Another version of the twistor-like approach to superparticles

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A worldsheet supersymmetric generalization of the $D=3$ Ferber–Shirafuji twistor-superparticle action is considered. © *1994 American Institute of Physics.*

Doubly supersymmetric, twistor-like formulations of superparticles,[1–6] superstrings,[7–9] and supermembranes,[10] have attracted considerable attention, in particular, because of the hope of breaking through the long-standing problem of the covariant quantization of these theories.

In the twistor-like approach the infinite-reducible fermionic κ-symmetry,[11,12] which causes the problem of covariant quantization,[13] is replaced by a local worldsheet supersymmetry which is irreducible by definition, and the theory is formulated as a superfield theory in a worldsheet superspace imbedded into a target superspace. Thus, a model of this kind possesses double supersymmetry.

Doubly supersymmetric dynamical systems (of more general physical contents) were considered earlier by several groups of authors, irrespective of the κ-symmetry problem.[14]

Several versions of doubly supersymmetric, twistor-like particles and heterotic strings have been constructed in $D=3,4$ and 6 dimensions of space–time, while in $D=10$ only one superfield formulation is known[6] and, unfortunately, the latter involves the infinite reducibility problem which arises for a new local symmetry.[6] It is crucial for eliminating the auxiliary degrees of freedom of the objects under consideration.[2)]

The main motivation of the present paper is, on the one hand, to develop a version of the twistor-like formulation, which would be free of the reducibility problem already at the superfield level, and, on the other hand, would look "twistor-like" as much as possible. The ideas suggested in this letter would allow one, we hope, to better utilize the powerful twistor techniques for deeper implementation of the twistors into the structure of supersymmetric theories.

The superfield twistor-like models of $N=1$ Brink–Schwarz superparticles in $D=3$, 4, 6, and 10, which were considered so far, are based on the doubly supersymmetric generalization of the following massless bosonic particle action:[1]

$$S=\int d\tau p_m(\dot{x}^m-\bar{\lambda}\gamma^m\lambda), \tag{1}$$

where p_m is the particle momentum, and λ^α is a commuting spinor variable which ensures the validity of the mass shell condition $p_m p^m = 0 = \dot{x}_m \dot{x}^m$ due to the Cartan-Penrose representation $\dot{x}^m = \bar{\lambda}\gamma^m\lambda$ of the light-like vectors in $D=3$, 4, 6, and 10 space–time dimensions.

The straightforward doubly supersymmetric generalization of (14) is[6]

$$S = \int d\tau d^{D-2}\eta P_{mq}(D_q X^m - D_q \bar{\Theta}\gamma^m\Theta), \tag{2}$$

where the number $n = D-2$ of the local worldline supersymmetries is equal to the number of the κ-symmetries in $D=3$, 4, 6, and 10; $D_q = (\partial/\partial\eta^q) + i\eta_q\partial_\tau$ is an odd supercovariant derivative in a worldline superspace (τ, η^q), $\{D_q, D_p\} = 2i\delta_{pq}\partial_\tau$ and (X^m, Θ^α) are worldline superfields which parametrize the "trajectory" of the superparticle in a target superspace. Bosonic spinor variables λ_q^α appear in (2) as superpartners of Grassmann coordinates $\theta^\alpha = \Theta^\alpha|_{\eta=0}$:

$$\lambda_q^\alpha = D_q\Theta^\alpha(\tau, \eta)|_{\eta=0}. \tag{3}$$

The analysis of the action[1,6] (2) shows that it describes a superparticle which is classically equivalent to the massless $N=1$ Brink–Schwarz superparticle in $D=3$, 4, 6, and 10.

As we have already mentioned, in $D=4$, 6, and 10 the action (2) possesses a local symmetry[6] under the following transformations of the Lagrange multiplier P_{mq}:

$$\delta P_{mq} = D_p \bar{\Xi}_{qp\tau}\gamma_m D_\tau\Theta, \tag{4}$$

where $\bar{\Xi}^\alpha_{qp\tau}$ is symmetric and traceless with respect to the indices p, q, and τ. This symmetry is infinitely reducible since P_{mq} is inert under the transformations (4) with

$$\bar{\Xi}^\alpha_{qp\tau} = D_s \bar{\Xi}^\alpha_{qp\tau s}, \tag{5}$$

where $\bar{\Xi}^\alpha_{qp\tau s}$ is again symmetric and traceless, and (5) is trivial if $\bar{\Xi}^\alpha_{qp\tau s} = D\ s\bar{\Xi}^\alpha_{qp\tau st}$, etc.

The reducibility of the transformations (4) is similar to the reducibility of the gauge symmetries of the antisymmetric gauge fields. The problem of reducible symmetries in these theories has stimulated a further development of the quantization procedure which was consistently followed for finite reducible symmetries,[17] to which the gauge transformations of the antisymmetric bosonic tensor fields belong. However, the general procedure for dealing with the infinite reducible symmetries is still unknown (see Ref. 18 and the bibliography cited there). This problem must therefore be avoided in every way. In the case under consideration we can try to find another form of the twistor-like superfield action for the superparticle.

To this end, let us choose a starting point the form of the twistor particle action which was considered by Ferber[19] and Shirafuji[20]

$$S = \int d\tau \bar{\lambda}\gamma_m\lambda\dot{x}^m. \tag{6}$$

For simplicity, we shall consider the case of $N=1$, $D=3$ superparticle.

To generalize (6) to the doubly supersymmetric case, we could naively [using Eq. (3)] write an action in the form

$$S=\int d\tau d\eta D\Theta_\alpha D\Theta_\beta DX^{\alpha\beta}, \tag{7}$$

where $X^{\alpha\beta}\equiv X^m\gamma_m^{\alpha\beta}$.

However, action (7) describes not an $N=1$, $D=3$ Brink–Schwarz superparticle, but rather a model with odd physical contents. The reason is that (7) is invariant under the following transformations:

$$\delta\Theta^\alpha=\epsilon_1^\alpha, \quad \delta X^{\alpha\beta}=\Theta^\alpha\epsilon_2^\beta+\Theta^\beta\epsilon_2^\alpha,$$

so that the target space is not the usual superspace, but one with additional θ translations.

Note that action (7) is part of a so-called spinning superparticle model which was considered several years ago.[14]

To construct a doubly supersymmetric action for describing an $N=1$ Brink–Schwarz superparticle, we must retain only one target space supersymmetry. The right action turns out to be as follows:

$$S=\int d\tau d\eta\Lambda_\alpha\Lambda_\beta(DX^{\alpha\beta}-iD\Theta^\alpha\Theta^\beta-iD\Theta^\beta\Theta^\alpha), \tag{8}$$

where $\Lambda_\alpha(\tau,\eta)$ is a commuting spinor superfield.

In addition to the $N=1$ target space supersymmetry and $n=1$ local worldline supersymmetry

$$\delta\eta=\frac{i}{2}D\Xi(\tau,\eta), \quad \delta\tau=\Xi+\frac{1}{2}\eta D\Xi, \quad \delta D=-\frac{1}{2}\partial\Xi D, \tag{9}$$

action (8) is invariant under bosonic transformations

$$\delta X^{\alpha\beta}=b(\tau,\eta)\Lambda^\alpha\Lambda^\beta, \quad \delta\Theta^\alpha=0=\delta\Lambda^\alpha \tag{10}$$

and under a superfield irreducible counterpart of the conventional fermionic κ-symmetry

$$\delta\Theta^\alpha=\kappa(\tau,\eta)\Lambda^\alpha, \quad \delta X^{\alpha\beta}=2i\,\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \quad \delta\Lambda^\alpha=0, \tag{11}$$

which resembles the fermionic symmetry of component twistor-like actions for super-p-branes[1,16] (the braces $\{\ldots\}$ denote symmetrization of the indices).

The algebra of the transformations (10), (11) is closed.

The equations of motion derived from (8) are

$$\Pi^{\alpha\beta}\Lambda_\beta\equiv(DX^{\alpha\beta}-2iD\Theta^{\{\alpha}\Theta^{\beta\}})\Lambda_\beta=0, \tag{12}$$

$$\Lambda_\beta D\Theta^\beta=0, \tag{13}$$

$$\Lambda_{\{\alpha}D\Lambda_{\beta\}}=0. \tag{14}$$

The general solutions of (12) and (13) are, respectively,

$$\Pi^{\alpha\beta}=\Psi(\tau,\eta)\Lambda^\alpha\Lambda^\beta, \tag{15}$$

$$D\Theta^\alpha=\alpha(\tau,\eta)\Lambda^\alpha. \tag{16}$$

At the same time, from (14) it follows that

$$D\Lambda^{\alpha}=0. \tag{17}$$

On the mass shell (15)–(17) the fermionic superfield Ψ and the bosonic superfield a tranform under (11), (10), and (9) as follows:

$$\delta\Psi=Db-\tfrac{1}{2}\partial_{\tau}\Xi\Psi-2ia\kappa, \quad \delta a=D\kappa-\tfrac{1}{2}\partial_{\tau}\Xi a. \tag{18}$$

Hence, we can fix a gauge

$$\Psi=0, \quad a=1, \tag{19}$$

at which (15) and (16) reduce to

$$\Pi^{\alpha\beta}=0, \tag{20}$$

$$D\Theta^{\alpha}=\Lambda^{\alpha}. \tag{21}$$

This gauge[3)] is conserved under the κ-symmetry which is reduced to the worldline supersymmetry

$$D\kappa-\tfrac{1}{2}\partial_{\tau}\Xi=D\left(\kappa+\frac{i}{2}D\Xi\right)=0. \tag{22}$$

As a result, the twistor superfield Λ^{α} can be expressed in terms of $D\Theta^{\alpha}$, and it does not carry independent degrees of freedom. In the gauge (19) the equations for $X^{\alpha\beta}$ and Θ^{α} coincide with those in the conventional twistor-like formulation[1,6] (2).

We conclude, therefore, that the doubly supersymmetric action (8) is classically equivalent to (2), and that it describes the massless $N=1$ superparticle.

The relationship between the two actions can be understood if we use the following line of reasoning. It was proved in Ref. 5 that for $n=1$ action (2) is classically equivalent to

$$S=\int d\tau d\eta(P_{\alpha\beta}\Pi^{\alpha\beta}-\tfrac{1}{2}EP_{\alpha\beta}P^{\alpha\beta}) \tag{23}$$

because of the existence of the following counterparts of the symmetry transformations (11) and (10):

$$\delta X^{\alpha\beta}=\tilde{b}P^{\alpha\beta}, \quad \delta E=D\tilde{b}, \quad \Theta^{\alpha}=0, \tag{24}$$

$$\delta X^{\alpha\beta}=2i\delta\Theta^{\{\alpha}\Theta^{\beta\}}, \quad \delta E=-2i\kappa_{\alpha}D\Theta^{\alpha}, \quad \delta\Theta^{\alpha}=\kappa_{\beta}P^{\beta\alpha}, \tag{25}$$

which allows us to set the Grassmann superfield $E(\tau,\eta)$ equal to zero globally on the worldline superspace.[4)]

At the same time, the variation of (23) with respect to $E(\tau,\eta)$ leads to the equation

$$P^{\alpha\beta}P_{\alpha\beta}=0, \tag{26}$$

which can be solved as

$$P_{\alpha\beta}=\Lambda_{\alpha}\Lambda_{\beta}, \tag{27}$$

where Λ_α is an arbitrary bosonic spinor superfield. Substituting (27) into (23), we obtain the action (8).

In conclusion we have constructed a version of the twistor-like formulation of the massless $N=1$, $D=3$ superparticle based on Eq. (8) with all symmetries of the model being irreducible. Action (8) looks very much like a worldline superfield generalization of the supertwistor action proposed by Ferber.[19]

We can even rewrite (8) in a complete supertwistor form, just like Shirafuji,[20] by introducing the second bosonic spinor component and the Grassmann component of the supertwistor:[19]

$$M^\alpha = X^{\alpha\beta}\Lambda_\beta, \quad \Upsilon = \Theta^\alpha \Lambda_\alpha. \tag{28}$$

Taking into account constraint (28), action (8) will then take the form

$$S = \int d\tau d\eta (\Lambda_\alpha DM^\alpha - D\Lambda_\alpha M^\alpha - 2i\Upsilon D\Upsilon).$$

Note that the transformations (11) resemble an extra, hidden, local, worldline supersymmetry, which relates Θ^α and Λ^α. It would be of interest to understand the nature and the role of this symmetry in more detail.

Action (8) admits a generalization to $D=4$ and 6 superparticles and possibly to heterotic strings within the line of the twistor-like formulation developed in Refs. 1, 4, 5, and 8, where the notion of a double Grassmann analyticity (for $D=4$) and a double harmonic analyticity (for $D=6$) have been explored. But the generalization to the case of $D=10$ twistor-like objects with irreducible local symmetries seems to be more subtle. Work on this subject is in progress.

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[2)]Note, however, that at the component level, when auxiliary fields are eliminated by gauge fixing and solving for relevant equations of motion, all remaining local symmetries are irreducible. This also takes place in a twistor-like Lorentz harmonic formulation of super-p-branes,[15,16] which were developed in parallel with the superfield twistor approach.

[3)]Note that the gauge choice $a=0$ in Eq. (19) is inadmissible, since from (15) and (16) it would follow that $(d/d\tau)X^m|_{\eta=0}=0$, which, in general, is incompatible with the boundary conditions $X^m(\tau_1)|_{\eta=0}=x_1$, $X^m(\tau_2)|_{\eta=0}=x_2$.

[4)]Note that in contrast to (11), the transformations of Eq. (25) correspond to an infinite reducible κ-symmetry.[11-13]

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