Cyclotron superradiance of a moving electron swarm under group synchronization conditions

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In the group synchronization regime, which is realized when the translational velocity of a swarm of electron oscillators is equal to the group velocity of the wave, it becomes possible to increase the growth rate of the superradiance instability and to increase the peak emission power. © *1994 American Institute of Physics.*

1. Superradiance in swarms of classical electron oscillators has recently attracted considerable attention. This process can be utilized to generate intense, ultrashort electromagnetic pulses.[1–7] In the present letter we analyze cyclotron superradiance under group synchronization conditions, such that the translational velocity of the electron swarm (or slab) is equal to the group velocity of an electromagnetic wave:

$$V_{\parallel}=V_{\rm gr}. \tag{1}$$

A situation of this sort is realized, for example, during waveguide propagation of electromagnetic radiation when the dispersion curves of the wave, $h=c^{-1}\sqrt{\omega^2-\omega_c^2}$, and of the electron flux, $\omega-hV_{\parallel}=\omega_H$, are tangent (Fig. 1a).[1)] In this case the cutoff frequency ω_c and the relativistic gyrofrequency $\omega_H=eH_0/mc\gamma$ satisfy

$$\omega_H=\omega_c\gamma_{\parallel}^{-1},$$

where $\gamma_{\parallel}=(1-V_{\parallel}^2/c^2)^{-1/2}$, $\gamma=(1-V_{\parallel}^2/c^2-V_{\perp}^2/c^2)^{-1/2}$, and $V_{\perp}=\beta_{\perp}c$ is the electron revolution velocity. The frequency of the radiation is therefore

$$\omega=\gamma_{\parallel}^2\omega_H. \tag{2}$$

For ultrarelativistic electrons this frequency may be substantially higher than the electron oscillation frequency. On the other hand, group synchronization regime (1) is favorable from the standpoint of increasing the growth rates of the superradiance instability, and it is extremely promising for first experimental observations of this process, as we will show below.

2. In the analysis below we work in a comoving frame of reference K', which is moving at the translational velocity of the electron swarm. According to Eq. (1), that velocity is equal to the group velocity. Using Lorentz transformations, we easily find that the longitudinal wave number h' and the transverse component of the magnetic field, $H'_{\perp}$, in the K' frame tend toward zero, and this process reduces to the radiation of a fixed ensemble of cyclotron oscillators at a quasicritical frequency (Fig. 1b).

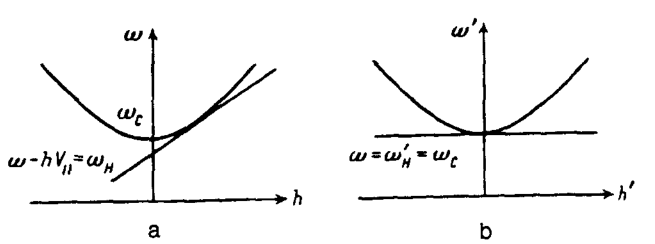

FIG. 1. Dispersion diagram of the group synchronization regime in the lab frame of reference (a) and in the comoving frame (b).

Under the assumption that the transverse structure of the radiation is the same as that of the $\mathbf{E}'_\perp(\mathbf{r}_\perp)$ waveguide mode, we write the electric field as

$$\mathbf{E}'=\mathrm{Re}[\mathbf{E}_\perp(\mathbf{r}_\perp)A'(z',t')\exp(i\omega_c t')], \tag{3}$$

where the evolution of the longitudinal field distribution $A'(z',t')$ is described, in accordance with the dispersion relation, by the inhomogeneous parabolic equation

$$i\frac{\partial^2 a}{\partial Z^2}+\frac{\partial a}{\partial \tau}=2if(Z)G\langle\hat{\beta}_+\rangle_{\Theta_0}. \tag{4}$$

The azimuthal self-phasing of the electrons, which arises in the course of the superradiance, is due to the dependence of the gyrofrequency on the energy of the particles. It is described by the equations of a nonisochronous oscillator:

$$\frac{\partial\hat{\beta}_+}{\partial\tau}+i\hat{\beta}_+(|\hat{\beta}_+|^2-\Delta-1)=ia. \tag{5}$$

Under the assumption that in the initial state the electrons are distributed uniformly in cyclotron-revolution phase, aside from fluctuations stemming from the small parameter r, we can write the initial conditions on system (4), (5) as follows:

$$\hat{\beta}_+|_{\tau=0}=\exp[i(\Theta_0+r\cos\Theta_0)],\quad \Theta_0\in[0,2\pi],\quad a|_{\tau=0}=0.$$

Here we are using dimensionless variables: $\hat{\beta}_+=(\beta'_x+i\beta'_y)/\beta'_{\perp 0}$ is the normalized transverse electron velocity;

$$a=(2eA'/mc\omega_c\beta'^3_{\perp 0})J_{m-1}(R_0\omega_c/c);\quad Z=z'\beta'_{\perp 0}\omega_c/c;\quad \tau=t'\beta'^2_{\perp 0}\omega_c/2;$$

$\Delta=2(\omega'_H-\omega_c)/\omega_c\beta'^2_{\perp 0}$ is the detuning of the unperturbed cyclotron frequency from the cutoff frequency (in the comoving frame);

$$G=\frac{1}{2\pi}\frac{eI_0}{mc^3}\frac{1}{\beta^4_{\perp 0}\beta_{\parallel 0}\gamma^3_\parallel}\frac{\lambda^2}{\pi R^2}\frac{J^2_{m-1}(R_0\omega_c/c)}{J^2_m(\nu_n)(1-m^2/\nu^2_n)}$$

is a form factor written under the assumption that the electron swarm is tubular with an injection radius R_0; I_0 is the total current in the laboratory frame; $\lambda=2\pi c/\omega_c=2\pi R/\nu_n$; R is the radius of the waveguide; m is the azimuthal index of the waveguide mode; and ν_n is the nth root of the equation $J_m(\nu)=0$. The function $f(Z)$ describes the distribution of the electron density along the longitudinal coordinate.

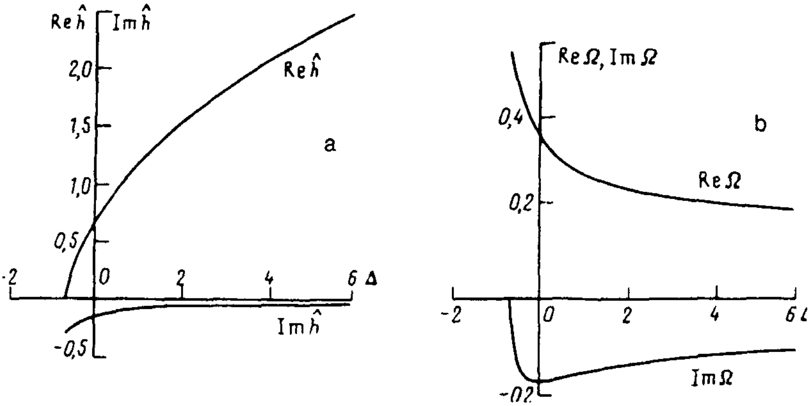

FIG. 2. Real part and the imaginary part of the longitudinal wave number (part a) and the growth rate and the electron frequency shift (b) versus the detuning parameter Δ. $\hat{G}=0.005$.

Below we consider the case in which the electron swarm is relatively short, and the condition

$$b'^2/c\lambda T' \ll 1 \tag{6}$$

holds; here $b'=b\gamma_{\parallel}$ is the length of the swarm in the comoving frame, and T' is a time scale of the onset of superradiance [the reciprocal growth rate; see (8)]. Under condition (6) we can set $f(Z)=B\delta(Z)$, where $B=\beta'_{\perp 0}b'\omega_c/c$, and $\delta(Z)$ is the Dirac δ-function.

To describe the initial linear stage of the superradiance, we write the radiated field in the form $a(Z,\tau)=a(0)\exp(-i\hat{h}|Z|+i\Delta\tau+i\Omega\tau)$ and linearize system of equations (4). We find the characteristic equation ($\hat{G}=GB$)

$$i\Omega^2\sqrt{\Omega+\Delta}+2\hat{G}\Omega=2\hat{G}, \tag{7}$$

which determines complex eigenfrequencies of the oscillations of the slab. At a sufficiently small particle density, $\hat{G}\ll 1$, in the tangency regime, $\Delta=0$, the growth rate is $\mathrm{Im}\Omega=(2\hat{G})^{2/5}\sin(\pi/5)$. Correspondingly, the growth rate of the superradiance instability is given in dimensional variables by

$$|\mathrm{Im}\omega'|=2^{-3/5}\omega_c\left(\sin\frac{\pi}{5}\right)\left(\frac{eI_0}{mc^3}\frac{\beta_{\perp 0}^2}{\beta_{\parallel 0}}\frac{\lambda b}{\pi R^2}\frac{J_{m-1}^2(R_0\omega_c/c)}{J_m^2(\nu_n)(1-m^2/\nu_n^2)}\right)^{2/5}. \tag{8}$$

Note that the superradiance instability does not involve a threshold. This is a consequence of the infinite lifetime of the electron oscillators in the region of the interaction with the electromagnetic field. There are fluxes of electromagnetic energy in both directions away from the electron slab, $\mathrm{Re}\hat{h}>0$, since (because of the electron shift $\mathrm{Re}\Omega>0$) the radiation frequency is above the critical frequency.

Figure 2 shows the normalized growth rate $\mathrm{Im}\Omega$, the electron frequency shift $\mathrm{Re}\Omega$, and the real ($\mathrm{Re}\hat{h}$) and imaginary ($\mathrm{Im}\hat{h}$) parts of the longitudinal wave number $\hat{h}=(\Omega+\Delta)^{1/2}$ versus the parameter Δ. We see that a detuning from the tangency regime

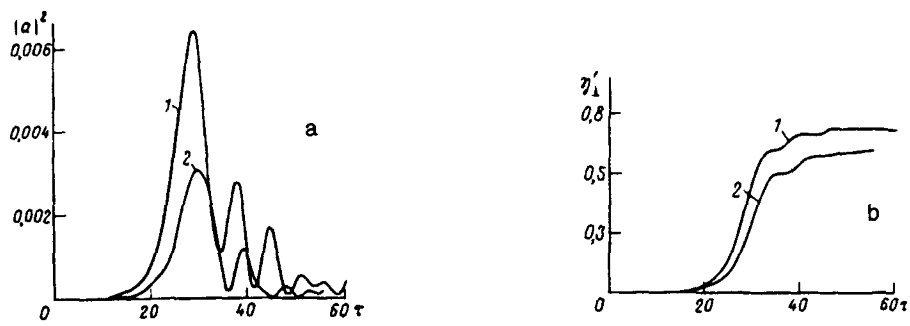

FIG. 3. Time evolution of (a) the square modulus of the amplitude and (b) the electron efficiency. 1—Group synchronization regime ($\Delta=0$); 2—deviation from the group synchronization regime ($\Delta=1$). $\hat{G}=0.005$.

leads to a decrease in the growth rates. While the instability is cut off at a large negative detuning ($\omega_H'<\omega_c$), the instability persists at arbitrarily large values of Δ in the region of positive detuings ($\omega_H'>\omega_c$) . Under the condition $\Delta\gg1$, the following asymptotic expressions hold:

$$\Omega=\hat{G}^{1/2}\Delta^{-1/4}(1-i),\quad \hat{h}=\Delta^{1/2}+\hat{G}^{1/2}\Delta^{-3/4}(1-i)/2. \tag{9}$$

These expressions correspond to a transition to a region of an intersection of the dispersion characteristics, in which there is no group synchronization.[5]

Figure 3 shows results of a numerical simulation of the nonlinear stage of the superradiance according to Eqs. (4). Shown here are plots of the time dependence of the square amplitude $|a|^2$ and the electron efficiency

$$\eta_\perp'=1-\frac{1}{2\pi}\int_0^{2\pi}|\beta_\perp'|^2 d\Theta_0.$$

We see that the bulk of the transverse oscillation energy of the electrons is transformed into the energy of electromagnetic radiation over a time on the order of a few times the reciprocal growth rate. The maximum field amplitude is reached in the case of exact group synchronization.

3. Let us look at the basic characteristics of the superradiance in the lab frame of reference. In the group synchronization regime, the frequencies of the radiation in the positive and negative directions along the z axis are approximately equal (within some small corrections which arise from the electron restructuring of the frequency. As a result, we have $\mathrm{Re}h'\neq0$ in the comoving frame). These frequencies are given by (2). The peak power of the superradiance passing through a fixed area outside the layer is given by

$$P=\frac{\pi}{8}|a|^2\left[\frac{m^2c^5}{e^2}\right]\beta_{\|0}\beta_{\perp0}^6\gamma_\|^6\frac{\pi R^2}{\lambda^2}\frac{J_m^2(\nu_n)(1-m^2/\nu_n^2)}{J_{m-1}^2(R_0\omega_c/c)}\,. \tag{10}$$

The change in the electron phase in the lab frame, averaged over phase, can be found from the efficiency in the comoving frame with the help of the relation

$$\langle 1-\gamma^2/\gamma_0^2\rangle_{\Theta_0}=\gamma_{\|0}^2\beta_{\perp 0}^2\eta_\perp' . \tag{11}$$

A high energy-transfer efficiency can thus be achieved in the lab frame at relatively small transverse velocities, $\beta_\perp \sim \gamma_{\|0}^{-1}$, because the energy of both the transverse and longitudinal motions of the electrons is converted into the energy of electromagnetic oscillations in this frame.

We conclude with an estimate of the pulse length and the power of the cyclotron superradiance under group synchronization conditions. We assume that the strength of the guiding magnetic field in the lab frame is $H_0=10.7$ kOe, the wavelength is $\lambda=1$ mm, the total current is $I_0=100$ A, the electron energy is 1 MeV ($\gamma=3$), the revolution frequency is $V_\perp=0.2$ s, and the length of the swarm is $b=3$ cm. At a waveguide radius $R=4$ mm and for the TE_{13} working mode, the form factor is $\hat{G}=0.05$. From Fig. 3 we then find $|a|^2_{\max}=6.4\times 10^{-3}$; this figure corresponds to a peak radiation power on the order of 50 MW. The length of the pulse at the level of e^{-1} of the maximum amplitude is on the order of ten rf oscillations.

[1)]The group synchronization regime can evidently also be realized during the motion of a swarm of electron oscillators in a plasma.

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