Low-temperature kinetics of normal systems with a fermion condensate: use in describing the normal phase of high-T_c superconductors

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The existence of an acoustic mode in electron systems with a fermion condensate is predicted. The resistance and thermal emf of such systems at temperatures $T \geqslant T_c$ are discussed. The behavior of these coefficients is quite different from that predicted by ordinary Fermi-liquid theory. © *1994 American Institute of Physics.*

The strongly correlated Fermi liquid is currently the subject of extremely active research in many-body theory. It was recently found[1–4] that at $T=0$ there can be a phase transition involving a restructuring of function $n_F(p)$, the momentum distribution of the quasiparticle. According to Landau's theory,[5] before the point of the transition this distribution function remains the same as in a Fermi gas: $n_F(p)=\theta(p_F-p)$, where p_F is the Fermi momentum, and $\rho=p_F^3/3\pi^2$. The transition occurs when the effective repulsion in the particle–hole channel reaches a threshold above which the necessary condition for stability is violated, and an energy benefit is realized upon a certain change in $n_F(p)$. The minimum of the energy functional $E_0[n(\mathbf{p})]$ is thus displaced inward from a corner point of the $[n]$ functional space. It has been suggested[1] that new stationary points of this functional at $T=0$ be sought by working from the equation for the minimum, which is as follows for an isotropic system:

$$\frac{\delta E_0[n(p)]}{\delta n(p)}=\mu, \quad p_i \leqslant p \leqslant p_f, \tag{1}$$

where μ is the chemical potential of the system. Outside this region, whose boundaries are determined from the equation itself and arise because $n(\mathbf{p})$ must be positive and must not exceed one anywhere, the old distribution n_F and the new one n_0 are the same. That this restructuring is a phase transition was demonstrated by a different approach by Volovik,[6] on the basis of a calculation of the topological charge of the Fermi system which he constructed before and after the point of the transition.

According to Ref. 5, the left side of (1) is the energy of a quasiparticle. If a solution of (1) exists, it must therefore describe a phase transition, in the course of which the quasiparticle system (according to the Landau–Luttinger theorem, the number of quasiparticles is equal to the number of particles[7]) decays into two subsystems. One of the two

has no special properties, while the other contains quasiparticles of the same energy $\varepsilon(p)=\mu$. Their group velocity is $d\varepsilon(p)/dp=0$. As a result, a peak $\sim\rho_c\delta(\varepsilon-\mu)$ arises in the density of states of the system, $\rho(\varepsilon)$. This peak is analogous to that which has been observed in liquid He^4 below the λ-point, which stems from a condensation of bosons with a zero momentum and an energy $\varepsilon(p=0)=\mu$. On the basis of this analogy, this subsystem was called a "fermion condensate" in Ref. 1.

The wave function corresponding to distribution (1) is multiply degenerate. This degeneracy is lifted when many-particle correlations, primarily pairing, are taken into account. As usual, a gap Δ forms in the spectrum of one-particle excitations, but this gap, like the superfluid transition temperature T_c, does not contain the standard BCS small factor $[\Delta_{BCS}\sim\exp(-1/\lambda)]$, and it is a linear function of the pairing constant λ (Ref. 1). The theoretical ratio $\eta_{theor}=2\Delta(T=0)/T_c$ is considerably larger than $\eta_{BCS}=3.52$ (Refs. 2–4). A similar difference is observed experimentally for high-T_c superconductors: η_{exp} reaches values of 6–8 (Refs. 8 and 9). Furthermore, the experimental one-particle spectrum of one of these superconductors, $Ba_2Sr_2CaCu_2O_{8+\delta}$ with $T_c=85$ K, as constructed in Ref. 10 on the basis of measurements of photoemission electron spectra, has a plateau which occupies up to 20% of the Brillouin zone. It lies (within the experimental errors) directly on the Fermi surface. Similar results were found in Ref. 11 for $YBa_2Cu_3O_{6.9}$ and $YBa_2Cu_4O_8$. If their superconducting properties are indeed determined by a fermion condensate, as suggested in Refs. 3 and 4, then the effect of this condensate should of course also be manifested at $T>T_c$, until temperature effects erase traces of the plateau in the $\varepsilon(p)$ spectrum.

In this letter we examine the low-temperature kinetics of an electron system with a fermion condensate. We find that at $T\geqslant T_c$ it is totally unlike an ordinary one-component Fermi liquid, being more reminiscent of an electron–ion plasma, except that the density of the heavy component—the condensate—is not imposed from the outside but is itself found from Eq. (1).

Let us find the spectrum of collective excitations in the simple model of a homogeneous and isotropic system in which the condensate is in the region $p_i<p<p_f$, outside the old Fermi surface, where the quasiparticles have a finite effective mass M^*. A similar model of a doubly connected Fermi surface [although with an ordinary Fermi filling, $n(p)=1$] was analyzed in Ref. 12 in order to determine the conditions under which an anomalous state of this sort can provide an energy benefit to the system. To calculate the spectrum of collective oscillations we use the Landau kinetic equation in the form

$$(\omega-\mathbf{k}\cdot\mathbf{v})\phi(\mathbf{p},\mathbf{k})-\mathbf{k}\cdot\frac{\partial n_0(p,T)}{\partial\mathbf{p}}\int F[\mathbf{p},\mathbf{p}_1,\varepsilon(p)=\mu,\varepsilon(p_1)=\mu]\phi(\mathbf{p}_1,\mathbf{k})d\tau_1=0. \quad (2)$$

Here $n_0(p,T)$ is a new quasiparticle distribution determined by the solution of (1) at $T\neq 0$, and $F(\mathbf{p},\mathbf{p}_1)$ is the amplitude of the interaction of the quasiparticles near the new, non-singly-connected Fermi surface. Expanding the solution $\phi(\mathbf{p},\mathbf{k})$ in spherical harmonics, we retain only the zeroth and the first harmonics, as in the ordinary hydrodynamic approximation:

$$\phi(\mathbf{p},\mathbf{k})=a(p)+b(p)\mathbf{p}\cdot\mathbf{k}/pk. \quad (3)$$

The reason for this procedure is that collisions suppress the other harmonics; these too survive because the number of particles and their resultant momentum are conserved.[13,14] The coefficients a and b are now nonzero not only near the point $p=p_F$ but also on the interval $p_i<p<p_f$, where they are constant, with values a_c and b_c, respectively. We can thus equate the diagonal part of the collision integral to zero, but the off-diagonal part, which is responsible for the scattering of normal excitations with $p\simeq p_F$ by condensate excitations, remains nonzero. We will study its effects separately. At this point, we ignore that part, as a small correction and rewrite the Landau equation for the coefficients a_k and b_k as follows:

$$
\begin{aligned}
&-\omega a_F+kv_F(1+f_1/3)b_F/3+kv_F\nu_1 f_1 b_c/9=0,\\
&-\omega a_c+kv_F(s_c+\nu_2\nu_1 f_1/3)b_c/3+kv_F\nu_2 f_1 b_F/9=0,\\
&-\omega b_F+kv_F(1+f_0)a_F+kv_F\nu_1 f_0 a_c=0,\\
&-\omega b_c+kv_F(s_c+\nu_1\nu_2 c f_0)a_c+kv_F\nu_2 f_0 a_F=0.
\end{aligned}
\tag{4}
$$

Here we have introduced $f_k=F_k p_F M^*/\pi^2$, $s_c=v_c/v_F$, $v_F=p_F/M^*$, and $\nu_1(T)=[p_f^3(T)-p_i^3(T)]/3p_F^3$ we have also introduced

$$
\nu_2(T)=-\frac{1}{p_F^2}\int_{p_i(T)}^{p_f(T)}\frac{\partial n_0(p,T)}{\partial p}p^2dp,\quad v_c(T)=-\frac{1}{p_F^3}\int_{p_i(T)}^{p_f(T)}\frac{\partial\varepsilon(p,T)}{\partial p}p^2dp. \tag{5}
$$

The numbers $\nu_1\sim\nu_2$ are $\sim n_c/\rho$, where n_c is the condensate at the optimum doping level,[10] $\nu_1\sim\nu_2\leqslant 0.2$.

Solving system (4), and carrying out some simple but lengthy calculations, we find a dispersion relation

$$
\omega^4-A\omega^2k^2+Bk^4=0, \tag{6}
$$

where

$$
\begin{aligned}
A&=v_F^2[(1+f_1/3)(1+f_0)/3+(s_c+\nu_1\nu_2 f_1/3)(s_c+\nu_1\nu_2 f_0)/3+2\nu_1\nu_2 f_0 f_1/9],\\
B&=v_F^4[s_c(1+f_1/3)+f_1\nu_1\nu_2][s_c(1+f_0)+f_0\nu_1\nu_2]/27.
\end{aligned}
\tag{7}
$$

Substituting in the Coulomb interaction $f_0=4\pi e^2 p_F M^*/\pi^2k^2$, and retaining only the leading terms in the limit $k\to 0$, we find two roots, one of which corresponds to ordinary plasma waves, while the second has a dispersion relation $\omega_s=c_s k$. Ignoring small corrections which stem from terms with v_c, we find

$$
c_s^2=v_F^2 f_1^0\nu_1^2\nu_2^2/9, \tag{8}
$$

where $f_1^0=f_1M/M^*$. The velocity c_s is quite low: For the experimental values[10] $\nu_1\sim\nu_2\sim 0.2$, the ratio of the corresponding Debye temperature T_D to the Fermi energy does not exceed 0.01, even if we ignore the increase in the effective mass M^* due to correlations (i.e., $T_d\sim 100$–200 K). The spectrum of plasmons in a 3D isotropic system with a fermion condensate thus has an acoustic branch. In a 2D system, the plasmons may be gapless even without a fermion condensate.[15,16]

Armed with the acoustic spectrum of excitations, we can calculate the corresponding component of the resistance $\rho(T)$. Using the τ approximation, and ignoring the conden-

sate contribution, because of its low mobility, we find $\rho(T) \sim M^*/ne^2\tau(T)$, where n is the density of charged particles at the old Fermi surface. We find the T dependence of τ by the same methods as in the case of phonons.[13,14] We find $\tau \sim 1/T$ and thus

$$\rho(T) \sim \frac{M^*T}{ne^2}. \tag{9}$$

If we make the simple assumption $n \sim x$, where x is the Sr content in the alloy $La_{2-x}Sr_xCuO_4$ we find that the dependence $\rho(x) \sim 1/x$, which follows from (9), agrees with experimental data[17] down to fairly small values of x. For a carrier density $n \sim 10^{21}$ cm^{-3} and a temperature $T \sim 200$ K we find $\rho \sim 0.2$ m$\Omega \cdot$cm, which is consistent with experiment in order of magnitude.

With decreasing x, the Coulomb parameter $\alpha = e^2/v_F \sim e^2/x^{1/3}$ increases; the region occupied by the condensate expands, and it ultimately absorbs the old Fermi surface. Almost no free carriers with a normal effective mass remain, and the conductivity of the system, now determined by the condensate mobility, falls off sharply, tending toward zero with decreasing T. At large values $x > x_c$, on the other hand, at which α is small and there is no fermion condensate, the resistance is proportional to T^2 at low T, in agreement with Landau's theory. In the boundary region in which there is not yet any condensate, but a "roton" minimum appears in the $\varepsilon(p)$ spectrum, preceding the condensation, an intensification arises in the ordinary electron–electron collision integral because of the small factor $d\varepsilon(p)/dp$, which is attributable to a scattering of pairs from the old Fermi surface to the vicinity of the new minimum.

To determine the exact transition law and compare it with experiment will require a more accurate consideration of the crystal structure, which we have ignored up to this point. There are points in the electron band spectrum—van Hove singularities—at which, at a certain occupation of the band, certain components of the quasiparticle group velocity $d\varepsilon(\mathbf{p})/d\mathbf{p}$ are anomalously small at the Fermi surface. The importance of these points for the theory of high-T_c superconductivity has been demonstrated repeatedly (see, for example, Refs. 11, 18, and 19), but so far the associated violation of the necessary conditions for stability and for the onset of a fermion condensate have been studied only in Refs. 4 and 20, in the schematic model of Nozieres,[2] which was intended for homogeneous systems.

In a more realistic approach to the problem, the quasiparticle group velocity near the Fermi surface can be calculated from the Landau–Pitaevskiĭ formula:

$$\frac{\partial\varepsilon(\mathbf{p},x)}{\partial\mathbf{p}} = \frac{\partial\varepsilon^0(\mathbf{p},x)}{\partial\mathbf{p}} + \int F_1(p,p_1,x)\frac{\partial n(p_1,x)}{\partial\mathbf{p}_1}\, d^l p_1/(2\pi)^l. \tag{10}$$

The integral here includes a summation over the band index; l is the dimensionality of the space. Expanding the amplitude (F) of the quasiparticle interaction at the Fermi surface in spherical harmonics, we find that odd Landau harmonics contribute to this equation in an anisotropic system. In the coordinate representation, the diagrams of the amplitude F are local, since they do not contain particle–hole pole contributions. One consequence is that the Landau harmonics fall off rapidly with increasing index. Second, the effect of the crystal field of the lattice on their magnitude is slight. In a rough approximation, we can

restrict this equation to the first harmonic, F_1, and we can assume that its density dependence is the same as that of the effective mass M^*. In a homogeneous system, this first harmonic is related unambiguously to this effective mass by a formula from Fermi-liquid theory: $M^*=M(1-1/3f_1^0)$, where $f_1^0=F_1 p_F M/\pi^2$. We are interested in only the normal component of the velocity, v_n, since the tangential component vanishes at the Fermi surface. It vanishes because the energy of a quasiparticle is constant, equal to μ, in this case. When the system is stable, the velocity v_n is positive, telling us that the energy of any quasiparticle which is outside the Fermi surface is greater than the energy of the quasiparticles inside it. We restrict the discussion to the case $l=2$, in which the band spectrum $\varepsilon^0(\mathbf{p})$ is described approximately by[21]

$$\varepsilon^0(\mathbf{p},x)=\beta(x)-\gamma(x)(\cos p_x+\cos p_y) \tag{11}$$

(we are assuming the lattice constant a to be one). When the occupation begins, this spectrum is quadratic in the momentum, and the Fermi surfaces are circles. The lattice has only a slight effect in this case. Closer to the middle of the band, the $\varepsilon^0(\mathbf{p},x)$ spectrum depends strongly on x. At half-filling, it has saddle points $(\pm\pi,0)$ and $(0,\pm\pi)$, at which the velocity v_n vanishes. Near these saddle points, the magnitude of the velocity is proportional to the distance from the saddle point, so the sign of v_n may change when the second term in (10) is taken into account. Whether this change actually occurs depends on the sign of F_1. For good metals—with dense electron systems—the sign of F_1 is negative: The Coulomb parameter $\alpha=Me^2/\pi p_F$ is small, and the Fock contribution to F_1 is predominant. Noting that the derivative $dn(p)/dp_n$ is negative along the entire Fermi surface in the usual case, we see that the signs of the two terms are the same and that $d\varepsilon(\mathbf{p})/d\mathbf{p}$ is positive even at a saddle point. The stability condition is not violated here. In sufficiently low-density, strongly correlated electron systems, however, α is greater than one even at the middle of the band, where the electron density is at a maximum. In the jellium model this situation corresponds to $r_s\geqslant 6$. Calculations which have been carried out[22,3] in this model show that the sign of F_1 is positive in such systems, and the ratio M^*/M becomes greater than one. In strongly correlated systems, the signs of the band contribution to v_n and of the contribution from the interaction are different. Near the saddle points, the former vanishes as the middle of the band is approached, while the latter depends weakly on the filling. In an anisotropic system, a new possibility for fermion condensation thus arises: With increasing carrier density, the normal component $d\varepsilon(\mathbf{p},x)/dp_n$ decreases, and at a certain critical filling x_{1c} it undergoes a first change in sign near one of the van Hove points. As a result, the stability conditions are violated, and a fermion condensate arises. While the first harmonic is fairly large, the distance from the point at which the condensation first begins to the saddle point may not be small at all. Since v_n is positive (although small) in the region adjacent to the condensate, we find a sort of expanded saddle point, as is observed experimentally.[11]

What happens as x increases? With increasing x, the region occupied by the condensate grows, and the number of free carriers decreases. The condensate region ultimately covers the entire Fermi surface. Once this occurs, the conductivity is determined exclusively by condensate particles, and the metal becomes a poor conductor, since no normal quasiparticles are left on the Fermi surface. We see that the picture in an anisotropic system in the region in which the van Hove points have an effect is in a sense the opposite of that which we would expect in an isotropic system, in which corresponding

phenomena occur as x decreases, not as it increases. Once the middle of the band has been passed, the first term in (10) begins to increase again, and there is the possibility (among others) that normal quasiparticles appear at the Fermi surface again—the insulator becomes a conductor again. This picture has certain features in common with the Hubbard model, although the initial premises of the two models are completely different.

We conclude with a discussion of how the thermoelectric coefficient α of a normal system with a fermion condensate should behave. In the isotropic case, this coefficient is given by an integral which contains the derivative $d_n(p)/dT$. The explicit expression for this coefficient is[14]

$$\alpha = -\frac{2e}{3T^2\sigma}\int \omega\left(\frac{d\varepsilon(p)}{d\mathbf{p}}\cdot\mathbf{L}\right)\frac{1}{\left[\exp\left(\frac{\omega}{T}\right)+1\right]\left[\exp\left(-\frac{\omega}{T}\right)+1\right]}\frac{d^3p}{(2\pi)^3}, \tag{12}$$

where $\omega=\varepsilon(p)-\mu$. The integrand is an odd function of the energy at the Fermi surface; this circumstance usually leads to an anomalous small factor: $\alpha\sim T/e\varepsilon_F$, where ε_F is the Fermi energy. In systems with a fermion condensate, however, the particle and hole excitations occupy different phase volumes near the Fermi surface. As a result, even if the vector $\mathbf{L}$ is positive, the corresponding integral does not vanish. There is another way to look at this problem: In a system with a fermion condensate there are two different subsystems and two different derivatives, dn_c/dT and dn/dT. Each term contains $d\mu/dT=S$, where S is the "residual entropy." Nozieres[2] has used the ordinary Fermi-liquid formula[2] to calculate this entropy:

$$S(T) = -\int \{n(p,T)\ln[n(p,T)]+[1-n(p,T)]\ln[1-n(p,T)]\}d^3p/(2\pi)^3, \tag{13}$$

where $n(p,T)$ is given by the solution of Eq. (1). Since the quantity $n(p,T=0)$ is now different from both zero and one, we find a finite, nonzero value $S\sim n_c/n$ in the low-temperature limit. At $T=0$, the system is actually in a superconducting state, and its entropy is zero. At $T\sim T_c$, however, the entropy increases rapidly, so that at $T>T_c$ expression (13) gives a value which is correct in order of magnitude (we will discuss how to correct the shortcomings of this formula in a separate paper). If this is so, then by using (13) we find the following expression from (12):

$$\alpha(T\sim T_c)\sim n_c/n. \tag{14}$$

This result—that the thermal emf is independent of T at low temperatures—means that the value of the Lorentz constant $K=\pi^2/3e^2$ changes in the Wiedemann–Franz law; this constant is now dependent on the ratio n_c/n. Recent experimental data[23–25] provide evidence that the thermal emf of high-T_c superconductors depends only weakly on T at low temperatures, $T>T_c$, and that this emf increases toward the middle of the band, i.e., as the condensate density increases. This behavior of the thermal emf can be utilized to estimate the entropy of the system. It indirectly answers the question of just why a quantum chaos arises in these materials at such low temperatures.

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