$\bar{\lambda}$ from QCD sum rules for heavy quarkonium

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An estimate of the mass difference between a heavy meson and a heavy quark, $\bar{\Lambda}=0.59\pm 0.02$ GeV, is derived in the leading approximation in the reciprocal mass of the heavy quark. This estimate is derived with the help of a specific set of QCD sum rules for heavy quarkonium. © *1994 American Institute of Physics.*

The difference between the masses of a heavy meson $(Q\bar{q})$ and a heavy quark (Q), i.e., $\bar{\Lambda}=m_{(Q\bar{q})}-m_Q$, is an exceedingly important parameter of the effective heavy-quark theory[1] (EHQT). That theory is based on a kinematic expansion in the reciprocal mass of the heavy quark. It plays an important role in describing the QCD dynamics in a study of, for example, semileptonic decays of heavy mesons, $B\to D^{(*)}l\nu$. At present, an estimate of the difference $\bar{\Lambda}$ based on QCD sum rules[2] for heavy mesons yields the value[3,4] $\bar{\Lambda}=0.57\pm 0.07$ GeV.

In this letter we show that, since $\bar{\Lambda}$ determines the threshold for a hadron continuum of the two-current correlation function in an analysis of the QCD sum rules for heavy quarkonium in the leading approximation, one can derive the estimate $\bar{\Lambda}=0.59\pm 0.02$ GeV, with a far better accuracy, by virtue of detailed experimental data on the spectroscopy of charmonium $(\bar{c}c)$ and bottomonium $(\bar{b}b)$.

In the set of sum rules[5] for the vector currents[2] which we use, we introduce the number $n(q^2)$ for nS levels in heavy quarkonium $[n(M_k^2)=k]$. The contribution of resonances can therefore be written in the form

$$\Pi_V^{(\mathrm{res})}(q^2)=\int\frac{ds}{s-q^2}\sum_n f_{Vn}^2M_{Vn}^2\delta(s-M_{Vn}^2),$$

$$=\int\frac{ds}{s-q^2}\,sf_{Vn(s)}^2\frac{dn(s)}{ds}\frac{d}{dn}\sum_k\theta(n-k). \tag{1}$$

Using the expectation value of the derivative of the step function, we find

$$\Pi_V^{(\mathrm{res})}(q^2)=\left\langle\frac{d}{dn}\sum_k\theta(n-k)\right\rangle\int\frac{ds}{s-q^2}\,sf_{Vn(s)}^2\frac{dn(s)}{ds}\,. \tag{2}$$

Assuming

$$\left\langle\frac{d}{dn}\sum_k\theta(n-k)\right\rangle\simeq 1, \tag{3}$$

we find that the contribution of resonances averaged in this manner is

$$\langle \Pi_V^{(\mathrm{res})}(q^2)\rangle \approx \int \frac{ds}{s-q^2}\, s f^2_{Vn(s)} \frac{dn(s)}{ds}\,. \tag{4}$$

With regard to the theoretical part of the QCD sum rules for vector currents, we note that (first) in the leading approximation in the reciprocal mass of the heavy quark we are ignoring the power-law corrections from quark–gluon condensates. They make only small contributions to the lepton constants ($\leqslant 15\%$; Ref. 2). Second, we do incorporate Coulomb-like α_S/v corrections, which are important in heavy quarkonium, with $v\to 0$, in the form of the factor

$$F(v)=\frac{4\pi}{3}\frac{\alpha_S}{v}\frac{1}{1-\exp\left(-\dfrac{4\pi\alpha_S}{3v}\right)}\,. \tag{5}$$

This factor correctly reproduces the $O(\alpha_S/v)$ contribution derived in QCD perturbation theory.[2] Near the threshold for the production of heavy quarks we thus have

$$\mathscr{I}m\,\Pi_{P,V}^{(\mathrm{pert})}(s)\simeq \alpha_S 8\mu^2, \tag{6}$$

where $\mu=m_Q m_{Q'}/(m_Q+m_{Q'})$ and $s\simeq M^2\simeq(m_Q+m_{Q'})^2$. Equating the theoretical part of the sum rules and the average contribution of resonances, and assuming that the contribution of the hadron continuum at $\sqrt{s}\geqslant m_{(Q\bar q)}+m_{(\bar Q' q)}\simeq m_Q+m_{Q'}+2\bar\Lambda$ is equal to the value calculated in QCD perturbation theory, we find

$$\frac{f_n^2}{M_n}=\frac{\alpha_S}{\pi}\frac{dM_n}{dn}\left(\frac{4\mu}{M}\right)^2. \tag{7}$$

As has been mentioned in the phenomenological potential models of heavy quarkonium,[6] the density of states of quarkonium does not depend on the flavors of the quarks,

$$\frac{dn}{dM_n}=\phi(n), \tag{8}$$

since the potential is approximately logarithmic. The quantity $\alpha_S\simeq 0.2$ varies slightly as we go from charmonium $[\alpha_S(\bar c c)\simeq 0.22]$ to bottomonium $[\alpha_S(\bar b b)\simeq 0.18]$, so we have the following scaling relation, within logarithmic corrections and in the leading order in the reciprocal mass of the heavy quark:[5,7]

$$\frac{f_n^2}{M_n}\left(\frac{M}{4\mu}\right)^2=\mathrm{const.} \tag{9}$$

In the case of quarkonium with a hidden flavor, $4\mu/M=1$, this relation becomes

$$\frac{f^2}{M}=\mathrm{const}, \tag{10}$$

which agrees well with empirical data on the ψ and Υ particles.

An integration by parts of the contribution of the resonances yields[8]

$$\frac{df_n}{f_n dn} = -\frac{1}{2n}, \tag{11}$$

so we have

$$\frac{f_{n_1}^2}{f_{n_2}^2} = \frac{n_2}{n_1}. \tag{12}$$

This result agrees well with the experimental values of the lepton constants of nS states which lie below the level of the hadron continuum.

Expressing f_n in terms of f_1, we find[9]

$$\frac{dM_n}{dn} = \frac{1}{n}\frac{dM_n}{dn}(n=1), \tag{13}$$

so we have

$$\frac{M_n - M_1}{M_2 - M_1} = \frac{\ln n}{\ln 2}, \quad n \geq 2, \tag{14}$$

in agreement with spectroscopy. We also find

$$M_2 - M_1 = \frac{dM_n}{dn}(n=1)\ln 2. \tag{15}$$

Evaluating the integrals of the sum rules for $q^2=0$, we find the following expression in the leading approximation:[10]

$$\frac{dM_n}{dn}(n=1) \simeq \frac{2\bar{\Lambda}}{\ln n_{\mathrm{th}}}, \tag{16}$$

where n_{th} is the number of nS levels below the continuum threshold. This number depends slightly on the flavors of the quarks.

The $1/m_Q$ expansion should be more effective for the b quarks. Assuming $n_{\mathrm{th}}=4$, as in the $(\bar{b}b)$ system, we thus find

$$M(2S) - M(1S) = \bar{\Lambda}. \tag{17}$$

Corresponding sum rules for pseudoscalar currents lead to the same results. Accordingly, ignoring the spin-dependent splitting, and using spectroscopic data on the ψ and Υ particles, we find

$$\bar{\Lambda} = 0.59 \pm 0.02 \ \text{GeV}. \tag{18}$$

The sum rules thus make it possible to establish several scaling relations and the fact that the density of the S-wave levels of quarkonium is independent of the flavors of the heavy quarks. On this basis we can estimate the parameter $\bar{\Lambda}$ within an error close to the uncertainty stemming from the role played by both nonperturbative power-law and logarithmic corrections.[3]

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[1]M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 292 (1987); **47**, 511 (1988); N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); B **237**, 527 (1990).
[2]M. A. Shifman *et al.*, Nucl. Phys. B **147**, 385 (1979); L. J. Reinders *et al.*, Phys. Rep. **127**, 1 (1985).
[3]I. I. Bigi *et al.*, Preprint CERN-TH.7171/94 (1994).
[4]M. Neubert, Phys. Rev. D **46**, 1076, 2212 (1992); A. F. Falk *et al.*, Nucl. Phys. B **388**, 363 (1992).
[5]V. V. Kiselev, Nucl. Phys. B **406**, 340 (1993).
[6]E. Eichten *et al.*, Phys. Rev. D **21**, 203 (1980); J. L. Richardson, Phys. Lett. B **82**, 272 (1979); W. B uchmüller and S.-H. Tye, Phys. Rev. D **24**, 132 (1981); C. Quigg and J. L. Rosner, Phys. Lett. B **71**, 153 (1977); A. Martin, Phys. Lett. B **93**, 338 (1980).
[7]V. V. Kiselev, Preprint IHEP 94–63, Protvino (1994).
[8]V. V. Kiselev, Preprint IHEP 94–71, Protvino (1994).
[9]V. V. Kiselev, Preprint IHEP 94–74, Protvino (1994).
[10]V. V. Kiselev, Preprint IHEP 94–75, Protvino (1994).

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