Precessional, relaxation, and elastic oscillations in a ferromagnet in the region of orientational phase transitions

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The effect of magnetization relaxation on the spectrum of precessional (spin) and elastic oscillations of a ferromagnet in the region of orientational phase transitions is examined theoretically. Near an orientational phase transitions, all oscillation modes (both spin and elastic modes) can become purely relaxation oscillations. A relaxation mode is a soft mode at the point of an orientational phase transition. Consequently, the 100% decrease in the sound velocity predicted by previous theories cannot occur. © *1994 American Institute of Physics.*

Magnetization oscillations in ferromagnets in a magnetically ordered state are spin waves in the dissipationless approximation.[1] Spin waves can also be thought of as a precession of the magnetization around the direction of the effective magnetic field. When dissipation in the magnetic subsystem is taken into account, the spin waves are damped. Since the dissipation is ordinarily slight, spin waves are treated as weakly damped waves. This is the situation if the ferromagnet is far from points of orientational phase transitions, with the real parts of the spin-wave frequencies $\omega_{\rm pr}$ being much larger than the imaginary parts $|\omega_r|$: $\omega_{pr} \gg |\omega_r|$. As is shown below, however, near an orientational phase transition (the point of such a transition is usually determined by the condition $\omega_{pr} \to 0$), the real part of the spin-wave frequency can become smaller than the imaginary part. This situation of course affects the spectrum of oscillations of the ferromagnet.

In this letter we take a theoretical look at how magnetization relaxation in a ferromagnet influences the precessional and elastic oscillations in the region of orientational phase transitions.

As an example we consider a biaxial ferromagnet which is isotropic in terms of elastic and magnetoelastic properties. For this case, the magnetic and magnetoelastic parts of the free-energy density are

$$F = F(M^2) + \frac{1}{2}\,\alpha\left(\frac{\partial M}{\partial x_i}\right)^2 + \frac{1}{2}\,\beta_1 M_x^2 + \frac{1}{2}\,\beta_2 M_y^2 + \frac{1}{2}\,\beta_3 M_z^2 - \mathbf{MH} + \frac{1}{2}\,b M_i M_k U_{ik}, \tag{1}$$

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where α, β_i, and b are the exchange constant, the anisotropy constant, and the magnetostriction constant, respectively; **M** is the magnetization of the ferromagnet; **H** is the external magnetic field, and $\hat{U}$ is the strain tensor.

Without any loss of generality, we will discuss the case $\mathbf{H}\|\mathbf{x}$ and the ground state of the ferromagnet, in which we have $\mathbf{M}\|\mathbf{H}$. This phase is stable under the conditions

$$\beta_2-\beta_1+H/M\geqslant 0, \quad \beta_3-\beta_1+H/M\geqslant 0. \tag{2}$$

In analyzing the dynamics of the magnetic and elastic subsystems, we start from the Landau–Lifshitz equations with a relaxation term in Hilbert form and also the elastic equation.[2,3] Here is the linearized system of equations of motion which determines the dynamics of the transverse components of the magnetization and the displacement vector of the ferromagnet in the phase $\mathbf{M}\|\mathbf{H}$, for magnetoelastic waves propagating along the **x** axis:

$$\omega m_{y,z}\mp(i\omega_{3,2k}+r\omega)m_{z,y}\pm\tfrac{1}{2}gM^2bku_{z,y}=0,$$

$$(\omega^2-\omega_t^2)u_{y,z}+\frac{1}{2\rho}ikbMm_{y,z}=0. \tag{3}$$

Here **m** and **u** are Fourier components of the oscillatory parts of the magnetization and the displacement vector of the ferromagnet, ρ is the density of the ferromagnet, r is a relaxation constant, $\omega_t=s_tk$, $s_t^2=\mu/\rho$, μ is a Lamé coefficient, and g is the gyromagnetic ratio. The characteristic frequencies of the magnetic subsystem are given by

$$\omega_{2,3k}=gM(\alpha k^2+\beta_{2,3}-\beta_1+H/M+h_t), \tag{4}$$

where the dimensionless magnetoelastic field is $h_t=b^2M^2/4\mu$. Note that at points at which the phase $\mathbf{M}\|\mathbf{H}\|\mathbf{x}$ becomes unstable—these points are determined by the signs of the equality in (2) and are points of a second-order orientational phase transition—there is a softening (in the case $h_t=0$) of the frequencies ω_{2k} and ω_{3k} or, in other words, of the precessional mode $\omega_{pr}=(\omega_{2k}\omega_{3k})^{1/2}$ and of one of the relaxation modes (either $\omega_{r1}=-ir\omega_{2k}$ or $\omega_{r2}=-ir\omega_{3k}$).

The dispersion relation for coupled oscillations is

$$\begin{aligned}(1+r^2)\omega^6+ir\omega^5(\omega_{2k}+\omega_{3k})-\omega^4[2\omega_t^2(1+r^2)+\omega_{2k}\omega_{3k}]\\ -2ir\omega^3\omega_t^2(\omega_{2k}+\omega_{3k}-\omega_{\rm me})+\omega^2\omega_t^2[\omega_t^2(1+r^2)+\omega_{2sk}\omega_{3k}\\ +\omega_{3sk}\omega_{2k}]+ir\omega\omega_t^4(\omega_{2sk}+\omega_{3sk})-\omega_t^4\omega_{2sk}\omega_{3sk}=0,\end{aligned} \tag{5}$$

where

$$\omega_{2,3sk}=\omega_{2,3k}-\omega_{\rm me}, \quad \omega_{\rm me}=gMh_t. \tag{6}$$

We first take a more detailed look at the spectrum of spin oscillations of the ferromagnet in the absence of magnetoelastic coupling ($\omega_{\rm me}$, $h_t=0$). In this case the dispersion relation becomes

$$(1+r^2)\omega^2+ir\omega(\omega_{2sk}+\omega_{3sk})-\omega_{2sk}\omega_{3sk}=0. \tag{7}$$

Its solution (under the condition $r\ll 1$) is

$$\omega_{1,2} = -\tfrac{1}{2} ir(\omega_{2sk} + \omega_{3sk}) \pm [\omega_{2sk}\omega_{3sk} - \tfrac{1}{4} r^2(\omega_{2sk} - \omega_{3sk})^2]^{1/2}. \tag{8}$$

We thus see that, far from points of orientational phase transitions, at which we have [according to (2), (4), and (6)] $\omega_{2so}=0$ (the transition $M_x \to M_x, M_y$) or $\omega_{3so}=0$ ($M_x \to M_x, M_z$), with $\omega_{2sk}\omega_{3sk} \gg r^2(\omega_{2sk} - \omega_{3sk})^2$, the total effect of the magnetization relaxation on the precessional oscillations reduces to a damping of the spin waves. This damping is slight. Near points of orientational phase transitions, e.g., at the point $\omega_{2so} \to 0$ (in this case we have $\omega_{2sk} \ll \omega_{3sk}$ if $k \to 0$), in contrast, the situation may change fundamentally. For example, in the case $\omega_{2sk}\omega_{3sk} \ll r^2(\omega_{2sk} - \omega_{3sk})^2$, solution (8) represents purely relaxation oscillations:

$$\omega_1 = -i\omega_{2sk}\omega_{3sk}/r(\omega_{3sk} - \omega_{2sk}), \quad \omega_2 = -ir\omega_{3sk}. \tag{9}$$

These frequencies determine the reciprocal relaxation times of the transverse components of the magnetization of the ferromagnet. The relaxation mode ω_1 is soft: Its frequency tends toward zero at the stability boundary of the phase as $k \to 0$. In the vicinity of orientational phase transitions, $\omega_{3so} \to 0$, the solution is given by (9), on whose right sides the subscripts 3 and 2 must be interchanged.

In the absence of magnetoelastic coupling, far from orientational phase transitions, the magnetization oscillations are thus weakly damped spin waves, while near orientational phase transitions the precessional nature of the motion of the magnetization may change to a purely relaxation motion. In the latter case, a relaxation mode is soft (its frequency is zero precisely at the transition point in the case $k=0$); this mode is involved in the orientational phase transition itself.

We now incorporate a magnetoelastic interaction. For definiteness we examine the spectrum of coupled oscillations in the region of the orientational phase transition $\omega_{2so} \to 0$. We first write a solution of dispersion relation (5) in the case $k=0$:

$$\omega_{1,2} = \pm[\omega_{20}\omega_{30} - \tfrac{1}{4} r^2(\omega_{20} - \omega_{30})^2]^{1/2} - \tfrac{1}{2} ir(\omega_{20} + \omega_{30}), \quad \omega_{3,4,5,6} = 0. \tag{10}$$

At the point of an orientational phase transition ($\omega_{2so}=0$), we find $\omega_{20} = \omega_{\mathrm{me}}$ from (2), (4), and (6). We thus see that when magnetoelastic coupling is taken into account, the solution $\omega_{1,2}$ describes a damped precessional motion of the magnetization, both far from and close to the orientational phase transition, since the condition $\omega_{\mathrm{me}}\omega_{30} > r^2(\omega_{\mathrm{me}} - \omega_{30})^2$ essentially always holds under the condition $r \ll 1$. The other four frequencies may describe both relaxation and elastic oscillations. To determine their nature, we write a solution of dispersion relation (5) for $k \neq 0$ (but $k \to 0$). This solution is

$$\begin{aligned} \omega_{1,2} &= \pm[\omega_{2k}\omega_{3k} - \tfrac{1}{4} r^2(\omega_{2k} - \omega_{3k})^2]^{1/2} - \tfrac{1}{2} ir(\omega_{2k} + \omega_{3k}), \\ \omega_{3,4} &= \pm \omega_t(1 - \omega_{\mathrm{me}}/\omega_{3k})^{1/2} - \tfrac{1}{2} ir\omega_t^2(2\omega_{3k} + \omega_{\mathrm{me}})/\omega_{3k}^2, \\ \omega_{5,6} &= \omega_t\{\pm[4\omega_{2k}\omega_{2sk} - r^2\omega_t^2]^{1/2} - ir\omega_t\}/2\omega_{2k}. \end{aligned} \tag{11}$$

These expressions were derived under the conditions $\omega_t \ll \omega_{2k}$, ω_{3k} and $r \ll 1$. It follows from (11) that the spectrum of coupled oscillations of the ferromagnet near an orientational phase transition with $k \neq 0$ consists of a weakly damped quasispin branch $\omega_{1,2}$, a weakly damped transverse quasielastic branch $\omega_{3,4}$, and a branch $\omega_{5,6}$, whose nature is determined by the relation between the quantities $\omega_{2k}\omega_{2sk}$ and $r^2\omega_t^2$. Under the condition $\omega_{2k}\omega_{2sk} \gg r^2\omega_t^2$, the branch $\omega_{5,6}$ is a weakly damped, transverse, quasielastic oscillation branch with quadratic dispersion [since it follows from (2), (4), and (6) that we have $\omega_{2sk}=gM\alpha k^2$ at the point of the orientational phase transition]:

$$\omega_{5,6}=\pm\omega_t(\omega_{2sk}/\omega_{2k})^{1/2}-\tfrac{1}{2}\, ir\omega_t^2/\omega_{2k}. \tag{12}$$

In the case $\omega_{2k}\omega_{2sk} \ll r^2\omega_t^2$, on the other hand, branches $\omega_{5,6}$ are purely relaxation oscillations (quasimagnetic and quasielastic) with a quadratic dependence on the magnitude of the wave vector:

$$\omega_5=-i\omega_{2sk}/r, \quad \omega_6=-ir\omega_t^2/\omega_{2k}. \tag{13}$$

In the limit $r\to 0$, we should use the exact formula in (11) instead of the first formula for ω_5.

In the limit $k\to 0$, the branch $\omega_{1,2}$ is an activation branch with a gap size which is governed, according to (11), by the magnetoelastic coupling and the magnetization relaxation. The other branches are of a nonactivation nature. One of the quasielastic oscillation branches ($\omega_{3,4}$) has a linear dispersion in the case $k\to 0$ in the vicinity of orientational phase transitions, with a slight dispersion of the propagation velocity [the factor $(1-\omega_{\rm me}/\omega_{3k})^{1/2}$ in (11)]. When magnetization relaxation is taken into account, on the other hand, this elastic branch is damped. The interaction between magnetic and elastic oscillations has the strongest effect on the dispersion relation on the second nonactivation branch of coupled oscillations, $\omega_{5,6}$. This branch may be both quasielastic and quasimagnetic. In both cases the dispersion relation for this branch is quadratic in k. Under the condition $\omega_{2k}\omega_{2sk} \gg r^2\omega_t^2$, the branch $\omega_{5,6}$ is quasielastic. Under the condition $\omega_{2k}\omega_{2sk} \ll r^2\omega_t^2$, the branches $\omega_{5,6}$ describe purely relaxation oscillations—respectively quasispin and quasielastic oscillations. These two modes soften as the orientational phase transition is approached.

Near an orientational phase transition, the condition $\omega_{2k}\omega_{2sk} \ll r^2\omega_t^2$ reduces to a condition imposed on the parameters of the problem. Since we have $\omega_{2k}\simeq\omega_{\rm me}$ and $\omega_{2sk}\simeq gM_0\alpha k^2$ in the limit $k\to 0$ and near an orientational phase transition, this condition can be written as $gM_0\alpha\omega_{\rm me} \ll r^2 s_t^2$ it actually reduces to a condition imposed on a damping parameter. For typical values of the constants for a ferromagnet ($g\simeq 1\times 10^7$ $\mathrm{Oe}^{-1}\cdot \mathrm{s}^{-1}$, $M_0\simeq 1\times 10^2$ Oe, $\alpha\simeq 1\times 10^{-12}$ cm^{-2}, $s_t\simeq 1\times 10^5$ cm/s, $b\simeq 1\times 10^2$, and $\mu\simeq 1\times 10^{12}$ $\mathrm{erg/cm}^3$), we find the following restriction on the damping parameter: $r\gg 10^{-4}$. This condition can definitely hold near specifically an orientational phase transition, since we know that the damping of spin waves strengthens sharply as an orientational phase transition is approached.[4]

Near the orientational phase transition $\omega_{3so}\to 0$, the spectrum is found from (10)–(13) by interchanging the 2 and 3.

In the region of orientational phase transitions, all types of motion (of both the magnetization and the lattice) may thus reduce to purely relaxation oscillations. In this case the transition involves specifically soft relaxation modes. When a magnetoelastic interaction is taken into account, there is always a weakly damped activation quasispin mode in the spectrum of coupled oscillations of the ferromagnet. A conversion of a softening quasielastic oscillation mode near an orientational phase transition into a purely relaxation mode can explain why experiments carried out to measure the sound velocity near an orientational phase transition have yet to observe the theoretically predicted 100% decrease in this velocity precisely at the point of the orientational phase transition or the dispersion of this velocity as the point of the orientational phase transition is approached.[5,6]

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