Aharonov–Bohm effect in the Luttinger liquid

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(Submitted 18 August 1994; resubmitted 29 September 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 9, 639–644 (10 November 1994)

In systems with the spin-charge separation, the period of the Aharonov–Bohm (AB) oscillation becomes half of the flux quantum. This effect related at least to the fact that two electrons are needed for the creation of holons (spinons). The effect is illustrated using the example of the Hubbard Hamiltonian with the help of bosonization including topological numbers. It exists also in the Luttinger liquid on two chains. The relation to a fractional $1/N$ AB effect, which can be associated with a modified Luttinger liquid, is discussed. © *1994 American Institute of Physics.*

In nearly all strongly correlated $1D$ electron systems there exists the phenomenon known as "spin-charge" separation.[1] It was also recently argued[2] that the spin-charge separation is not only inherent to $1D$, but also occurs in the two-dimensional systems related to HTSC. The degrees of freedom associated with a single electron are split into two independent spin and charge degrees of freedom associated with single-particle gapless excitations: spinons and holons as in a $1D$ Luttinger liquid.[3]

We show[4] that the properties of strongly correlated systems are associated with a new type of AB effect; specifically, the period of the AB effect decreases and becomes half of the period of the AB oscillations for the free electrons.[5] This is valid for all systems in which the spin-charge separation exists. The spinon and holon excitations are created by two single-electron operators which are associated with the spin and charge density fluctuations. This is also the reason why the period of the AB oscillation is halved. With holons as well as with spinons two types of topological numbers are associated bound with some selection rules defined by the parity of the total number of electrons.[3] As a result, all properties are parity dependent and the parity effect exists in the Luttinger liquid of spinful electrons. However, the period of the oscillation for the Hubbard ring in the limit of $U \rightarrow \infty$ decreases N_e times, where N_e is the number of electrons on the ring.[6,7] The other important feature of this effect is the absence of the parity effect, which exists for free electrons[5,8] and for interacting fermions.[5,9,10] The absence of the parity effect is connected with the $1/N_e$ decrease of the AB period. The system in the strong-coupling regime can be described by a modified Luttinger liquid.

To illustrate the decrease of the AB period we consider the Hubbard Hamiltonian

$$H=t\sum_{i,\sigma}(a^{+}_{i\sigma}a_{(i+1)\sigma}+\text{H.c.})+U\sum_{i}n_{i\uparrow}n_{i\downarrow}, \qquad (1)$$

where t and U are hopping integral and the constant of the on-site Coulomb interaction between electrons, respectively. First, we go to the continuum limit and then apply the bosonization, where the Hamiltonian is[11]

$$H_e = it \sin k_F \sum_s \int_0^L dx[\Psi_{s-}(x)\partial_x\Psi_{s-}(x) - \Psi_{s+}(x)\partial_x\Psi_{s+}(x)]$$

$$+U\int_o^L dx[:j_{0\uparrow}::j_{0\downarrow}: + \Psi^+_{\uparrow+}(x)\Psi^+_{\uparrow-}(x)\Psi_{\downarrow+}(x)\Psi_{\downarrow-}(x) + \text{H.c.}], \tag{2}$$

where $\Psi_{s\pm}$ are left and right movers and $j_{0\uparrow} = \Psi^+_{\uparrow+}(x)\Psi_{\uparrow+}(x) + \Psi^+_{\uparrow-}(x)\Psi_{\uparrow-}(x)$. The analogous expression is written for the current $j_{0\downarrow}$ of down-spin fermions.

We take into account the periodical boundary (PB), and twisted boundary (TB) conditions, when the Hubbard ring is located in a transverse magnetic field. In both cases the fermion field $\Psi_{\beta\alpha}(x)$ can be represented as: $\Psi_{\beta\pm}(x) = 1/\sqrt{2\pi\alpha} \times \exp[\pm i\sqrt{4\pi}\Phi_{\beta\pm}(x)]$, where α is the cut parameter and the boson fields $\Phi_{\beta\pm}$ for PB conditions can be represented as: $\Phi_{\beta\pm}(x) = \Phi_\beta(x) \pm \int_{-\infty}^x \pi_\beta(x')dx'$. Here $\pi_\beta(x)$ are the variables conjugate to Φ_β. In terms of these fields, the Hubbard Hamiltonian takes the form

$$H = \int_0^L dx\left(\frac{t \sin k_F}{2}[\pi^a_\uparrow + (\partial_x\Phi_\uparrow)^2 + (\uparrow\to\downarrow)]\right.$$

$$\left.+U\left[\frac{\partial_x\Phi_\uparrow\partial_x\Phi_\downarrow}{\pi} + \frac{1}{2\pi^2\alpha^2}\cos[\sqrt{4\pi}(\Phi_\uparrow - \Phi_\downarrow)]\right]\right). \tag{3}$$

On the ring the variables π_β and Φ_β are multivalued. It is, therefore, convenient to decompose them into single-valued variables and topological quantum numbers which are related to the winding numbers on the ring:

$$\Phi_{\beta\pm}(x) = \Phi_\beta(x) \pm \int_{-\infty}^x \pi_\beta(x')dx' + (N_\beta \pm J_\beta)\frac{\sqrt{\pi}x}{2L}, \tag{4}$$

where the new variables $\pi_\beta(x)$ and Φ_β are single-valued and N_β, J_β are topological numbers associated with the charge and current on the ring. These numbers are connected by the selection rules, which depend on the parity of the total number of electrons, N_e. Imposing the periodical boundary conditions, we obtain the following selection rules: $(-1)^{(N_\beta \pm J_\beta)} = (-1)^{(N_e - 1)}$, which is a simple generalization of the selection rule for the Luttinger liquid of spinless fermions.[10] Implicitly, these selection rules dictate that if the number of electrons is odd, then the number N_β is even and the number J_β is odd, or the number N_β is odd and the number J_β is even. On the other hand, if the number of electrons is even, then the number N_β is even and the number J_β is even, or the number N_β is odd and the number J_β is odd.

For the case of TB conditions, we introduce different flux values for the up- and down-spin electrons f_β, where the shift has the form $\Phi_{\beta\pm}(x) \Rightarrow \Phi_{\beta\pm}(x) \pm \sqrt{\pi}f_\beta x/L$. We separate the theory into two parts, introducing the spin and charge fields $\varphi_s = (\Phi_\uparrow - \Phi_\downarrow)/2$ and $\varphi_c = (\Phi_\uparrow + \Phi_\downarrow)/2$, the fluxes of the electrical and magnetic field $f_s = (f_\uparrow - f_\downarrow)/2$ and $f_c = (f_\uparrow + f_\downarrow)/2$, and the topological numbers $N_s = N_\uparrow - N_\downarrow$,

$J_s = J_\uparrow - J_\downarrow$, $N_c = N_\uparrow + N_\downarrow$, $J_c = J_\uparrow + J_\downarrow$. In terms of the topological numbers and the single-valued variables π_β and Φ_β, the Hamiltonian can be split into two parts $H = H_c + H_s$, where

$$H_c = A_c \int_0^L dx[\pi_c^2 + (\partial_x \varphi_c)^2] + \frac{A_c \pi}{16L}[(J_c + 4f_c)^2 + N_c^2], \tag{5}$$

$$H_s = A_s \int_0^L dx[\pi_s^2 + (\partial_x \varphi_s)^2] + \frac{A_s \pi}{16L}[(J_s + 4f_s)^2 + N_s^2]$$
$$+ \frac{U}{2\pi^2 \alpha^2} \int_0^L dx \cos\left[\sqrt{16\pi}\left(\frac{\varphi_s}{A_s} + \frac{\sqrt{\pi} N_s x}{4A_s L}\right)\right] \tag{6}$$

are associated with the charge and spin degrees of freedom, respectively, and $A_{c/s}^2 = t \sin k_F \pm U/\pi$. The choice of integer numbers J_c, N_c, J_s, and N_s is dictated by the selection rules described above. For example, if the number of electrons N_e is odd, then and the numbers $N_\uparrow$ and $N_\downarrow$ have different parities; i.e., one of these numbers is odd and the other is even, since $N_c = N_e = N_\uparrow + N_\downarrow$. This means that the numbers $J_\uparrow$ and $J_\downarrow$ also have different parities and the number J_c is odd. The fact that the Hamiltonian for the charge degrees of freedom is split into two parts and the number J_c consists of the sum of the two topological numbers $J_\uparrow$ and $J_\downarrow$ is the reason why the "holon" Hamiltonian has the flux period $f_T = 1/2$, rather than the conventional period $f_T = 1$.

In the case where N_e is even, the selection rules indicate that the numbers $N_\uparrow$ and $N_\downarrow$ have the same parity. This means that the AB effect is half-flux quantum periodic and the energy-flux dependence is described by parabolic segments with the minima located at the flux equal to integers and half-odd integers [see Eq. (5)]. We thus have a new parity effect in which there is a difference in the behavior of the odd and even numbers of electrons, i.e., there is a shift in the energy-flux dependence by a quarter of the elementary flux quantum. This behavior is in contrast with the parity effect for spinless fermions,[5,8,9] where the shift is a half of the flux quantum. A similar situation occurs for an Aharonov–Cashier effect.

To calculate the current of the Hubbard ring, we change the problem to the Lagrangian formalism, drop the irrelevant spin degrees of freedom, and consider only the holon Lagrangian L_c and the action S_c. In the Lagrangian formalism our fields φ_c depend on the space and time variables $\varphi_c = \varphi_c(x,t)$ and satisfy the PB conditions for both variables. The multivalued field $\varphi_c(x,t)$ can be split into a single-valued field $\tilde{\varphi}_c(x,t)$ and into terms related to the winding numbers n and m, with the help of the relation $\varphi_c(x,t) = \tilde{\varphi}_c(x,t) + \sqrt{\pi} xn/(2L) + \sqrt{\pi} tm/(2L)$. In the Lagrangian for the charge degrees of freedom,

$$L_c = -\int_0^L dx[\dot{\varphi}_c^2/(4A_c) + A_c(\partial_x \varphi_c)^2] + i\frac{\sqrt{\pi}}{4L}(J_0 + 4f)\int_0^L \dot{\varphi} dx, \tag{7}$$

the single valued field $\tilde{\varphi}_c(x,t)$ can therefore be separated. The contribution of the orbital motion to the partition function Z of the ring can be calculated with the help of the continual integral over the single-valued field $\tilde{\varphi}_c(x,t)$ and the sums over the winding numbers n and m:[12,13]

$$Z_c=\int D\tilde{\varphi}_c\sum_{n,m}\exp[-S_c(\tilde{\varphi}_c,J_0,n,m)], \tag{8}$$

where the action has the form

$$S_c=\int_0^L\int_0^\beta dxdt\left[\dot{\varphi}_c^2/(4A_c)+A_c(\partial_x\varphi_c)^2-i\sqrt{\pi}\,\frac{\dot{\varphi}_c(J_0+4f)}{4L}\right]. \tag{9}$$

After the summation over the winding numbers the partition function Z_c takes the form $Z_c=Z_0\Theta_3(z_J,q_J)\Theta_3(z_m,q_m)$, where $\Theta_3(x,y)$ is the theta function and Z_0 is the partition function which is associated with the single-valued field $\tilde{\varphi}_c$;

$$z_J=\frac{(J_0+4f)\pi}{16},\quad q_J=\exp\left(-\frac{\pi L}{16A_c\beta}\right),\quad z_m=0,\quad q_m=\exp\left(-\frac{\pi A_c\beta}{4L}\right).$$

We derive the low- and high-temperature asymptotic relation for the free energy $F=-T\log Z_c$. In the case of the low-temperature limit $\beta\rightarrow\infty$ we have

$$\Delta F=\frac{\pi A_c(J_0+4f)^2}{16L}, \tag{10}$$

which is a flux-dependent term of Eq. (5), where J_0 is even or odd, which corresponds to even or odd number of particles, respectively, on the ring. In the case where $\pi L/16A_c\beta\gg 1$ or $\beta\rightarrow 0$, the contribution of the orbital motion is

$$\Delta F=-2T\exp\left(-\frac{\pi LT}{16A_c}\right)\cos\left(\frac{\pi(J_0+4f)}{8}\right). \tag{11}$$

After several recent experimental studies[14] the problem of the persistent current has been theoretically studied extensively (see Refs. 7 and 15 and the bibliography cited). The study of the problem was stimulated by the discrepancy between the amplitude of the current estimated theoretically and observed experimentally. The experiments indicate that this amplitude is several orders of magnitude larger than the theories predict.

The persistent current at zero temperature is

$$J_p=-\partial F/\partial f=-2\pi\frac{V_F}{L}\left(f+\frac{J_0}{4}\right), \tag{12}$$

where $-1/4-J/4\leqslant f\leqslant -J/4+1/4$, and $V_F=A_c$ which increases with U. This means that the current with electron–electron interaction is enhanced. At high temperatures this enhancement is even larger:

$$J_p=-\pi T\exp\left(-\frac{\pi LT}{16V_F}\right)\sin\left[\frac{\pi}{2}\left(f+\frac{J_0}{4}\right)\right]. \tag{13}$$

Because of the exponential prefactor, the current decreases appreciably with the temperature but increases exponentially with U. The characteristic temperature where the current is still visible is about $T_c \sim V_F/L$, which is the interlevel distance of the size quantization. The described enhancement is inconsistent with the arguments of Ref. 16 but agrees with the numerical simulations.[17]

In the strong-coupling limit $U \to \infty$, the problem can be diagonalized with the help of the Bethe ansatz. The spectrum obtained originally by the author[6] has the form

$$K_n = \frac{2\pi I_n}{L} + \frac{2\pi}{L}\frac{\Sigma J_\alpha}{N} + \frac{2\pi f}{L}. \tag{14}$$

The (half) integers I_n and J_α are holon and spinon quantum numbers, respectively. If we introduce the notation $\phi = \Sigma J_\alpha / N$, then this equation will look like the spectrum of spinless fermions in the flux $f + \phi$. In the continuum limit of this spectrum we can write an effective Hamiltonian of spinless fermions

$$H = \int_0^L [\psi^+(x)(K_{f\phi}^2 - k_F^2)\psi(x)]dx, \tag{15}$$

where $k_F = \pi N/L$, $K_{f\phi} = K + 2\pi\phi/L + 2\pi f/L$, and K is the momentum operator.

For comparison of the weak- and strong-coupling cases we represent the holon Hamiltonian (15) in the bosonized form. With the help of the Loss result[10] the Hamiltonian of the charge degrees of freedom takes the form

$$H = V_F \int_0^L [\pi^2 + (\partial_x \varphi)^2]dx + \frac{V_F \pi}{L}[N^2 + (J + 2\phi + 2f)^2]. \tag{16}$$

In comparison with Eq. (5), there appears the fictitious flux ϕ, which has fractional values $\phi = p/N$. Without an external magnetic field the selection rules have the form $(-1)^{N+J} = (-1)^{N_e + 1 + \phi}$, where the value ϕ can be equal to 0 or 1. The latter value means that the topological numbers N and J, which in the Luttinger liquid are coupled, now become decoupled. This stems from the fact that the parity of N_e plays no role, since we can change the value ϕ from 0 to 1 and the value J by 2 without changing the energy. This indicates a violation of the conventional Luttinger liquid, where the topological numbers N and J are coupled by the parity of N_e.

The parity effect appears at a finite value of U. The solution will then have a structure similar to Eq. (14) plus the energy of the spin-wave excitations. Therefore, for an odd number of particles in the bosonized form the Hamiltonian is

$$H = V_F \int_0^L [\pi^2 + (\partial_x \varphi)^2]dx + \frac{V_F \pi}{L}[N^2 + (J + 2\phi + 2f)^2] + \frac{V_F^2}{LU}|\sin 2\pi\phi|. \tag{17}$$

We see that, in addition to N and J, the topological quantum numbers, there appears a new term which is the internal energy of the field ϕ and the energy of the spin-wave excitations. Now the value of the field ϕ cannot take any rational number; i.e., the finite U lifts the degeneracy and the parity effect appears.

For an even number of particles we must change in Eq. (17) the $\sin 2\pi\phi$ to $\cos 2\pi\phi$. At zero external magnetic flux the selection rules take the conventional form $(-1)^{N+J} = (-1)^{N_e - 1}$, which dictates that $\phi = 0$. This corresponds to the maximum of the spin-wave excitation spectrum; i.e., with the external magnetic flux f there appears the spin-wave excitations (nonzero ϕ, which compensates f). This again indicates a violation of the conventional Luttinger liquid properties, where the field ϕ does not exist. This question needs special attention.

Thus, in a weak coupling the magnetization is a half-flux periodical function. The amplitude of the oscillations increases with U. When the spin and charge degrees of freedom are separated and composite particles (holons and spinons in our case) are created, half-flux periodical oscillations of the AB type occur (see, for comparison, Ref. 18). Therefore, the period of the AB oscillations in any strongly correlated systems always decreases.

This effect does not exist for interacting spinless fermions on a single-channel ring.[5,19] But the effect arises when the ring consists of two or many chains.[20]

The reason for the effect is similar to that for the Hubbard ring. We can prove exactly that the AB effect has the period of the half-flux quantum for any interactions which are not larger than the Fermi energy. When the interaction is comparable with the Fermi energy, the continuum approach is not applicable and there can occur a fractional $1/N_e$ AB effect[6] or a fractional M/N_e AB effect.[21] Thus, if in real HTSC materials in a normal state there occurs a spin-charge separation, the AB effect must have a half-flux quantum period in units of the elementary flux quantum. To observe such predictions in HTSC might be a good challenge for experimentalists.

I thank Yu Lu, Y. Avishai, A. Luther, A. Nersesyan, M. Takahashi, I. Krive, T. Ando, S. Katsumoto, M. Kohmoto, M. Ueda, K. Kawarabayashi, S. M. Manning, F. Assaad, and M. Kohno for useful discussions. I also thank the Ministery of Education, Science, and Culture of Japan and the International Center for Theoretical Physics for support, and ICTP and ISSP for their hospitality.

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