$D=3$ Chern-Simons-Higgs SUSY system with an anomalous magnetic momentum

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(Submitted 27 June 1994; resubmitted 12 September 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 7, 494–497 (10 October 1994)

The $D=3$ Chern-Simons-Higgs SUSY system with an anomalous magnetic momentum and the effects of spontaneous gauge and supersymmetry breaking have been investigated. © *1994 American Institute of Physics.*

Low-dimensional models of field theories, which are now being studied extensively, are sufficiently attractive for many reasons. Apart from purely pedagogical interest, dimensional lowering intrinsically simplifies the problem and allows us to obtain faster understanding of the basic questions. Such models, moreover, can be used for concrete physical applications, for example, in the case of the fractional quantum Hall effect.

The questions connected with particle gauge interaction with an anomalous magnetic momentum in $D=3$ space–time dimensions and the construction of the Chern–Simons superfield theory have been raised earlier in many papers (see, for example, Refs. 1–3). This paper is an attempt to bridge these formulations and the supersymmetry breaking in the Chern–Simons supersymmetric field theory with an anomalous magnetic momentum.

Suggested master action of this type of theory has the form

$$S=\int d^3xd^2\theta\left(\bar{\tilde{\nabla}}_\alpha\bar{\Phi}\nabla^\alpha\Phi-\mathrm{Im}\Phi\bar{\Phi}-\frac{1}{2}\bar{W}_\alpha W^\alpha+\frac{1}{4}k\bar{\Gamma}_\alpha W^\alpha\right). \tag{1}$$

Here and below we use the notation given in Ref. 2:

$$\Phi(x,\theta)=\phi(x)+i\bar{\theta}\psi(x)+i\bar{\theta}\theta F(x),$$

where $\phi(x)$, $\psi^\alpha(x)$, and $F(x)$ are the complex fields in the Wess–Zumino $D=3$ superfield multiplet,

$$\nabla^\alpha\Phi=(D^\alpha+e\Gamma^\alpha)\Phi,\quad \tilde{\nabla}^\alpha\bar{\Phi}=(D^\alpha-e\Gamma^\alpha)\bar{\Phi},$$

$$D^\alpha=(\partial/\partial\bar{\theta}_\alpha)+i(\gamma^l\theta)^\alpha\partial_l,\quad \bar{D}_\alpha=\varepsilon_{\alpha\beta}D^\beta$$

are the spinorial covariant derivatives,

$$\Gamma^\alpha(x,\theta)=\bar{\theta}_\beta(\gamma^m)^{\beta\alpha}v_m+i\bar{\theta}\theta\lambda^\alpha$$

is the vector gauge supermultiplet in the WZ gauge, and

$$W^\alpha=-\frac{i}{2}\bar{D}^\beta D^\alpha\Gamma_\beta$$

is the irreducible submultiplet of the gauge fields in the theory.

Action (1) thus contains the kinetic and mass terms of the Wess–Zumino multiplet that interacts with the gauge Maxwell field, the kinetic term of the gauge field, and the superfield analog of the Chern–Simons term.

After integration over the Grassman variables, action (1) transform to

$$S=\int d^3x\Big(\bar{F}F-(\partial_m-iev_m)\phi(\partial^m+iev^m)\bar{\phi}+i\bar{\psi}_\alpha(\gamma^m)^\alpha_\beta(\partial_m-iev_m)\psi^\beta$$

$$+e(\bar{\psi}_a\lambda^\alpha\phi+\bar{\lambda}_\alpha\psi^\alpha\bar{\phi})+m(\bar{F}\phi+F\bar{\phi})-\frac{i}{2}m\bar{\psi}_\alpha\psi^\alpha-\frac{i}{2}\bar{\lambda}_\alpha(\gamma^m)^\alpha_\beta\partial_m\lambda^\beta$$

$$-\frac{1}{4}f_{mn}f^{mn}+\frac{i}{4}k\bar{\lambda}_\beta\lambda^\beta+\frac{1}{2}k\varepsilon^{kln}v_k\partial_l v_n\Big), \qquad (2)$$

where $f_{mn}=\partial_m v_n-\partial_n v_m$.

For investigation of the spontaneous $U(1)$ gauge symmetry breaking we introduce the term which describes the matter–field self-interaction

$$-i\frac{1}{2}g(\Phi\bar{\Phi})^2. \qquad (3)$$

After eliminating the auxiliary fields F and $\bar{F}$, we then obtain a system with a ϕ-dependent part of the potential:

$$V(\phi)=|\phi|^2\left(\frac{1}{2}g|\phi|^2+m\right)^2, \qquad (4)$$

which has a minimum under nonzero vacuum expectation of $|\phi|2$,

$$\langle|\phi|^2\rangle=-2m/g\equiv\mu. \qquad (5)$$

The form potential (4), in a sense typical of the $D=3$ Chern-Simons-Higgs system, was reproduced earlier in Ref. 2, starting with self-dual requirements.

The presence of an anomalous magnetic momentum in the theory modifies the interaction scheme from a minimal, the field-current type, to a nonminimal scheme. In this connection, the vector potential $v_m\to v_m+l\varepsilon_{mnl}f^{nl}$, where ε_{mnl} is the Levi–Civita tensor, and consequently the vector covariant derivative $D_m=\partial_m-iev_m$ turn into $\tilde{D}_m=\partial_m-iev_m-il\varepsilon_{mnl}f^{nl}$. The key point of our reasoning is the consideration of the theory with a modified covariant vector derivative of the form

$$\tilde{\partial}_m=\partial_m-il\varepsilon_{mnl}f^{nl} \qquad (6)$$

and the standard vector potential v_m. In this connection, it is very important to note that the superalgebra does not change; i.e.,

$$\{\tilde{\bar{D}}_\alpha,\tilde{D}^\beta\}=2i(\gamma^m)^\beta_\alpha\tilde{\partial}_m, \qquad (7)$$

where $\tilde{D}^\beta=\partial/\partial\bar{\theta}_\beta+i(\gamma^m\theta)^\beta\tilde{\partial}_m$.

After an appropriate modification, action (2) becomes

$$S=\int d^3x\Bigg\{\phi D_m\bar{D}^m\bar{\phi}+(il\phi f_m\bar{D}^m\bar{\phi}+\text{H.c.})+l^2\phi f_m f^m\bar{\phi}+i\bar{\psi}_\alpha(\gamma^m)^\alpha_\beta D_m\psi^\beta$$
$$-\frac{i}{2}m\bar{\psi}_\alpha\psi^\alpha+l\bar{\psi}_\alpha(\gamma^m)^\alpha_\beta f_m\psi^\beta-\frac{i}{2}\bar{\lambda}_\alpha(\gamma^m)^\alpha_\beta\partial_m\lambda^\beta+\frac{i}{4}k\bar{\lambda}_\alpha\lambda^\alpha$$
$$-\frac{1}{4}f_{mn}f^{mn}+\frac{1}{2}k\varepsilon^{mnk}v_k\partial_m v_n+V(\psi,\phi)\Bigg\}, \tag{8}$$

where $f_m=\varepsilon_{mnk}f^{nk}$, $D_m=\partial_m-iev_m$, and

$$V(\phi,\psi)=|\phi|^2\left(\frac{1}{2}g|\phi|^2+m\right)^2-\frac{1}{4}ig(\psi_\alpha\psi^\alpha\bar{\phi}^2-4\bar{\psi}_\alpha\psi^\alpha\phi\bar{\phi}+\bar{\psi}_\alpha\bar{\psi}^\alpha\phi^2).$$

Spontaneous $U(1)$ symmetry breaking of the superpotential

$$V(\phi,\psi)|_{\min}=V(\mu,0)$$

reduces the following effective action of the vector gauge field:

$$S=\int d^3x\left[-\left(\frac{1}{4}-\frac{l^2}{2}\mu\right)f_{mn}{}^2+\left(2le^2\mu+\frac{1}{2}k\right)\varepsilon^{mnl}v_m\partial_n v_l+\mu e^2 v_m v^m\right]. \tag{9}$$

The resulting conclusions are similar to those of Ref. 3: If the Chern–Simons term is absent in the original action, it arises as a result of spontaneous $U(1)$ symmetry breaking; it has two critical points of the parameter μ, which drastically changes the dynamics of the vector field. One of them is $\mu=1/2l^2$, where the Maxwell kinetic term vanishes. At $\mu>1/2l^2$ this term has an irregular sign and leads to the unitarity breaking in this sector. If the Chern–Simons term exists in the action before spontaneous breaking, it will have a different critical point $\mu=-k/4le^2$, where the CS term vanishes, while the usual action of the massive vector field remains.

Let us now examine the spontaneous supersymmetry breaking in such a theory. It occurs more naturally in the case of $N=2$ SUSY, where it is possible to introduce the "chiral" superfields and to consider the formulation of the Ogievetsky–Sokatchev type with nonconstrained prepotentials. Such a trick is difficult to realize for $N=1$ theories, but, as is shown in Ref. 4, the $N=2$ SUSY action may be achieved in terms of $N=1$ superfields in the form

$$S=S_0+\int d^3x d^2\theta\bar{\theta}\theta\left(\frac{1}{4}kB^2+\xi B\right), \tag{10}$$

where S_0 is the action (1). The presence of the additional field B, which can be introduced manually in the case $N=1$, is a consequence of the hidden $N=2$ SUSY and allows us to bring the Fayet–Iliopoulos term into action.

From the equation of motion over B,

$$\frac{1}{2}kB+\xi=0, \tag{11}$$

we find that B has nonzero vacuum expectation,

$$\langle B\rangle = -2\xi/k, \tag{12}$$

and that it leads to the nonlinear transformation law for the vector field superpartner, which becomes the Goldstone fermion (see, for example, Ref. 5).

It is important to note that in the additional SUSY-breaking part of the action (10) the constant k is contained in the Chern–Simons term, since by manually introducing this part the coefficient in front of B^2 can be chosen arbitrarily. However, choosing the constant k allows us to establish a direct correlation between spontaneous symmetry breaking and the presence of the Chern–Simons term: In the limit $k \to 0$, *the spontaneous breaking of the SUSY phase does not exist.*

This conclusion is true only for a classical description. In the quantum case a possibility of a dynamical SUSY breaking arises because of the quantum fluctuation of the coupling fields over the vacuum state and the compensation of some terms in the effective action as a result of variation of the constants. These and other basic question require a further investigation.

The authors wish to thank D. P. Sorokin for stimulating discussions. One of us (A.N.) thanks N. A. Ukhova and V. P. Berezovoy for support at a critical time. A.N. acknowledges support from International Soros Foundation, Grant RY 9000, and partial support from Ukrainian State Committee on Science and Technology, Grant 2.3/664.

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