Parton interpretation of quark fragmentation into hadrons with different spins

A. Anselm and M. G. Ryskin
Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg, 188350, Russia

M. Anselmino
Dipartimento di Fisica Teorica, Universitá di Torino and INFN, Via P. Giuria 1, I-10125 Torino, Italy

F. Murgia
Istituto Nazionale de Fisica Nucleare, Sezione di Cagliari, Via A. Negri 18, I-09127 Cagliari, Italy

(Submitted 31 August 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 7, 486–493 (10 October 1994)

The spin density matrix of the hadron h, which is created via quark fragmentation in the process $e^-e^+ \to q\bar{q} \to h+X$, has been calculated. In the case of $h=\Lambda$ the experimental data could possibly elucidate the problem of s-quark contribution to the spin of the Λ hyperon, which can be compared with the case of the proton (the "spin crisis"). For hadrons with spin 1/2 the parton description, which uses only probabilities, generally works much better than for vector particles, since in the former case the nondiagonal matrix elements of the hadron spin density matrix are suppressed by a small factor $p_T/(z\sqrt{s})$, where p_T is the hadron transverse momentum inside the jet.

1. Introduction

The spin content of baryons, in terms of their constituents, is far from being clearly understood. The last deep inelastic scattering experiments,[1-3] which probed the internal structure of polarized nucleons with polarized leptons, have upset our original, simple, spectroscopic picture of three valence quarks that share the baryon spin (and other quantum numbers), while gluons and sea quarks contribute only to the baryon momentum. Oribital angular momentum, gluon, and nonperturbative contributions have also been invoked and taken into account to explain the experimental data on polarized nucleon structure functions; the quarks alone do not seem to be able to account for the proton spin.[4]

Any further experimental information on the internal spin structure of baryons is therefore of great importance and usefulness in our attempts to better understand the subtleties of quark and gluon bound states. In this respect, as has been known for some time,[5] and recently proposed again,[6,7] the production of Λ particles in the e^-e^+ annihilations seems to be particularly interesting. According to the SU(6) quark model, the Λ hyperon has a very simple spin flavor wave function, since its spin carried by the s quark, while the remaining ud pair is in the $S=I=0$ state. The s quarks in the e^-e^+ annihilations ($e^-e^+ \to s\bar{s}$) at high energies are strongly polarized; in particular, at the Z_0 pole, the (negative) longitudinal polarization of s quarks can be as high as $P \sim -0.9$ and virtually independent of the production angle. Fast Λ's produced in the e^-e^+ annihilations may be though as the direct result of s-quark fragmentations; moreover, their weak decays allow a precise measurement of their polarization; it would therefore be useful to propose a study[6,7] of the correlation between the (observed) Λ polarization and the (computable) s-quark polarization to determine the extent to which the latter is transferred to the former.

If the s quark and the Λ polarizations turned out to be uncorrelated, than a "Λ spin crisis" would add to the "proton spin crisis,"[8] an intriguing result indeed. There are already several experimental indications that this actually is the case in the puzzling polarization data of large pT Λ's (and other hyperons), which are inclusively produced in several processes as fragments of unpolarized nucleons.[9] In all of them it appears that unpolarized quarks (inside the unpolarized nucleons) give rise to strongly polarized Λ's, a situation which cannot be understood at the constituent level.

In this paper we consider the problem at a more general level by studying the production of a hadron h in the e^-e^+ annihilations via the two-step process $e^-e^+ \to q\bar{q} \to h + X$; we relate the helicity density matrix of the h hadron, $\rho_{\lambda_h \lambda'_h}(h)$, to the helicity density matrix of the $q\bar{q}$ pair, $\rho_{\lambda_q \lambda_{\bar{q}} \lambda'_q \lambda'_{\bar{q}}}(q,\bar{q})$, computed within the standard model, via some "fragmentation amplitudes" $D_{\lambda_h \lambda_X; \lambda_q \lambda_{\bar{q}}}(q\bar{q} \to \Lambda + X)$. This generalizes the approach of Refs. 6 and 7, since it also takes into account the nondiagonal elements of the $\rho(h)$ matrix and allows for the final-state interactions among the $q\bar{q}$ pairs. These final-state interactions, which are normally ignored in the (successful) computation of the jet cross sections and angular distributions, might be important when more subtle quantities, like spin correlations, are involved. Our results reproduce the usual ones,[6,7] when only the independent fragmentation of one quark and the diagonal helicity density matrix elements are considered.

Our approach, a coherent fragmentation picture versus an incoherent one, was proposed in a previous paper,[10] for the production of vector mesons in the purely electromagnetic case (e.g., $e^-e^+ \to D^a + X$, via one-photon annihilation). A related experimental measurement, in which the value of $\rho_{1,-1}(D^*)$ was very small, was also performed.[11] The large errors, however, do not allow us to draw any definite conclusion. We think that further tests of the importance of coherence effects, especially in the light of so many unexpected and subtle spin effects, are important.

For the production of spin-1/2 hadrons it turns out, however, that all nondiagonal spin density matrix elements are strongly suppressed if the particles inside the jet are well collimated. More precisely, we find that $\rho_{\lambda_h \lambda'_h}(h)(h, S_h = 1/2) \sim 2p_T/(z\sqrt{s})$ for $\lambda_h \neq \lambda'_h$, where p_T is the transverse momentum of the hadron with respect to the jet (the original quark) direction, and $z\sqrt{s}/2$ is the hadron longitudinal momentum. For high-energy spin-1/2 hadrons we obtain the usual independent quark fragmentation results, which seem to be well founded. Surprisingly, this is not necessarily true for spin-1 vector particles.[10]

In Sec. 2 we discuss the formalism and give the general expression of the matrix elements $\rho_{\lambda_h \lambda'_h}(h)$ in our scheme. In Sec. 3 we consider the case of spin-1/2 and spin-1 hadrons; we also recall how to measure these matrix elements; e.g., via the angular distribution of the Λ decay products in the Λ rest frame. In the last section we discuss our results and the differences between different spin particles; we point out how clarifying measurements could be actually performed at LEP or LSC.

2. The $e^-e^+ \to q\bar{q} \to h+X$ process and the helicity density matrix of the particle h

Following Ref. 10 we consider the spin state of a hadron h inclusively produced in the e^-e^+ annihilation. Such a process should proceed via the usual two-step process: First, a $q\bar{q}$ pair is created $(e^-e^+ \to q\bar{q})$, which then annihilates into the observed hadron plus other unobserved particles $(q\bar{q} \to h+X)$. While the first process can be perturbatively computed on the basis of the standard model, the second one is essentially unknown and is parametrized using phenomenological fragmentation functions. The helicity density matrix of the hadron h can then be written as follows:

$$\rho_{\lambda_h\lambda_h'}(h)=\frac{1}{N_h}\sum_{q,X,\lambda_X,\lambda_q,\lambda_{\bar{q}},\lambda_q',\lambda_{\bar{q}}'} D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}\rho_{\lambda_q\lambda_{\bar{q}},\lambda_q'\lambda_{\bar{q}}'}(q,\bar{q})D^*_{\lambda_h'\lambda_X;\lambda_q'\lambda_{\bar{q}}'}, \tag{2.1}$$

where $\rho_{\lambda_q\lambda_{\bar{q}};\lambda_q'\lambda_{\bar{q}}'}(q,\bar{q})$ is the helicity density matrix of the $q\bar{q}$ state:

$$\rho_{\lambda_q\lambda_{\bar{q}};\lambda_q'\lambda_{\bar{q}}'}(q,\bar{q})=\frac{1}{N_{q\bar{q}}}\sum_{\lambda_{e^-},\lambda_{e^+};\lambda_{e^-}',\lambda_{e^+}'} M_{\lambda_q\lambda_{\bar{q}};\lambda_{e^-}\lambda_{e^+}}(e^-,e^+)$$

$$\times M''_{\lambda_q'\lambda_{\bar{q}}';\lambda_{e^-}'\lambda_{e^+}'}. \tag{2.2}$$

The M's are the helicity amplitudes for the $e^-e^+ \to q\bar{q}$ process and the D's are the fragmentation *amplitudes* for the process $q\bar{q} \to h+X$; they are related to the usual fragmentation *functions* D_q^h by

$$\sum_{X,\lambda_X,\lambda_h,\lambda_q,\lambda_{\bar{q}},\lambda_q',\lambda_{\bar{q}}'} D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}\rho_{\lambda_q\lambda_{\bar{q}};\lambda_q'\lambda_{\bar{q}}'}D^*_{\lambda_h\lambda_X;\lambda_q'\lambda_{\bar{q}}'}=D_q^h. \tag{2.3}$$

In Eq. (2.1) the summation is carried out over all quark flavors q. In Eqs. (2.1) and (2.3) the Σ_{X,λ_X} remains for the phase space integration and for the sum over the spins of all unobserved particles which are grouped into the state X. The normalization factors N_h and $N_{q\bar{q}}$ are such that $\mathrm{Tr}(\rho)=1$. In particular,

$$N_h=\sum_q D_q^h. \tag{2.4}$$

Finally, the spin state of the initial e^-e^+ system is described by the helicity density matrix $\rho_{\lambda_{e^-}\lambda_{e^+};\lambda_{e^-}'\lambda_{e^+}'}(e^-,e^+)$. For unpolarized e^- and e^+ we have

$$\rho_{\lambda_{e^-}\lambda_{e^+};\lambda_{e^-}'\lambda_{e^+}'}(e^-,e^+)=\tfrac{1}{4}\delta_{\lambda_{e^-}\lambda_{e^-}'}\delta_{\lambda_{e^+}\lambda_{e^+}'}. \tag{2.5}$$

Equation (2.1) differs from the standard, independent, quark-fragmentation approach in that the hadronization process is $q\bar{q} \to h+X$, rather than $q \to h+X$. The (necessary) $q\bar{q}$ interactions are taken into account in the fragmentation amplitudes. If one assumes that

$D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar q}}$ is independent of $\bar q$ and ignores the quantum numbers of $\bar q$ and all its fragmentation products [so that one can set $\lambda_{\bar q} = \lambda'_{\bar q}$ in $\rho(q,\bar q)$ and use $\Sigma_{\lambda_{\bar q}}\rho_{\lambda_q\lambda_{\bar q};\lambda'_q\lambda_{\bar q}} = \rho_{\lambda_q\lambda'_q}(q)$], Eq. (2.1) becomes

$$\rho_{\lambda_h\lambda'_h}(h)=\frac{1}{N_h}\sum_{q,X,\lambda_X,\lambda_q,\lambda'_q} D_{\lambda_h\lambda_X;\lambda_q}\,\rho_{\lambda_q\lambda'_q}(q)D^*_{\lambda'_h\lambda_X;\lambda'_q}. \tag{2.6}$$

Furthermore, if the final hadron momentum is parallel to the quark momentum, then the angular momentum conservation in the forward direction requires that for each $D_{\lambda_h\lambda_X;\lambda_q}(q\to h + X)$

$$\lambda_h+\lambda_X=\lambda_q. \tag{2.7}$$

Recalling that when quark masses are ignored, the helicity density matrix of the quark produced in the $e^-e^+\to q\bar q$ annihilation is diagonal [$\rho_{\lambda_q\lambda'_q}(q) = 0$ for $\lambda_q\neq\lambda'_q$] and we find that only the diagonal terms in Eq. (2.6) are retained. This gives rise to the usual probabilistic formula:[5]

$$\rho_{\lambda_h\lambda_h}(h)\frac{1}{N_h}\sum_{q,\lambda_q}\rho_{\lambda_q\lambda_q}(q)D^{h,\lambda_h}_{q,\lambda_q}, \tag{2.8}$$

where $D^{h,\lambda_h}_{q,\lambda_q} = \Sigma_{X,\lambda_X}|D_{\lambda_h,\lambda_X;\lambda_q}|^2$.

This must be true in general,[10] as can be seen from Eqs. (2.1) and (2). The $q\bar q$ helicity density matrix can be computed if $\rho(e^-,e^+)$ [Eq. (2.5) for unpolarized beams] and the center-of-mass annihilation amplitudes $M_{\lambda_q\lambda_{\bar q};\lambda_{e^-}\lambda_{e^+}}$ are known. In lowest order of the perturbative standard model, ignoring the quark masses and taking into account the weak (Z_0) and the electromagnetic (γ) contributions, they are given for unpolarized initial leptons (l) on the basis of the standard model parameters[12] by

$$\begin{aligned}M_{\lambda_q\lambda_{\bar q};\lambda_{l^-}\lambda_{l^+}}(s,\theta)&=i\,\delta_{\lambda_{l^-},-\lambda_{l^+}}\delta_{\lambda_q,-\lambda_{\bar q}}\\ &\times\left\{\left[e_qe^2-\frac{1}{4}g_Z^2\,c_V^lc_V^q\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right](1+4\lambda_{l^-}\lambda_q\cos\theta)\right.\\ &+\frac{1}{4}g_Z^2\,\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}[2c_V^lc_A^q(\lambda_{l^-}\cos\theta+\lambda_q)\\ &\left.+\,2c_A^lc_V^q(\lambda_{l^-}+\lambda_q\cos\theta)-c_A^lc_A^q(\cos\theta+4\lambda_{l^-}\lambda_q)]\right\},\end{aligned} \tag{2.9}$$

where $\sqrt{s}$ is the l^-l^+ c.m. energy, θ is the q production angle, and e_q is the quark charge. From Eqs. (2.2), (2.5), and (2.9) we see that the only nonzero elements of $\rho_{\lambda_q\lambda_{\bar q};\lambda'_q\lambda'_{\bar q}}(q,\bar q)$ are

$$\begin{aligned}&\rho_{+-;+-}(q\bar q)=1-\rho_{-+;-+}(q\bar q),\\ &\rho_{+-;-+}(q\bar q)=\rho^*_{-+;+-}(q\bar q).\end{aligned} \tag{2.10}$$

Inserting Eq. (2.10) into Eq. (2.1), we obtain

$$\rho_{\lambda_h\lambda_h'}(h)=\frac{1}{N_h}\sum_{X,\lambda_X}\{[D_{\lambda_h\lambda_X;+-}D^*_{\lambda_h'\lambda_X;+-}-D_{\lambda_h\lambda_X;-+}D^*_{\lambda_h'\lambda_X;-+}]\rho_{+-;+-}$$

$$+[D_{\lambda_h\lambda_X;+-}D^*_{\lambda_h'\lambda_X;-+}+D_{\lambda_h\lambda_X;-+}D^*_{\lambda_h'\lambda_X;+-}]\mathrm{Re}[\rho_{+-;-+}]$$

$$+i[D_{\lambda_h\lambda_X;+-}D^*_{\lambda_h'\lambda_X;-+}-D_{\lambda_h\lambda_X;-+}D^*_{\lambda_h'\lambda_X;+-}]\mathrm{Im}[\rho_{+-;-+}]$$

$$+D_{\lambda_h\lambda_X;-+}D^*_{\lambda_h'\lambda_X;-+}. \tag{2.11}$$

The above expression for $\rho_{\lambda_h\lambda_h'}(h)$ can be further simplified by exploiting the fact that the hadronization process $(q\bar{q}\to h+X)$ is parity invarient. The parity relations for the c.m. helicity amplitudes[13] then yield

$$\sum_{\lambda_X} D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}D^*_{\lambda_h'\lambda_X;\lambda_q'\lambda_{\bar{q}}'}=(-1)^{2S_h-\lambda_h-\lambda_h'}D_{-\lambda_h\lambda_X;-\lambda_q-\lambda_{\bar{q}}}D^*_{-\lambda_h'\lambda_X;-\lambda_q'-\lambda_{\bar{q}}'}. \tag{2.12}$$

It appears from Eq. (2.11) that the nondiagonal elements of $\rho(q\bar{q})$, which are usually disregarded in the independent quark fragmentation scheme, contribute to $\rho_{\lambda_h\lambda_h}(h)$ and $\rho_{\lambda_h\lambda_h'}(h)(\lambda_h\neq\lambda_h')$; in particular, the latter can be different from zero. However, even if the fragmentation amplitudes $D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}$ are essentially unknown, we know from experiment that at lowest perturbative order in the e^-e^+ annihilation the hadron production proceeds via the creation of two collimated particle jets, each of which retains the original q and $\bar{q}$ directions. The $D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}$ can then be regarded as the center-of- mass helicity amplitudes for the process $q\bar{q}\to h+X$, *essentially in the forward direction*, i.e., with the hadron momentum $\mathbf{h}$ almost parallel to the quark momentum $\mathbf{q}$ (and $\bar{\mathbf{q}}$ parallel to $\mathbf{X}$).

From the well-known forward behavior of the c.m. helicity amplitudes[13] we then have

$$D_{\lambda_h\lambda_X;\lambda_q\lambda_{\bar{q}}}D^*_{\lambda_h'\lambda_X;\lambda_q'\lambda_{\bar{q}}'}\sim\left(\sin\frac{\theta_h}{2}\right)^{|\lambda_h-\lambda_X-\lambda_q+\lambda_{\bar{q}}|+|\lambda_h'-\lambda_X-\lambda_q'+\lambda_{\bar{q}}'|}, \tag{2.13}$$

where θ_h is the angle between the hadron momentum, $\mathbf{h}=z\mathbf{q}+\mathbf{p}_T$, and the quark momentum $\mathbf{q}$; i.e.,

$$\sin\theta_h=\frac{2p_T}{z\sqrt{s}}, \tag{2.14}$$

where we have used $|\mathbf{q}|=\sqrt{s}/2$.

The bilinear combinations of the fragmentation amplitudes, which contribute to $\rho(h)$, then will not be suppressed by powers of $[p_T/(z\sqrt{s})]$ only if the exponents in Eq. (2.13) are zero, which involves

$$\lambda_h-\lambda_h'=(\lambda_q-\lambda_{\bar{q}})-(\lambda_q'-\lambda_{\bar{q}}'). \tag{2.15}$$

Equations (2.11)–(2.14) hold, in general, for the production of a hadron h with spin S_h. In the next section we specialize them to the cases $S_h=1/2$ and $S_h=1$.

3. The process $e^-e^+\to q\bar{q}\to h+X$ with $S_h=1/2$ and $S_h=1$

From Eqs. (2.11) and (2.12) we have the following expressions for the helicity density matrix of hadrons with spin $S_h=1.2$:

$$\rho_{++}(S_h=1/2)=\frac{1}{N_h}\sum_{X,\lambda_X,q}\{|D_{+\lambda_X;-+}|^2+[|D_{+\lambda_X;+-}|^2-|D_{+\lambda_X;-+}|^2]\rho_{+-;+-}$$
$$+2\,\mathrm{Re}[D_{+\lambda_X;+-}D^*_{+\lambda_X;-+}]\mathrm{Re}[\rho_{+-;-+}]$$
$$-2\,\mathrm{Im}[D_{+\lambda_X;+-}D^*_{+\lambda_X;-+}]\mathrm{Im}[\rho_{+-;-+}]\}, \tag{3.1}$$

$$\mathrm{Re}[\rho_{+-}(S_h=1/2)]=\frac{1}{N_h}\sum_{X,\lambda_X,q}\{\mathrm{Re}[D_{+\lambda_X;+-}D^*_{-\lambda_X;+-}](2\rho_{+-;+-}-1)$$
$$-\mathrm{Im}[D_{+\lambda_X;+-}D^*_{-\lambda_X;-+}-D_{+\lambda_X;-+}D^*_{-\lambda_X;+-}]\mathrm{Im}[\rho_{+-:-+}]\}, \tag{3.2}$$

$$\mathrm{Im}[\rho_{+-}(S_h+1/2)]=\frac{1}{N_h}\sum_{X,\lambda_X,q}\{\mathrm{Im}[D_{+\lambda_X;+-}D^*_{-\lambda_X;+-}]$$
$$+\mathrm{Im}[D_{+\lambda_X;+-}D^*_{-\lambda_X;-+}$$
$$+D_{+\lambda_X;-+}D^*_{-\lambda_X;+-}]\mathrm{Re}[\rho_{+-;-+}]\}. \tag{3.3}$$

In this case, however, none of the nondiagonal bilinear combinations appearing in the above equations satisfy Eq. (2.15). In other words, in the limit $p_T/(z\sqrt{s})\to 0$ we obtain the usual probabilistic independent quark fragmentation results, (2.8):

$$\rho_{++}(S_h=1/2)=\frac{1}{N_h}\sum_q[\rho_{++}(q)D^{h,+}_{q,+}+\rho_{--}(q)D^{h,+}_{q,-}],$$

$$\rho_{+-}(S_h=1/2)=0,$$

where we have used $\rho_{+-;+-}(q\bar{q})=\rho_{++}(q)$. Note that the corrections to Eq. (3.4) are on the order of $[p_T/(z\sqrt{s})]^2$ for ρ_{++} and $[p_T/(z\sqrt{s})]$ for ρ_{+-}.

This conclusion does not hold for the production of vector particles,[10] $S_h=1$. In such a case, Eq. (2.15) can be satisfied by setting $\lambda_h=-\lambda'_h=1$ and $\lambda_q-\lambda_{\bar{q}}=\lambda'_{\bar{q}}-\lambda'_q=1$, and even in the limit $(p_T/(z\sqrt{s}))\to 0$ we are left with nonzero nondiagonal matrix elements:

$$\mathrm{Re}[\rho_{1,-1}(S_h=1)]=\frac{1}{N_h}\sum_{X,\lambda_X,q}D_{1\lambda_X;+-}D^*_{-1\lambda_X;-+}\mathrm{Re}[\rho_{+-;-+}],$$

$$\mathrm{Im}[\rho_{1,-1}(S_h=1)]=\frac{1}{N_h}\sum_{X,\lambda_X,q}D_{1\lambda_X;+-}D^*_{-1\lambda_X;-+}\mathrm{Im}[\rho_{+-;-+}]. \tag{3.5}$$

The helicity density matrix elements can be measured by looking at the two-body decay of the hadron h in its helicity rest frame, $h \to A + B$. Let us consider as two most common examples the decays of a spin-1/2 hadron into a spin-1/2 plus spin-0 hadrons. (e.g., $\Lambda \to p\pi^-$, $\Sigma^+ \to p\pi^0$, $\Lambda_c \to \Lambda\pi^+$) and of a spin-1 hadron into two spin-0 hadrons (e.g., $\rho \to \pi\pi$, $K^* \to K\pi$, $D^* \to D\pi$). In the former case the normalized angular distribution of the final particle A (e.g., p for the $\Lambda \to p\pi$ decay) is given by

$$W(\theta_A,\varphi_A)=\frac{1}{2\pi}\left\{\frac{1}{2}-\frac{1}{2}\ \alpha \cos\ \theta_A+\alpha\rho_{++}(S_h=1/2)\cos\ \theta_A+\ \alpha\mathrm{Re}[\rho_{+-}(S_h=1/2)]\right.$$
$$\left.\times\sin\ \theta_A\ \cos\ \varphi_A-\alpha\mathrm{Im}[\rho_{+-}(S_h=1/2)]\sin\ \theta_A\ \sin\ \varphi_A\right\}, \qquad (3.6)$$

where θ_A and φ_A are respectively the polar and azimuthal angle of particle A in the rest frame of the decaying spin-1/2 hadron; α is the known weak decay parameter (e.g., $\alpha=0.642\pm0.013$ for the $\Lambda \to p\pi^-$ decay).

In the case of a spin-1 $\to$ spin-0 + spin-0 decay we have

$$W(\theta_A,\varphi_A)=\frac{3}{4\pi}\left\{\frac{1}{2}(1-\rho_{0.0})+\frac{1}{2}\ (3\rho_{0.0}-1)\cos^2\theta_A\right.$$
$$-\ \frac{1}{\sqrt{2}}\sin\ 2\theta_A\ \cos\ \varphi_A\ \mathrm{Re}[\rho_{1.0}-\rho_{0.-1}]$$
$$+\ \frac{1}{\sqrt{2}}\sin\ 2\theta_A\ \sin\ \varphi_A\ \mathrm{Im}[\rho_{1.0}-\rho_{0.-1}]$$
$$\left.-\ \sin^2\ \theta_A\ \cos\ 2\varphi_A\ \mathrm{Re}[\rho_{1.-1}]+\sin^2\ \theta_A\sin\ 2\varphi_A\ \mathrm{Im}[\rho_{1.-1}]\right\}. \qquad (3.7)$$

In the case in which the production process of the hadron h is parity invarient ($e^-e^+ \to h+X$ at $\sqrt{s} \ll M_Z$, when the weak effects can be ignored) we have additional parity relations between the matrix elements of $\rho(h)$. Equations (3.6) and (3.7) then simplify respectively to

$$W(\theta_A,\varphi_A)=\frac{1}{2\pi}\left\{\frac{1}{2}\ -\alpha\ \mathrm{Im}[\rho_{+-}]\sin\ \theta_A\ \sin\ \varphi_A\right\}, \qquad (3.8)$$

$$W(\theta_A,\varphi_A)=\frac{3}{4\pi}\left\{\frac{1}{2}(1-\rho_{0.0})+\frac{1}{2}(3\rho_{0.0}-1)\cos^2\ \theta_A\right.$$
$$\left.-\sqrt{2}\ \sin\ 2\theta_A\ \cos\ \varphi_A\ \mathrm{Re}[\rho_{1.0}]-\sin^2\theta_A\ \cos\ 2\varphi_A\rho_{1.-1}\right\}. \qquad (3.9)$$

A measurement of the spin density matrix elements of the D^* mesons produced in the e^-e^+ annihilation at $\sqrt{s}=29$ GeV, according to Eq. (3.9), was carried out in Ref. 11. The data seem to be consistent with zero values of $\rho_{\lambda\lambda'}$ $(D^*)(\lambda\neq\lambda')$. Because of large errors, however, no definite conclusion has yet been made.

4. Conclusions

The study of spin properties of hadrons produced in the e^-e^+ annihilations should supply a rich and interesting basis for understanding the relationship between the spin of the constituents and that of the composite mesons and baryons. Such understanding is at the moment, after the "proton spin crisis" triggered by DIS experiments, rather confused and in need of much better theoretical and experimental information.

In the high- energy e^-e^+ experiments, which are being performed at LEP and SLC, quarks are always produced, via electroweak interactions, with a large polarization; to what extent can this be extended to the final hadrons is an open question which should be addressed and hopefully answered in the near future. The usual approach uses a simple probabilistic scheme, according to which a polarized quark fragments independently into the final hadron, and our ignorance at the hadronization process is hidden in the phenomenological fragmentation functions which should be measured via inclusive $e^-e^+ \to h+X$ cross sections.

Such a scheme is quite successful in the measurements of jet angular distributions and unpolarized cross sections, because the number and direction of the original quarks reflect accurately the number and direction of the observed hadrons. This might not be true when more subtle quantities, like spin observables, are involved. Many mysterious spin effects in several physical processes prove to be a challenge for the existing theories, because the spin variables often involve subtle interference effects between different amplitudes, rather than simple squared moduli, like unpolarized cross sections.

In this paper we have considered a more general scheme, i.e., $e^-e^+ \to q\bar{q}$ and $q\bar{q} \to h+X$, as a two-step process to describe hadron production in e^-e^+ annihilations. The second part, $q\bar{q} \to h+X$, is treated as an effective two-body into two-body process, which takes into account the $q\bar{q}$ interactions and the total spin states of the initial $q\bar{q}$ system; they can be easily computed on the basis of the perturbative standard model.

Our general result for the s pin density matrix of the hadron h is given by Eq. (2.11); it differs from the usual independent quark fragmentation. For example it allows nonzero nondiagonal matrix elements. However, upon consideration of additional experimental information, like the narrowness of the observed jets, it turns out that many of the contributions to Eq. (2.11) are negligible at high energies. Surprisingly, whether one obtains the usual probabilistic results or not depends on the spin of the final hadron h: For spin-1/2 particles this is indeed the case, whereas for spin-1 vector particles we can still be left with nondiagonal matrix elements. Only a direct measurement of these matrix elements would settle this question conclusively.

The fact that for spin-1/2 hadrons the independent fragmentation of a quark turns out to be a well-founded model is encouraging information in view of the proposed analysis of the Λ-hyperon polarization: To a good approximation a fast Λ, produced in the e^-e^+ annihilation, is generated by a strongly polarized s quark. Its independent fragmentation into Λ should certainly determine the spin of the latter; whether entirely, i.e., with the s quark and the Λ polarization equal, or only partially, is yet to be determined. The answer depends on the particular features of the Λ wave function and might not be trivial, as the proton spin crisis has taught us. The conclusion that $q\bar{q}$ interactions do not affect the analysis is the first reassuring step in the difficult task ahead of us.

One of us (M.A.) would like to thank the Petersburg Nuclear Physics Institute, where part of this work was done, for warm hospitality.

[1] I. Ashman *et al.*, Phys. Lett. B **206**, 364 (1988); Nucl. Phys. B **328**, 1 (1989).
[2] B. Adeva *et al.*, Phys. Lett. B **302**, 533 (1993).
[3] R. Arnold *et al.*, SLAC preprint SLAC PUB 5101 (1993).
[4] For reviews see, e.g., G. Altarelli, in *The Challenging Questions,* Proceedings of the 1989 Erice School, A. Zichichi Ed. (Plenum, New York, 1990); J. Ellis, Nucl. Phys. A **546**, 447 (1992).
[5] See, e.g., N. S. Craigie *et al.*, Phys. Rep. **99**, 69 (1983).
[6] G. Gustafson and J. Häkkinen, Lund. Preprint LU TP 92-29 (1992).
[7] M. Burkardt and R. L. Jaffe, MIT Preprint CPT# 2186 (1993).
[8] M. Anselmino and E. Leader, Z. Phys. C **41**, 239 (1988).
[9] See, e.g., K. Heller, in *Spin and Polarization Dynamics in Nuclear and Particle Physics*, Eds. A. O. Barut *et al.* (World Scientific, Singapore, 1990).
[10] M. Anselmino *et al.* Z. Phys. C **29**, 135 (1985).
[11] S. Abachi *et al.*, Phys. Lett. B **199**, 535 (1987).
[12] See, e.g., D. Griffiths, *Introduction to Elementary Particles* (Harper & Row, New York, 1987).
[13] See, e.g., C. Bourrely *et al.*, Phys. Rep. **59**, 95 (1980).

Published in English in the original Russian journal. Reproduced here with stylistic changes by the Translation Editor.