Universal theory of strings and 2D (super)gravity

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An explicit mapping of two different descriptions of 2D quantum gravity and corresponding symmetry algebras is constructed. These two descriptions are an $sl(2)$-covariant formalism and an $N=2$ supersymmetry description, which generalizes the DDK formalism. It is thus demonstrated that the ideas of universal string theory can be expanded to the gravitational sector. © *1994 American Institute of Physics.*

The David–Distler–Kawai (DDK) recipe[1,2] for the interaction of matter with gravity in a conformal gauge is, as we now know,[3,4] part of a formulation based on topological invariance. Another well-known fundamental property of 2D gravity is the existence of an $sl(2)$-covariant formalism for its description in the chiral gauge.[6–10] It would be interesting to see how two different ("complementary") properties of the same physical theory are related to each other. We will show below how to construct a *mapping* between two formulations of 2D quantum gravity. The DDK formulation turns out to be a "factor-space" $sl(2)$ formulation in terms of a topological conformal theory. A similar relation holds for corresponding supersymmetry expansions. This situation turns out to be analogous to the ideas of universal string theory,[13–20] but applied to the *gravitational* sector of the theory. Furthermore, the constructions below can be interpreted as embeddings of (non)critical string theories in the Wess–Zumino–Witten (WZW) model (see also Ref. 21, where a slightly different technique was developed).

A topological conformal algebra can be realized by dressing ordinary matter with a gravity multiplet, i.e., with ghosts bc and a scalar field ϕ (Liouville).[3,4] There is a version of this construction in which the ghosts are assigned a spin of one,[3] and the topological generators are

$$\mathcal{T}=T-\tfrac{1}{2}(\partial\phi)^2+\frac{\alpha_0}{\sqrt{2}}\partial^2\phi-b^{[1]}\partial c^{[1]},\quad \mathcal{J}=b^{[1]},$$

$$\mathcal{G}=c^{[1]}\left(T-\tfrac{1}{2}(\partial\phi)^2+\frac{\alpha_0}{\sqrt{2}}\partial^2\phi\right)-b^{[1]}\partial c^{[1]}c^{[1]}+\sqrt{2}\alpha_+\partial c^{[1]}\partial\phi$$
$$+\tfrac{1}{2}(1-2\alpha_+^2)\partial^2 c^{[1]},$$

$$\mathcal{H}=-b^{[1]}c^{[1]}-\sqrt{2}\alpha_+\partial\phi, \tag{1}$$

where $\alpha_+ = 1\sqrt{k+2}$, $\alpha_- = -\sqrt{k+2}$, and $\alpha_0=\alpha_+ + \alpha_-$. Adding the additional scalar

v_* to this system, we find a representation of the current algebra $sl(2)_k \oplus u(1)_{BC}$, where $u(1)_{BC}$ actually designates the BC ghost system. The currents, constructed in accordance with

$$J^+=e^{\sqrt{2}\alpha_+(v_*-\phi)},\quad J^0=b^{[1]}c^{[1]}+\sqrt{2}\alpha_+\partial\phi-\left(\frac{\alpha_-}{\sqrt{2}}+\sqrt{2}\alpha_+\right)\partial v_*,$$

$$J^-=\left\{\alpha_-^2\left(T-\tfrac{1}{2}(\partial\phi)^2+\frac{\alpha_0}{\sqrt{2}}\partial^2\phi\right)\right.$$
$$\left.+2b^{[1]}\partial c^{[1]}-\alpha_-^2\partial(b^{[1]}c^{[1]})+\sqrt{2}\alpha_-\partial\phi\times b^{[1]}c^{[1]}\right\}e^{-\sqrt{2}\alpha_+(v_*-\phi)}, \tag{2}$$

generate a Kac–Moody algebra $sl(2)_k$, and the ghosts BC are

$$B=c^{[1]}e^{\sqrt{2}\alpha_+(v_s-\phi)},\quad C=b^{[1]}e^{-\sqrt{2}\alpha_+(v_*-\phi)}. \tag{3}$$

The BRST-invariant prime states of a topological conformal algebra are characterized by a topological charge $U(1)$:

$$\mathscr{H}_0|\mathrm{h}\rangle_{\mathrm{top}}=\mathrm{h}|\mathrm{h}\rangle_{\mathrm{top}},\quad \mathscr{L}_{\geq 0}|\mathrm{h}\rangle_{\mathrm{top}}=\mathscr{H}_{\geq 1}|\mathrm{h}\rangle_{\mathrm{top}}=\mathscr{G}_{\geq 1}|\mathrm{h}\rangle_{\mathrm{top}}=\mathscr{J}_{\geq 0}|\mathrm{h}\rangle_{\mathrm{top}}=0. \tag{4}$$

They are constructed in a specific given representation in the following way:

$$|h(r,s)\rangle_{\mathrm{top}}=|r,s\rangle\otimes|p_M(r,s)\rangle_L\otimes|0\rangle^{[1]}, \tag{5}$$

where $|p_M(r,s)\rangle_L$ is a state in the Liouville theory with a momentum $p_M(r,s)=-(1/\sqrt{2})[\alpha_+(r-1)+\alpha_-(s-1)]$. The allowed $sl(2)_k$-senior weights with spin

$$j(r,s)=\frac{r-1}{2}-(k+2)\frac{s-1}{2} \tag{6}$$

can now be found from the topological prime states through tensor multiplication by a state with a definite momentum in the theory, ∂v_*:

$$|j(r,s)\rangle_{sl(2)}\otimes|0\rangle_{BC}=|r,s\rangle\otimes|p_M(r,s)\rangle_L\otimes|0\rangle^{[1]}\otimes|-p_M(r,s)\rangle_*. \tag{7}$$

In this manner, ordinary matter is incorporated in the WZW theory.

A supersymmetric expansion of this construction is interesting. We describe $N=1$ matter by means of the energy–momentum tensor T_m and its superpartner G_m:

$$T_m(z)T_m(w)=\frac{d/2}{(z-w)^4}+\frac{2T_m(w)}{(z-w)^2}+\frac{\partial T_m(w)}{z-w},$$

$$T_m(z)G_m(w)=\frac{3/2G_m(w)}{(z-w)^2}+\frac{\partial G_m(w)}{z-w},$$

$$G_m(z)G_m(w)=\frac{2d/3}{(z-w)^3}+\frac{2T_m(w)}{z-w}, \tag{8}$$

where $d=\frac{15}{2}-3\alpha_-^2-3\alpha_+^2$ and $\alpha_-=-1/\alpha_+$. The interaction with $N=1$ supergravity in the conformal gauge is described by introducing a super-Liouville field with components ϕ (a scalar) and ψ (a Majorana–Weyl fermion) and fermion and boson ghosts bc and $\beta\gamma$. This theory actually has a (twisted) $N=3$ symmetry.[4] As in the boson case, there is a "dual" version of the theory, in which the ghosts bc have a spin of one, and the ghosts $\beta\gamma$ have a spin of one-half. As before, this theory has an $N=3$ supersymmetry. The energy–momentum tensor in this realization of the $N=3$ superalgebra is

$$\mathcal{T}=T_m-\tfrac{1}{2}\partial\phi\partial\phi+\tfrac{1}{2}(\alpha_+-\alpha_--4x)\partial^2\phi-\tfrac{1}{2}\partial\psi\psi-b\partial c-\tfrac{1}{2}\beta\partial\gamma+\tfrac{1}{2}\partial\beta\gamma, \tag{9}$$

and the corresponding supergenerators are

$$\begin{aligned}&\mathcal{G}^+=b,\quad \mathcal{G}^0=-G_m+b\gamma-\partial c\beta-\psi\partial\phi+(\alpha_+-\alpha_--4x)\partial\psi,\\&\mathcal{G}^-=4cT_m+2\gamma G_m-b\gamma\gamma-4b\partial cc-2c\beta\partial\gamma-2c\partial\phi\partial\phi+2c\partial\beta\gamma-2c\partial\psi\psi\\&\quad+(-2\alpha_-+2\alpha_+-8x)c\partial^2\phi+2\psi\gamma\partial\phi+4x\psi\partial\gamma-8x\partial c\partial\phi+4\partial c\beta\gamma\\&\quad+(2\alpha_--2\alpha_++4x)\partial\psi\gamma-8x^2\partial^2c.\end{aligned} \tag{10}$$

Working exclusively from the condition for closure of the $N=3$ algebra, we conclude that two values of the parameter x are possible: $x=\pm\frac{1}{2}\alpha_\pm$. Starting from this point, we choose the value $x=\frac{1}{2}\alpha_+$ for definiteness. The current algebra $osp(1|2)$ is then constructed as in the boson case, after the theory is multiplied by the one scalar field ∂v_* with the energy–momentum tensor

$$T_*=\tfrac{1}{2}\partial v_*\partial v_*+\frac{\alpha_++\alpha_-}{2}\partial^2 v_*. \tag{11}$$

Specifically, the construction for the $osp(1|2)$ currents is

$$\begin{aligned}&J^{++}=e^{2\alpha_+(\phi-v_*)},\quad J^+=\sqrt{2}\psi e^{\alpha_+(\phi-v_*)},\\&J^0=\tfrac{1}{2}\alpha_-\partial\phi+bc+\tfrac{1}{2}\alpha_-\sqrt{7\alpha_+^2-2}\beta\gamma,\\&J^-=\left(\alpha_-\sqrt{\frac{1+\alpha_+^2}{2}}\psi\partial\Phi+\sqrt{2}bc\psi\mp\frac{\alpha_-}{\sqrt{2}}G_m+\frac{\alpha_-^2-2}{\sqrt{2}}\partial\psi\right)e^{-\alpha_+(\phi-v_*)},\\&J^{--}=\frac{\alpha_-^2}{2}(-\tfrac{1}{2}[1+\alpha_+^2)\partial\Phi\partial\Phi+\tfrac{1}{2}(3\alpha_+-\alpha_-)\sqrt{1+\alpha_+^2}\partial^2\Phi+2\alpha_+\sqrt{1+\alpha_+^2}bc\partial\Phi\\&\quad-(1-3\alpha_+^2)b\partial c-(1+\alpha_+^2)\partial bc+(1+\alpha_+^2)T_{\rm eff}]e^{-2\alpha_+(\phi-v_*)},\end{aligned} \tag{12}$$

where we have used

$$\begin{aligned}&\partial\Phi=\frac{\alpha_+}{\sqrt{1+\alpha_+^2}}[-(2\alpha_-+3\alpha_+)\partial\phi+(\alpha_-+3\alpha_+)\partial v_*-\alpha_-\sqrt{7\alpha_+^2-2}\beta\gamma],\\&T_{\rm eff}=\frac{1}{1+\alpha_+^2}T_m\pm\frac{\alpha_+}{1+\alpha_+^2}G_m\psi+\frac{1-2\alpha_+^2}{2(1+\alpha_+^2)}\partial\psi\psi.\end{aligned} \tag{13}$$

The corresponding fermion ghosts (BC) and boson ghosts ($\beta\gamma$) are given by

$$B=be^{-2\alpha_+(\phi-v_*)},\qquad C=ce^{2\alpha_+(\phi-v_*)},$$
$$\beta=\beta e^{\sqrt{7\alpha_+^2-2}(\phi-v_*)},\quad \gamma=\gamma e^{-\sqrt{7\alpha_+^2-2}(\phi-v_*)}. \tag{14}$$

Treating the ghosts in (14) and the $osp(1|2)$ currents as independent fields, we introduce several energy–momentum tensors for them:

$$T_{\text{ghosts}}=-B\partial C+\tfrac{1}{2}\partial\beta\gamma-\tfrac{1}{2}\beta\partial\gamma,$$
$$T_{\text{Sug}}=2\alpha_+^2(J^0J^0+\tfrac{1}{2}J^{++}J^{--}+\tfrac{1}{2}J^{--}J^{++}+\tfrac{1}{4}J^+J^--\tfrac{1}{4}J^-J^+)+\partial J^0. \tag{15}$$

Through a direct calculation we can then test the "completeness condition"

$$T_{\text{Sug}}+T_{\text{ghosts}}=\mathscr{T}+T_*, \tag{16}$$

which means that descriptions of the system in terms of "composite" fields $(B,C,\beta,\gamma,J^{++},J^+,J^0,J^-,J^{--})$ and in terms of "elementary" fields $(T_m,G_m,b,c,\beta,\gamma,\partial\phi,\psi,\partial v_*)$ are equivalent. The $N=1$ matter, dressed with supergravity and one more boson current, is thus incorporated in the $osp(1|2)$ current algebra.

We can demonstrate certain applications of these constructions [(2) and (12)] to 2D gravity and supergravity (see also Ref. 5). We begin with supergravity. We recall that it allows an $osp(1|2)$ description.[11,12] The matter sector contains some $N=1$ theory with generators T_m' and G_m' which satisfy relations (8) with d replaced by $d'=\frac{15}{2}+3\alpha_-^2+3\alpha_+^2$. When supergravity is incorporated, $osp(1|2)$ currents arise, as does a set of boson and fermion ghosts: $b^{[2]}c^{[2]}$ (reparametrized ghosts), $\beta^{[\frac{3}{2}]}\gamma^{[\frac{3}{2}]}$ (their superpartners), $b^{[1]}c^{[1]}$ (spin-1 ghosts corresponding to the current gauge J^{++}), and $\beta^{[\frac{1}{2}]}\gamma^{[\frac{1}{2}]}$ (their spin-$\frac{1}{2}$ superpartners). We also need[11] one more Majorana–Weyl fermion χ with an energy–momentum tensor $T_\chi=\frac{1}{2}\partial\chi\chi$. The total central charge is $d'+c_{\text{Sug}}-26+11-2-1+\frac{1}{2}=0$ [where $c_{\text{Sug}}=10-3\alpha_+^2-\alpha_-^2$ is the central charge of Sugawara tensor (15)].

Now combining the $osp(1|2)$ currents with the ghosts $b^{[1]}c^{[1]}$ and $\beta^{[\frac{1}{2}]}\gamma^{[\frac{1}{2}]}$ (and identifying the latter with BC and $\beta\gamma$, respectively), we can express them in terms of $N=3$ ingredients and v_* matter by means of representations (12) and (14). As a result, we find the following set of fields with central charges:

matter′	χ	matter	bc	$\beta\gamma$
$\frac{15}{2}+3\alpha_+^2+3\alpha_-^2$	$\frac{1}{2}$	$\frac{15}{2}-3\alpha_+^2-3\alpha_-^2$	-2	-1
$\partial\phi$	ψ	∂v_*	$b^{[2]}c^{[2]}$	$\beta^{[\frac{3}{2}]}\gamma^{[\frac{3}{2}]}$
$-5+3\alpha_+^2+3\alpha_-^2$	$\frac{1}{2}$	$7-3\alpha_+^2-3\alpha_-^2$	-26	11

(17)

Here we have $d'+d=15$, so matter′ + matter + $b^{[2]}c^{[2]}+\beta^{[\frac{3}{2}]}\gamma^{[\frac{3}{2}]}$ form a supersymmetric Distler–Kawai sector in which matter′ and matter play dual roles of "matter proper" and Liouville matter (we recall that both of these theories are $N=1$-supersymmetric).

Furthermore, there is one more theory with a central charge of zero, constructed by means of the fields

$$\begin{array}{cccccc} bc & \partial\phi & \partial v_* & \chi & \beta\gamma & \psi \\ -2 & (-5+3\alpha_+^2+3\alpha_-^2) & (7-3\alpha_+^2-3\alpha_-^2) & \frac{1}{2} & -1 & \frac{1}{2} \end{array} \tag{18}$$

Significantly, the scalar ∂v_* and the "extra" ghost χ are combined here in a supersymmetric $N=1$ theory, which we call "$*$-matter," in order to distinguish it from the $N=1$ matter from (8):

$$T_{m*}=\tfrac{1}{2}\partial v_*\partial v_*+\tfrac{1}{2}(\alpha_++\alpha_-)\partial^2 v_*+\tfrac{1}{2}\partial\chi\chi,\quad G_{m_*}=\partial v_*\chi+(\alpha_++\alpha_-)\partial\chi. \tag{19}$$

This $*$-matter becomes part of a (twisted) $N=3$ algebra by means of formulas like (9) and (10), but with T_m and G_m replaced by T_{m_*} and G_{m_*}. We call this algebra an "$*$-$N=3$ algebra" to distinguish it from the $N=3$ algebra in which unbosonized matter participates. The various algebras are interrelated by

$$T^{\mathrm{KPZ}}=\overbrace{t^{[2]}+t^{[\frac{3}{2}]}+T'_m}^{\text{super-DDK}}+\overbrace{\underbrace{\underbrace{T_m+T_\phi+t_{bc}+T_\psi+t_{\beta\gamma}}_{\mathcal{T}_{N=3}}+\overbrace{T_*+T_\chi}^{T_{m*}}}_{T_{\mathrm{Sug}}+t_{BC}+t_{\beta\gamma}}}^{\mathcal{T}_{N=3*}} \tag{20}$$

Reading the right side as $t^{[2]}+t^{[\frac{3}{2}]}+T'_m+T_{\mathrm{Sug}}+t_{BC}+t_{\beta\gamma}+T_\chi$, we have an $osp(1|2)$ description of supergravity. Another interpretation of the same theory, as can be seen from the diagram, is to treat it as *super*-DDK$+\mathcal{T}_{N=3_*}$. In the purely boson case, we have, in a corresponding way, a partitioning of $sl(2)$ gravity into the DDK sector and the $*$-topological (twisted $N=2$) theory. The equivalence of the DDK and Knizhnik–Polyakov–Zamolodchikov (KPZ) description requires that this "extraneous" topological theory be trivial. A crucial circumstance is that the $*$-matter which enters the corresponding $*$-algebra is bosonized explicitly in terms of a free scalar. This representation representation for the $*$-matter gives rise to screening operators. Among them is a fermion screening operator which commutes with the BRST operator. This fermion screening operator is not present for superalgebras "proper," obtained by dressing matter "proper," which is *not* assumed to be bosonized. In contrast, it is in the sector of the $*$-matter that the fermion screening operator can serve as independent BRST operators, in accordance with the explanation in Ref. 4. The choice of a "physical" BRST operator is determined by the requirements imposed on the theory. In this sense, the choice is fixed by external considerations (one might say that the theory should be redefined[1)]; Ref. 22). In the present context, all states of $*$-algebras can be rendered BRST-trivial by choosing the cohomologies of a *binary* BRST complex as physical states.

Inverting the construction, i.e., starting with the supersymmetric DDK sector, we can multiply it in a tensor manner by a twisted $*$-$N=3$ theory. This procedure gives rise to a hidden $osp(1|2)$ symmetry and allows us to construct a formalism of $osp(1|2)$ supergravity. Interestingly, the field χ required to supersymmetrize ∂v_* matter remains in the $osp(1|2)$ formalism and becomes an "additional ghost" from Ref. 11.

Abusing the notation slightly, we can write Eq. (16) as the following coset construction for a hidden $N=3$ algebra:

$$\frac{osp(1|2)\oplus u(1)\oplus u(1)\oplus u(1)}{u(1)}=\mathscr{T}_{N=3}, \tag{21}$$

where $\mathscr{T}_{N=3}$ represents the entire $N=3$ algebra, and the $u(1)$ in the denominator is generated by the current ∂v_*. At the same time, $u(1)$ in the numerator represents ghosts in accordance with the "crude" correspondence (fermion ghosts) $\leftrightarrow$ (one scalar), (boson ghosts)$\leftrightarrow$ (two scalars). An invariant description of this coset (without the use of an explicit construction in terms of "elementary" fields) uses a nonlocal current $\partial\ \log J^{++}$ (which becomes a local one in our specific representation). This current can be specified by means of its operator products: the zeroth operator product with itself and with J^{++} and J^+ and also

$$\partial \log J^{++}(z)J^0(w)=\frac{1}{(z-w)^2},\quad \partial\ \log J^{++}(z)j^-(w)=\frac{1}{(z-w)^2}J^+(J^{++})^{-1}(w),$$

$$\partial \log J^{++}(z)J^-(w)=\frac{1-\alpha_-^2}{(z-w)^3}(J^{++})^{-1}(w)-\frac{1}{(z-w)^2}(2J^0-k\partial\ \log J^{++})$$
$$\times(J^{++})^{-1}(w), \tag{22}$$

where the *level* k is given by $k=\dfrac{\alpha_-^2-3}{2}$. Using $\partial\ \log J^{++}$, we can determine the $u(1)$ current in the denominator of the coset construction from

$$\partial v_*=2\alpha_+BC-\sqrt{7\alpha_+^2-2}\beta\gamma-2\alpha_+J^0-\tfrac{1}{2}(\alpha_-+3\alpha_+)\partial\ \log J^{++}. \tag{23}$$

(Currents corresponding to the ingredients of the supergravity, bc, $\beta\gamma$, and $\partial\phi$, can also be expressed in terms of $\beta\gamma$, BC, J^0, and $\partial\ \log J^{++}$.)

It is also a straightforward matter to construct *states* of the $osp(1|2)$ algebra by working from the states of matter (in the Neveu–Schwarz sector). Let us assume a prime state of algebra (8) with the standard dimensionality:

$$\Delta(r,s)=\tfrac{1}{8}[\alpha_+^2(r^2-1)+\alpha_-^2(s^2-1)+2-2rs]. \tag{24}$$

A prime state of $N=3$ algebra is constructed in accordance with

$$|\mathrm{h}(r,s)\rangle=|r,s\rangle\otimes|p_M(r,s)\rangle_L\otimes|0\rangle_\psi\otimes|0\rangle_{bc}\otimes|0\rangle_{\beta\gamma}, \tag{25}$$

where $p_M(r,s)=\tfrac{1}{2}(r-1)\alpha_++\tfrac{1}{2}(s-1)\alpha_-$. From this $|\mathrm{h}(r,s)\rangle$ we then construct the $osp(1|2)$ state of senior weight simply as

$$|j(r,s)\rangle=|\mathrm{h}(r,s)\rangle\otimes|-p_M(r,s)\rangle_*. \tag{26}$$

We have constructed representations of the $sl(2)$ and $osp(1|\ 2)$ current algebras, found by dressing (super)minimal matter, and we have demonstrated embeddings of certain noncritical string theories in the WZW model. We have also constructed some explicit mappings which make it possible to establish the equivalence of various formalisms for 2D quantum (super)gravity. It would be interesting to study the correspondence between the approach proposed here and Ref. 21, where there are constructions which seem at first glance to be different. This study was supported in part by grant #MQM000 from the International Science Foundation and by a grant from the Russian Fund for Fundamental Research.

[1)]For example, a minimal model dressed with gravity can be converted into a topological minimal model by adding a fermion screening current to the BRST current.[4,22]

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