Quantum oscillations of a new type in two-dimensional electron systems in the vicinity of the percolation threshold

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A new type of conductivity quantum oscillations are predicted in two-dimensional electron systems at carrier concentrations close to the zero-magnetic-field percolation metal–insulator transition. The off-diagonal conductivity σ_{xy} changes nonmonotonically with magnetic field. In particular, these oscillations may be responsible for the reentrant transitions between a quantum-Hall effect state and an insulating state, which were recently observed in several two-dimensional electron systems. Influence of the Zeeman splitting on the oscillations is predicted. © *1994 American Institute of Physics.*

It is generally accepted (see, for example, Ref. 1) that in a high magnetic field H the energy spectrum of a two-dimensional electron system consists of magnetic levels that are broadened by a disorder in a sample. Degeneracy of each level is eH/hc. (Here e is the electron charge, c is the velocity of light, and h is Planck's constant). In the presence of only long-range potential fluctuations [in comparison with the magnetic length $l=\hbar c/eH)^{1/2}$] magnetic levels, which are locally sharp, are inhomogeneously broadened. Most of the electronic states of a level are localized; only a small number of them are extended in the vicinity of the level center. The extended states of a magnetic level (i) provide quantized off-diagonal conductivity, $\sigma_{xy}=e^2/h$, when occupied, and (ii) give rise to a nonzero diagonal conductivity, σ_{xx}, when they coincide with the Fermi level. Increasing the magnetic field at a fixed carrier concentration results in a decrease of several occupied magnetic levels. The conductivity σ_{xy} will then monotonically decrease, changing step by step between the quantized values $\sigma_{xy}=ie^2/h$, where i is the number of magnetic levels under the Fermi level. These steps are periodic in the inverse magnetic field. In accordance with the generally accepted notation, the states of a two-dimensional electron system with a quantized nonzero σ_{xy} are the integer quantum Hall effect states and a state with $\sigma_{xy}=0$ is an insulating state. (In both types of states $\sigma_{xx}=0$). Transitions between different states occur via a metal phase[2] and are accompanied by peaks in the diagonal conductivity σ_{xx}.

In this paper we first consider magnetic quantization for noninteracting electrons in the case of a regular long-range potential modulation in the vicinity of the zero-magnetic-field percolation metal–insulator transition. We show that in this case a new type of quantum oscillations should appear. These oscillations result in a nonmonotonic dependence of σ_{xy} on the magnetic field and, in particular, describe a reentrant insulator–quantum Hall effect transition. Such transitions were observed recently in two-dimensional electron systems of Si-MOSFETs,[3–6] GaAs/AlGaAs heterostructures,[7,8] and

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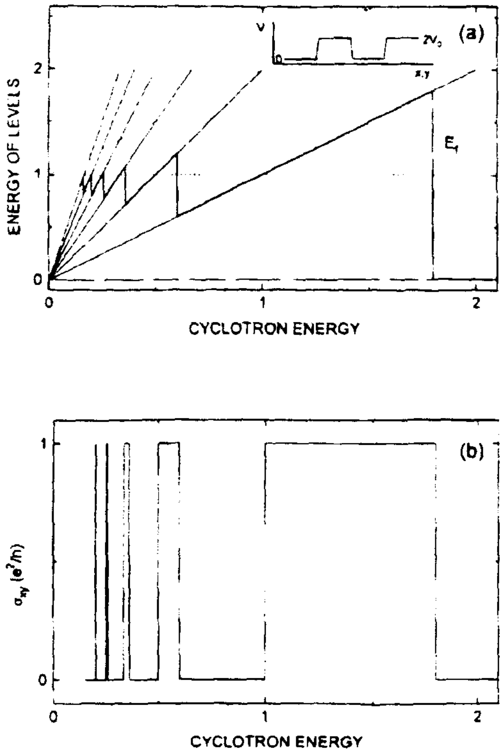

FIG. 1. (a) Fermi energy E_f (wide solid line) versus the cyclotron energy. Lower magnetic sublevels are denoted by narrow solid lines and the extended states are represented by dotted lines. The chosen value of the Fermi energy $E_f^0=0.9$ corresponds to the zero-magnetic-field insulating state. All the energies are given in units of V_0; $g\mu H=\hbar\omega_c$. Inset: variation of the model potential with coordinates x,y in the plane of the electron system. (b) Calculated off-diagonal conductivity σ_{xy} versus the cyclotron energy.

Si/SiGe heterostructures.[9] We can argue, therefore, that our results for the regular potential can also be valid for commonly used samples.

To illustrate the possibility of oscillations of the new type, we start with an exactly solvable model of a long-range potential, which looks like a chessboard with potentials $V=0$ and $V=2V_0$ in the white and black squares, respectively. The potential V varies between 0 and $2V_0$ in the transition regions, which are narrow in comparison with the size of a square (see the inset in Fig. 1), but still much larger than the magnetic length. In principle, such a potential can be easily produced by modern lithographic technique. In the chosen potential a zero-magnetic-field percolation threshold occurs at the Fermi energy $E_f^0=V_0$. The extended states of the inhomogeneously broadened magnetic levels $E_n^{\pm}=(n+1/2)\hbar\omega_c\pm g\mu H/2+V$ are located at energies[1] $E_{pn}^{\pm}=(n+1/2)\hbar\omega_c \pm g\mu H/2 + V_0$. Here n is an integer, $\omega_c=eH/m^*c$ is the cyclotron frequency (m^* is the carrier effective mass), g is the g factor, and μ is the Bohr magneton. In the case of narrow transition regions we can ignore their influence on the density of states which is then approximated for each magnetic level by two peaks at energies $E_{n1}^{\pm}=(n+1/2)\hbar\omega_c\pm g\mu H/2$ and $E_{n2}^{\pm}=E_{n1}^{\pm}+2V_0$. Below we shall refer to these peaks as magnetic sublevels. The degeneracy of each sublevel is $eH/2hc$. The position of the Fermi level at zero temperature can then be easily calculated as a function of the magnetic field. The energies of the lower magnetic sublevels, those of the extended states, and the Fermi energy are shown in Figs. 1a and 2a versus the cyclotron energy $\hbar\omega_c$, which is proportional to the magnetic field. All energies are presented in units of V_0. The zero-

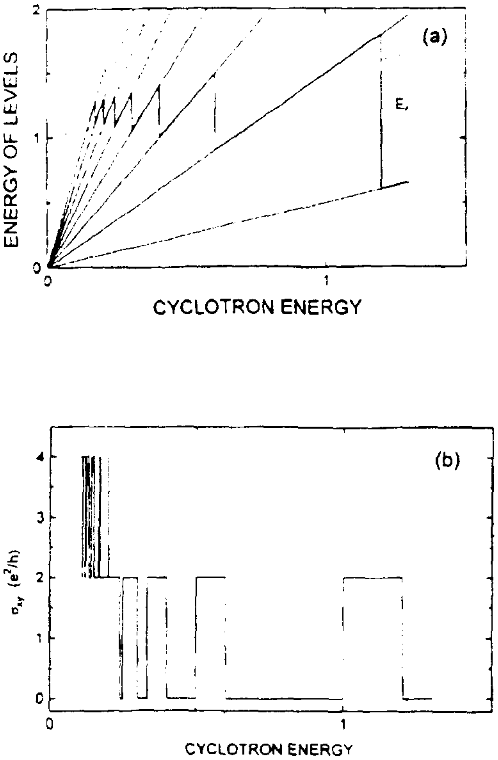

FIG. 2. The same as in Fig. 1 for another set of parameters: $g=0$ and $E_f^0=1.2$. The zero-magnetic-field state is a metal.

magnetic-field state is an insulator in Fig. 1 ($E_f^0 < V_0$) and a metal in Fig. 2. Figures 1 and 2 correspond to different Zeeman splittings (the splitting is equal to the cyclotron splitting in Fig. 1 and is zero in Fig. 2). For the value of E_f^0/V_0 we are interested in energies lower than $2V_0$; the magnetic sublevels corresponding to the black squares of the chessboard are not shown. Except for the lowest level in Fig. 1, the energies of the two levels with different spins coincide. The value of σ_{xy} (Figs. 1b and 2b) was calculated as the number of the extended states under the Fermi level, multiplied by e^2/h. Our results for σ_{xy} give sequences of jumps between two different σ_{xy} values as a function of the magnetic field. In particular, they describe the reentrant quantum Hall effect—the insulator transition. These jumps originate from the crossing of the extended states by the Fermi level and should be accompanied by peaks in σ_{xx}. Comparison of Figs. 1 and 2 demonstrates the role of the spin splitting. For example, in systems with $g=0$ (Fig. 2) the transition from the zero-magnetic-field insulating state to a quantum-Hall-effect state is impossible. It is clear that the effects considered above exist only in the vicinity of the percolation threshold in zero magnetic field. At $E_f^0 \gg V_0$, we obtain the usual picture of oscillations.

It should be noted that the steepness of the potential in the intermediate regions is not of crucial importance for our results. It is shown in Fig. 3, where the conductivity σ_{xy} is given for the potential $V=V_0[1-\cos(2\pi x/a)\cos(2\pi y/a)]$. Here a is a space scale of the potential variation ($a \gg l$).

We cannot expect the existence of such regular potentials in commonly used

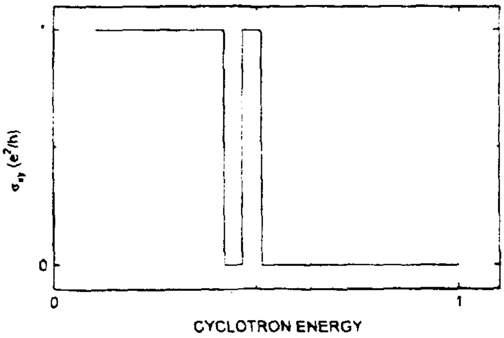

FIG. 3. Variation of the conductivity σ_{xy} in the magnetic field for the potential V of the form $V=V_0\quad[1-\cos(2\pi x/a)\cos(2\pi y/a)]$; $g\mu H=\hbar\omega_c$, $E_f^0=1.06$.

samples. Nevertheless, we believe that the above-considered effects can exist in such samples. The most important point for our consideration are the small fluctuations of the bottoms of different potential minima in comparison with the total amplitude of the potential fluctuations ($2V_0$ in our case). It was shown in Ref. 10 that such a potential should exist in heterostructures with thick spacer layers in the vicinity of a percolation metal–insulator transition. In this case the screening is nonlinear and the sample is divided into regions occupied by electrons (electron "lakes"), where the potential fluctuations are screened, and those where the electrons are absent and the amplitude of potential fluctuations is large. In this case the transition regions are narrow in comparison with the electron "lake" size and the above-presented consideration of magnetic quantization in the "lakes" is directly applicable. The point which depends on the total form of the potential fluctuations is the value of the percolation energy. Note that in systems with a steep potential in the transition regions the self-consistent potential should depend weakly on the magnetic field. The electron space distribution virtually does not change with the magnetic field, when the Fermi level coincides with one of the magnetic sublevels. Additional peculiarities in the screening are possible only when the exact integer number of sublevels is occupied. But this can modify the dependence of the Fermi energy on the magnetic field only in the vicinity of the Fermi level jumps between magnetic sublevels (we expect these jumps to be broadened).

Short-range disorder and finite temperature smear the oscillations of the Fermi level and those of σ_{xy}. In particular, transitions between different quantum Hall effect states should occur through an intermediate metal state.

A nonmonotonic dependence of σ_{xy} on the magnetic field has so far been observed only for transitions between different quantum Hall effect states and an insulating state. In an increasing magnetic field the following sequences of σ_{xy} have been reported (in units of e^2/h): (i) (6,0,2,0,1,0) in the electron channels of Si MOSFETs,[3–6] where each Landau level consists of four levels corresponding to two spin orientations and two different valleys of the Si bulk energy spectrum, (ii) (0,2,0) in the electron channels of GaAs/AlGaAs heterostructures,[7,8] (iii) (3,0,1) in the p-channels of Si/SiGe heterostructures.[9] No repetition of transitions between two different quantum Hall effect states was observed until now. Our results definitely show that in the reentrant insulator the quantum Hall effect transitions can be naturally explained in terms of the percolation effects. Experimental results supporting the percolation mechanism of the metal–insulator transitions in Si MOSFETs have been recently published in Ref. 2. Our model explains the following experimental results. (i) the reentrant behavior is observed only at carrier concentrations in the vicinity of the metal–insulator transition at $H=0$, (ii) the reentrant behavior is not observed for sweeping carrier concentration. In addition, we predict that reentrant behavior should be much easier to observe in systems with a large Zeeman splitting, in agreement with Ref. 9. In particular, we predict that in the electron channels of Si MOSFETs the modulation of the phase boundary of the low-density insulating state, reported in Refs. 2, 3, and 6, will increase in amplitude in a tilted magnetic field.

Finally, in samples with regular modulation of the potential we made predictions of the reentrant transitions between different quantum Hall effect states and between these states and the insulating state. We argued that the same effects can exist in commonly used samples with long-range potential fluctuations. The latter statement is confirmed by recent observations of the reentrant quantum Hall effect–insulator transitions.

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