful for studying the physics of nonlinear wave processes. The parametric resonance of such waves in an alternating magnetic field, $h\cos\omega_p t$, directed parallel to a static field has been studied quite thoroughly. In this case the photons of the microwave pump generate pairs of half-frequency quasiparticles with equal but oppositely directed wave vectors ($\omega_p=\omega_{\mathbf{k}}+\omega_{-\mathbf{k}}$). Just above the excitation threshold $h_{c0}=\gamma/V$ (γ is the linear relaxation rate of the quasiparticle, and V is the coefficient of the coupling with the pump field), a dynamic order characterized by only two parameters arises in the system. One of these parameters is the number of excited pairs of magnons (or quasiphonons); the other is the phase of these entities with respect to the pump field.[1–3] This nonequilibrium Bose condensate, of a macroscopic number of excited quasiparticles, is evidently a forced oscillation of the nonlinear medium at the frequency of the excited field.

The situation becomes much more complicated if an alternating external field in the frequency interval $(\omega_p-\Delta\omega/2,\ \omega_p+\Delta\omega/2)$ is applied and if the pump frequency spread, $\Delta\omega$, exceeds the value of γ for the excited waves. That a nonequilibrium Bose condensate can form in this case is not at all obvious, since a noisy field has no distinctive phase and by itself is incapable of establishing a coherent state in the excited system.

A parametric resonance of spin waves in the case of a noisy pump has been studied by Mikhaĭlov and Uporov[4] and Cherepanov.[5] The equations which they derived for the average threshold power of the noise field, P_c, at which a nonlinear absorption arises in the system (on the average over time), become the same under the condition $\Delta\omega\gg\gamma$. For arbitrary values $\Delta\omega\ll\omega_p$ one can write

$$(h_c/h_{c0})^2-1=\Delta\omega/\gamma, \tag{1}$$

where $h_c\propto P_c^{1/2}$ is an integral characteristic of the amplitude of the noisy microwave field.

On the other hand, these papers differ fundamentally in their view of the behavior of the system just above the excitation threshold. It was asserted in Ref. 4 that the primary mechanism operating to limit the absorption is a nonlinear dissipation, while phase correlations of the waves are ineffective. In Ref. 5, in contrast, it was stated that strong phase correlations should arise in pairs of excited waves (a phase limitation mechanism), as in the case of a monochromatic pump. This question has remained open, since no corresponding experiments have been carried out. We might add that in yet another theoretical paper[6] there has been an analysis of a noisy pump of magnons with $\Delta\omega \sim \omega_p$. It was shown there that in this case there is the possibility of a nonequilibrium Bose condensation of magnons at the bottom of the spin-wave band.

Another possibility for an incoherent effect of external fields on a spin-wave system has been studied by Zautkin *et al.*[7] They studied the parametric excitation of magnons by a monochromatic microwave field in yttrium iron garnet in the case of a noisy modulation of the spin-wave spectrum. They stated that this case corresponds to a nonmonochromatic pump with a fluctuating phase. It was concluded on the basis of a qualitative agreement between the theoretical equations and the experimental results on the nonlinear magnetic susceptibility above the threshold that phase correlations are responsible for the primary mechanism which operates to limit the excitation level. On the other hand, there was no direct confirmation that there are strong phase correlations in the wave system. In a later study[8] of the problem regarding the threshold $\tilde{h}_c$ for the disruption of phase correlations in a wave system excited by a monochromatic field upon the application of a noisy modulation of the spectrum, the formula

$$(\tilde{h}_c/h_{c0})^2 - 1 = (2 + \Delta\omega/\gamma)\Delta\omega/\gamma \qquad (2)$$

was derived. The measurements of the threshold for parametric excitation carried out in Ref. 7 can be described identically, within the experimental errors, by Eq. (1) (which was used in Ref. 7) and Eq. (2).

In this letter we examine the behavior of a system of nonequilibrium quasiphonons in the cases of 1) a noisy pump with $\gamma \leqslant \Delta\omega \ll \omega_p$ and 2) a monochromatic microwave pump with a noisy modulation of the spectrum of magnetoelastic waves. Our primary purpose here is to search for states with strong phase correlations (a nonequilibrium Bose condensate) in this system.

Experimental procedure

A noisy field with $\omega_p/2\pi = 1200$–1250 MHz and $\Delta\omega/2\pi \leqslant 3$ MHz was produced in antiferromagnetic $FeBO_3$ (with a Néel temperature of 348 K) in two ways:

1) The noise signal from a G2-37 generator was used for frequency modulation of an RG4-04 microwave source. The pump spectrum was studied with an S4-49 spectrum analyzer and was found to be approximately Gaussian. The spectral width $\Delta\omega$ was measured within 10% at half-maximum.

2) The microwave source operated under ordinary conditions. The signal from the noise generator in the frequency band 0–600 kHz was fed through a wide-band amplifier to a modulation coil wound coaxially with the spiral microwave cavity. The value of $\Delta\omega$ was found as in Ref. 7: $\Delta\omega = (H_n V)^2/\gamma = \gamma(H_n/h_{c0})^2$. The error in the measurement of the spectral density of the noise field, H_n^2, was 10%.

A sample with a volume of $1.2 \times 3.5 \times 7\ \text{mm}^3$ was placed in the spiral microwave cavity, which had a low Q (~ 300), so that the edge of the spectrum of the noisy pump would not be cut off. Experiments were carried out at $T=77$ and 293 K in a field $H=80$–400 Oe. The static magnetic field and all the alternating magnetic fields were parallel. The relaxation rate of the quasiphonons which were excited was $\gamma \sim 2\pi \cdot 0.1$ MHz.

The microwave pumping was carried out in a pulsed regime with a repetition frequency of 10–500 Hz and a pulse length of 20–1000 μs. On the pump pulses which were transmitted through the cavity and detected we observed the onset of a nonlinear absorption above a certain threshold power. The relative error of the measurements of this threshold was less than 5%.

To find the nonequilibrium Bose condensate we used two methods, based on the observation of collective effects arising from this state.

The first method was to observe the modulation response α_m. This method was developed for, and has been widely used for, studying the properties of a nonequilibrium Bose condensate of magnons which arises in the case of monochromatic pumping.[2] A weak rf magnetic field $H_m \cos \omega_m t$ was applied to the sample; this weak field modulated the spectrum of quasiphonons. In this case, oscillations of the amplitude and phase of the nonequilibrium Bose condensate around their equilibrium values arise (these are "collective oscillations"). They lead to a modulation of the microwave power absorbed by the sample, with an amplitude ΔP and a frequency ω_m. The onset of this amplitude modulation ($\Delta P = \alpha_m H_m$) indicates the existence of a nonequilibrium Bose condensate in the sample.

To find α_m, we detected the microwave signal transmitted through the cavity and sent it to a tuned microvoltmeter, tuned to the frequency ω_m. This signal was then sent to a lock-in detector; the output signal from the detector was fed to the Y input of an x,y chart recorder.

The second method was to observe electromagnetic radiation from the system of excited quasiphonons. The effect, which we observed just recently (and which will be described in detail in a separate paper), is as follows: After the pump is turned off, the nonequilibrium Bose condensate produced by the monochromatic microwave pump causes characteristic electromagnetic radiation from the sample. The intensity of this radiation is not monotonic in time; its frequency is close to the pump frequency.

Results and discussion

A study showed that the picture of the nonlinear absorption of microwave power is qualitatively the same in the cases of noisy pumps produced by frequency modulation and by phase modulation (a noisy modulation of the spectrum): One should distinguish two thresholds, $h_c^{(1)} < h_c^{(2)}$. The first corresponds to the onset of nonlinear microwave absorption, and the second to the formation of a nonequilibrium Bose condensate.

Figure 1 shows a chart recording of the amplitude of the modulation-response signal versus the total pump power. Also shown here are oscilloscope traces of the microwave signal transmitted through the cavity. We see that there is no nonlinear absorption at point A. On trace B there are some "spikes" of nonlinear absorption above the threshold $h_c^{(1)}$. With increasing pump power, this absorption increases on the average (trace C), but

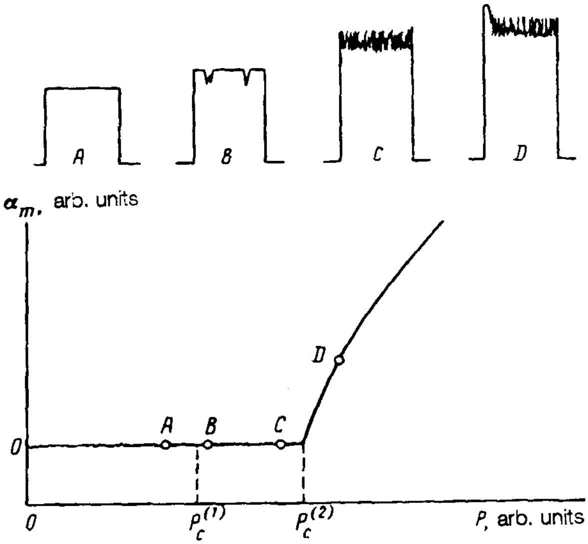

FIG. 1. Modulation-response signal versus the pump power $P \propto h^2$ (this is a chart recording) and oscilloscope traces of the microwave pulses transmitted through the cavity at points A, B, C, and D. The pulse length is 100 μs; H_m<0.1 Oe.

there is still no modulation-response signal, although nonequilibrium quasiphonons do exist (on the average) in the sample. As the microwave power is raised further, at $h > h_c^{(2)}$, we observe a characteristic decay, as in the case of a monochromatic pump. At the same time, a modulation response arises, signifying the presence (on the average) of phase correlations in the system, i.e., signifying the formation of a nonequilibrium Bose condensate. The ratio of thresholds, $h_c^{(2)}/h_c^{(1)}$, increases with increasing width $\Delta\omega$ and reaches a value of 2. The ratio of $h_c^{(2)}$ to the threshold in the absence of noise reaches $h_c^{(2)}/h_{c0} \simeq 5$.

The formation of a nonequilibrium Bose condensate above the threshold $h_c^{(2)}$ was also detected from the electromagnetic emission from the sample, observed after the end of the pump pulse (Fig. 2). We wish to stress that at $h < h_c^{(2)}$ there is no such radiation, while above the threshold the radiation has approximately the same intensity and the same induction-signal decay time as in the case of the emission of a Bose condensate excited by a monochromatic microwave field. It can thus be concluded that the nonequi-

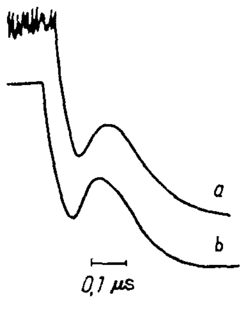

FIG. 2. Oscilloscope traces of the trailing edge of the microwave pulses, along with a signal representing the electromagnetic radiation from the sample. a—Noisy pump; b—monochromatic pump.

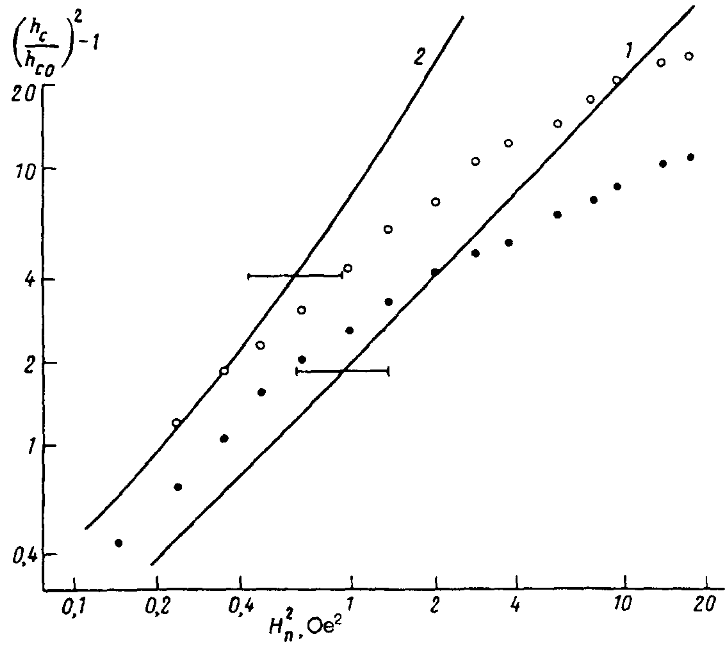

FIG. 3. Relative increase in the thresholds during noisy modulation of the phonon spectrum. ●—For $h_c^{(1)}$; ○—for $h_c^{(2)}$. $H=153$ Oe, $\omega_p=2\pi\times1220$ MHz, $T=77$ K, $\Delta\omega=2\pi\times600$ kHz. Curves 1 and 2 are drawn from Eqs. (1) and (2), respectively, without adjustable parameters.

librium Bose condensate of quasiphonons which arises in the case of a noisy pump has approximately the same frequency width as in the case of a monochromatic pump.

Figure 3 shows the thresholds $h_c^{(1)}$ and $h_c^{(2)}$ versus the spectral density of the noisy field in the case of noisy modulation of the quasiphonon spectrum. Curves 1 and 2 are calculated from Eqs. (1) and (2) without adjustable parameters, but with the help of the experimental value of h_{c0}^2. Since the error in the absolute measurements of h_{c0}^2 is 50%, the theoretical curves have the same error along the abscissa, as shown in Fig. 3. We see that at $h_c/h_{c0}<2$ the experimental results can be described satisfactorily by dynamic theories: 1) for the threshold for nonlinear microwave absorption and 2) for the threshold for the appearance (and disappearance) of strong phase correlations in the system. Thereafter, both theories describe an increase in the pump threshold which is faster than that observed in our experiments.

A functional dependence of $(h_c/h_{c0})^2-1$ on $\Delta\omega$ like that in Fig. 3 was found for the case of a noisy pump produced by frequency modulation. In this case, however, Eq. (1)—which correctly describes the functional dependence of $h_c^{(1)}$ on $\Delta\omega$ at $\Delta\omega\leqslant5\gamma$—predicts a threshold for nonlinear absorption which is several times the experimental values. Figure 4 shows the relative increase in $h_c^{(1)}$ as a function of the amplitude of the static magnetic field. The curve is drawn from Eq. (1) with the help of the values found for the quasiphonon relaxation rate from the threshold level of the

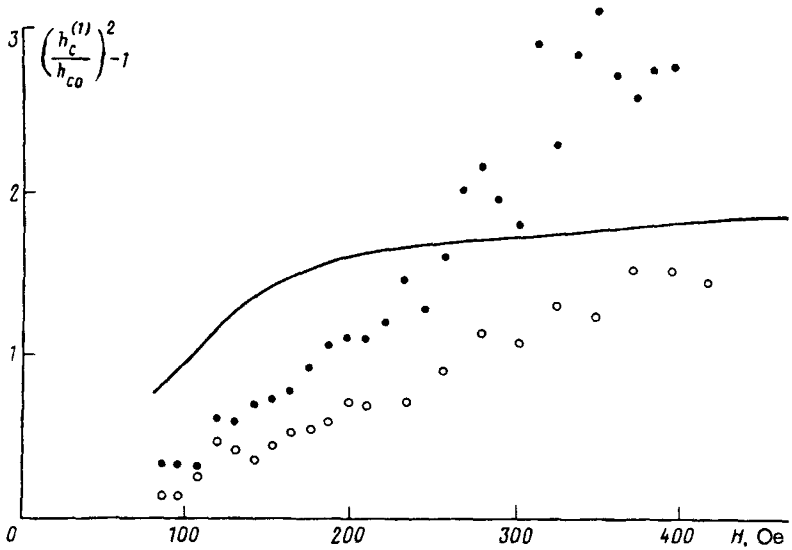

FIG. 4. Relative increase in the threshold power of the noisy pump versus the strength of the static magnetic field. ○—$\Delta\omega=2\pi\times0.5$ MHz; ●—$\Delta\omega=2\pi\times1$ MHz. $T=77$ K, $\omega_p=2\pi\times1220$ MHz. The curve corresponds to a calculation from Eq. (1) with a coefficient of 0.2 for the first case and a coefficient of 0.1 for the second.

monochromatic pump. We see that theory (1) fails to describe the experimental results in terms of both the functional dependence on $\gamma=\gamma(H)$ and the absolute values (the values on the theoretical curve must be multiplied by a factor of 5 in the case $\Delta\omega=2\pi\times0.5$ MHz and by a factor of 10 in the case $\Delta\omega=2\pi\times1$ MHz).

In summary, these experiments have established that a coherent state (a nonequilibrium Bose condensate) forms during parametric excitation of a nonlinear wave system (of quasiphonons) by 1) a noisy microwave pump and 2) a monochromatic microwave pump with a noisy modulation of the spectrum. The existing theory gives a satisfactory description of the results found in the second case, at small amplitudes of the noisy modulation.

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