Direct calculation of the slope of the QCD pomeron trajectory

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The diffraction slope of the generalized BFKL pomeron amplitude was found to have a conventional Regge growth $B(s)=B(0)+2\alpha'_{\rm IP}\log(s)$. This proves that the generalized BFKL pomeron is described by the moving j-plane singularity. The slope $\alpha'_{\rm IP}$ is estimated in terms of the correlation radius for the perturbative gluons. © *1994 American Institute of Physics.*

1. Introduction

Whether the QCD pomeron is described by the fixed or moving singularity in the complex j plane remains one of the topical issues. The purpose of this letter is to prove that the generalized BFKL pomeron[1–4] is a moving cut. We present the first direct calculation of the slope $\alpha'_{\rm IP}$ for the pomeron trajectory.

The early works on the BFKL (Balitskii-Fadin-Kuraev-Lipatov[5]) pomeron focused on the idealized scaling regime with fixed strong coupling α_S=const and infinite gluon correlation radius R_c. In this regime, the BFKL pomeron is described by a fixed cut in the complex angular momentum plane $-\infty<j\leq\alpha_{\rm IP}(0)=1+\Delta_{\rm IP}$. However, because of the diffusion property of the Green's function of the scaling BFKL equation,[5] the scaling regime is not self-consistent. Recently, considerable progress has been made in the understanding of the BFKL pomeron in the framework of the dipole cross-section representation introduced in Ref. 6. In our previous papers[1–4] we derived the generalized BFKL equation for the dipole cross section in a realistic model with the running (and freezing) strong coupling $\alpha_S(r)$ and with the finite correlation radius R_c of the perturbative gluons. While the property of the cut in the j plane is retained, we found that the running $\alpha_S(r)$ and the finite R_c have a strong effect on the spectrum and solutions of our generalized BFKL equation. The crucial observation is that the intercept $\Delta_{\rm IP}$ and the asymptotic behavior of the dipole cross section are controlled by interactions at the dipole of size $r\sim R_c$. We also found that the recovery of the conventional multiperipheral pattern is likely at asymptotic energies, which suggests a Regge growth of the diffraction cone. In this letter we confirm the latter observation and show that indeed the pomeron trajectory has a finite slope, $\alpha'_{\rm IP}\propto R_c^2$.

The starting point of our analysis is the generalization of our BFKL equation[1–4] to the profile function of the dipole cross section $\Gamma(r,\mathbf{b})$. Defining the impact parameter $\mathbf{b}$ with respect to the center of the parent $q-\bar{q}$ dipole, and repeating the derivation,[1-4] we obtain

$$\frac{\partial\Gamma(\xi,r,\mathbf{b})}{\partial\xi}=\mathscr{K}\otimes\Gamma(\xi,r,\mathbf{b})$$

$$=\frac{3}{8\pi^3}\int d^2\boldsymbol{\rho}_1\ \mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\boldsymbol{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\boldsymbol{\rho}_2}{\rho_2}\right|^2$$

$$\times\left[\Gamma\left(\xi,\rho_2,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_1\right)+\Gamma\left(\xi,\rho_1,\mathbf{b}+\frac{1}{2}\boldsymbol{\rho}_2\right)-\Gamma(\xi,r,\mathbf{b})\right],\qquad(1)$$

where $\boldsymbol{\rho}_2=\boldsymbol{\rho}_1-\mathbf{r}$, the arguments of the running QCD charge $g_S(r)=\sqrt{4\pi\alpha_S(r)}$ are $R_i=\min\{r,\rho_i\}$, $K_1(x)$ is the generalized Bessel function, and $R_c=1/\mu_G$ is the correlation radius for the perturbative gluons. Here we use the standard definition of the profile function when

$$A(s,t)=2is\int d^2\mathbf{b}\,\exp(-i\mathbf{qb})\Gamma(\mathbf{b}),$$

and the dipole cross section is $\sigma(\xi,r)=2\int d^2\mathbf{b}\Gamma(\xi,r,\mathbf{b})$. We shall discuss the reduction of (2) to the equation for the diffraction slope

$$B(\xi,r)=\frac{1}{2}\langle\mathbf{b}^2\rangle=\lambda(\xi,r)/\sigma(\xi,r),\quad \lambda(\xi,r)=\int d^2\mathbf{b}\,\mathbf{b}^2\ \Gamma(\xi,r,\mathbf{b}).$$

The diffraction slope for the dipole of size r evidently contains a purely geometrical contribution $(1/8)r^2$, which comes from the elastic form factor of the dipole. It is therefore more convenient to consider $\eta(\xi,r)=\lambda(\xi,r)-\frac{1}{8}r^2\sigma(\xi,r)$, whose equation takes the form

$$\frac{\partial\eta(\xi,r)}{\partial\xi}=\frac{3}{8\pi^3}\int d^2\boldsymbol{\rho}_1\ \mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\boldsymbol{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\boldsymbol{\rho}_2}{\rho_2}\right|^2$$

$$\times\left\{\eta(\xi,\rho_1)+\eta(\xi,\rho_2)-\eta(\rho,r)+\frac{1}{8}(\rho_1^2+\rho_2^2-r^2)[\sigma(\rho_2,\xi)+\sigma(\rho_1,\xi)]\right\}$$

$$=\mathscr{K}\otimes\eta(\xi,r)+\beta(\xi,r),\qquad(2)$$

where the inhomogeneous terms are

$$\beta(\xi,r)=\mathscr{L}\otimes\sigma(\xi,r)=\frac{3}{64\pi^3}\int d^2\boldsymbol{\rho}_1\ \mu_G^2\left|g_S(R_1)K_1(\mu_G\rho_1)\frac{\boldsymbol{\rho}_1}{\rho_1}-g_S(R_2)K_1(\mu_G\rho_2)\frac{\boldsymbol{\rho}_2}{\rho_2}\right|^2$$

$$\times(\rho_1^2+\rho_2^2-r^2)[\sigma(\rho_2,\xi)+\sigma(\rho_1,\xi)].\qquad(3)$$

Here the crucial point is that the homogeneous Eq. (2) is precisely our generalized BFKL equation for the dipole cross section

$$\frac{d\sigma(\xi,r)}{d\xi}=\mathscr{K}\otimes\sigma(\xi,r),\qquad(4)$$

which enables us to prove on the generic grounds that $\alpha'_{\mathrm{IP}}=\frac{1}{2}dB(\xi,r)/d\xi\neq 0$.

The proof goes as follows: In Refs. 3 and 4 we have shown that the generalized BFKL operator $\mathcal{K}$ has a continuous spectrum, which corresponds to the cut in the j plane. Let $-\infty<\nu<\infty$ be the "wave number" which labels the eigenfunctions $E(\nu,r)\exp[\Delta(\nu)\xi]$ in Eq. (4) with the eigenvalue $\Delta(\nu)$. For guidance, in the scaling limit $\alpha_S=$const and $R_c\to\infty$ we have $E(\nu,r)=r\exp[i\nu\log(r^2)]=\sigma_{\mathrm{IP}}(r)\exp[i\nu\log(r^2)]$ with the orthogonality condition[3-5]

$$\delta(\nu-\mu)=\frac{1}{2\pi}\int\frac{d\log(r^2)}{[\sigma_{\mathrm{IP}}(r)]^2}E^*(\nu,r)E(\mu,r), \tag{5}$$

and ν is the wave number of the plane waves in the $\log(r^2)$ space. The properties of the eigenfunctions $E(\nu,r)$ in the case of the running $\alpha_S(r)$ and the finite R_c are discussed in Refs. 3, 4, and 7.

Now we proceed with the solution of the inhomogeneous equation (2). If $G(\nu,r)=\mathcal{L}\otimes E(\nu,r)=\int dwg(\nu,w)E(w,r)$, we can write the inhomogeneous term (3) as follows:

$$\beta(\xi,r)=\mathcal{L}\otimes\sigma(\xi,r)=\int d\nu E(\nu,r)\int dwf(w)g(w,\nu)\exp[\Delta(w)\xi]. \tag{6}$$

We search for a solution of the form $\eta(\xi,r)=\int d\nu\tau(\xi,\nu)E(\nu,r)\exp[\Delta(\nu)\xi]$. Making use of the property of the eigenfunctions $\mathcal{K}\otimes E(\nu,r)=\Delta(\nu)E(\nu,r)$, we find

$$\frac{\partial\tau(\xi,\nu)}{\partial\xi}\exp[\Delta(\nu)\xi]=\int dw\, f(w)g(w,\nu)\exp[\Delta(w)\xi] \tag{7}$$

and

$$\eta(\xi,r)=\int d\nu\tau(\xi=0,\nu)E(\nu,r)\exp[\Delta(\nu)\xi]+\int_0^\xi d\xi'\int d\nu\, E(\nu,r)$$
$$\times\exp[\Delta(\nu)(\xi-\xi')]\int dw\, f(w)g(w,\nu)\exp[\Delta(w)\xi']. \tag{8}$$

Here $\tau(\xi=0,\nu)$ describes a solution of the homogeneous equation (2) and is determined by the initial condition $\eta(\xi=0,r)$.

The singularity structure of $g(w,\nu)$ can be found by considering the large-r behavior of $G(\nu,r)=\mathcal{L}\otimes E(\nu,r)$. Because of the exponential decrease of the Bessel function $K_1(x)\propto\exp(-x)$, the integration in (3) is dominated by the two contributions from $\rho_1\lesssim R_c$, $\rho_2\approx r$, $\rho_2\lesssim R_c$, $\rho_1\approx r$. For definiteness, consider the former case. Note that in this regime we have $(\rho_1^2+\rho_2^2-r^2)\approx\rho_1^2$ and $E(\nu,\rho_2)\approx E(\nu,r)$, which gives the contribution of the form $2G_1E(\nu,r)$ to $\mathcal{L}\otimes E(\nu,r)$. Evidently, it corresponds to the singular term $g_1(w,\nu)=2G_1\delta(w-\nu)$. The contribution from the term $\propto\rho_1^2E(\nu,\rho_1)$ to $\mathcal{L}\otimes E(\nu,r)$ does not depend on r at large r and corresponds to $g_2(w,\nu)=G_2(\nu)\delta(w)$. In addition to these singular terms, $g(w,\nu)$ also has a smooth component $g_3(w,\nu)$.

Evidently, the $2G_1\delta(w-\nu)$ component of $g(w,\nu)$ gives contribution to $\eta(\xi,r)$ of the form

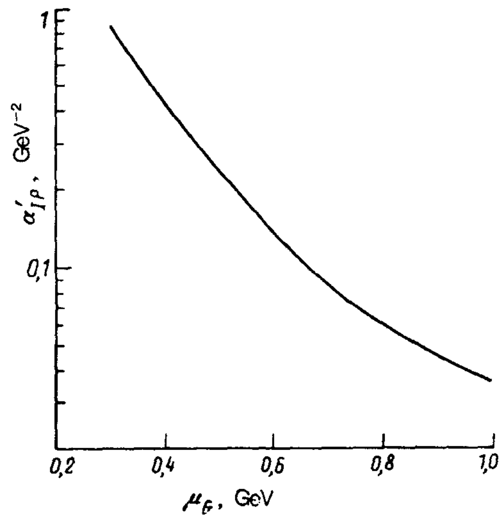

FIG. 1. The slope of the pomeron trajectory α'_{IP} as a function of the inverse correlation radius $\mu_G = 1/R_c$ for the perturbative gluons.

$$\eta_1(\xi,r) = 2G_1 \int_0^{\xi} d\xi' \int d\nu \, f(\nu) E(\nu,r) \exp[\Delta(\nu)\xi] = 2G_1 \xi \sigma(\xi,r), \tag{9}$$

which gives preccisely the Regge growth of the diffraction slope $B(\xi,r)$ with $\alpha'_{\mathrm{IP}} = G_1$. We have an explicit estimate for the slope of the pomeron trajectory

$$\alpha'_{\mathrm{IP}} \sim \frac{3}{16\pi^2} \int d^2 \mathbf{r} \; \alpha_S(r) \mu_G^2 r^2 K_1^2(\mu_G r) \propto \frac{3}{64\pi} R_c^2 \; \alpha_S(R_c). \tag{10}$$

The effect of $g_2(w,\nu) = G_2(\nu,w)$ can be evaluated making use of the explicit form of $E(\nu,r)$ (Refs. 3, 4, and 7). It also contributes to the slope of the pomeron trajectory $\alpha'_{\mathrm{IP}} \sim G_2(0) \sim G_1$. The smooth part of $g(w,\nu)$ does not contribute to the slope of the pomeron trajectory.

In the numerical calculation of the slope α'_{IP} we start with the dipole–dipole cross section and the corresponding diffraction slope, as described in Refs. 5 and 6. We calculate the ξ dependence of the dipole cross section $\sigma(\xi,r)$ and of the diffraction slope $B(\xi,r)$ and verify that, as $\xi \to \infty$, the effective intercept $\Delta_{\mathrm{eff}}(\xi, r) = \partial \log \sigma(\xi,r)/\partial \xi$ and the effective slope $\alpha'_{\mathrm{eff}}(\xi,r) = \partial B(\xi,r)/\partial \xi$ tend to the limiting values Δ_{IP} and α'_{IP}, respectively, which are independent of the size of the projectile and target color dipoles. We take the running coupling with the infrared freezing[3,4] $\alpha_S(r) \le \alpha_S^{(fr)} = 0.82$. The dependence of the slope α'_{IP} on $\mu_G = 1/R_c$ is shown in Fig. 1. The simple estimate (10) is close to these numerical results.

In summary, we have shown that the generalized BFKL pomeron[1–4] is the moving cut in the complex angular momentum plane. We derived a simple analytical estimate (10) for the slope α'_{IP} of the pomeron trajectory and found the dependence of the slope on the gluon correlation radius by an accurate numerical solution of our generalized BFKL equation (2) for the diffraction slope.

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