Dynamics of laser cooling of atoms below the recoil temperature

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A new analytic description is offered for the laser cooling of atoms below the recoil temperature by the method of velocity-selective coherent population trapping. This description is based on a solution of the quantum kinetic equation over the entire range of atomic momenta. In the particular case in which the low rate (Γ_{12}) of the relaxation of the low-frequency atomic coherence is ignored, confirmation is found for analytic results derived in the Levi–Gnedenko statistical theory. The cooling efficiency falls off as $t^{-3/4}$ at times $t \gg \Gamma_{12}^{-1}$. © *1994 American Institute of Physics.*

The cooling of atoms by velocity–selective coherent population trapping makes it possible to achieve temperatures below the recoil limit $T_R = \hbar\omega_R/k_B$, where k_B is the Boltzmann constant, $\omega_R = \hbar k^2/2M$ is the recoil frequency, $\hbar k$ is the momentum of the resonant photons which are absorbed and emitted, and M is the mass of the atom. The width of the atomic wave packet is thus greater than the wavelength of the light acting on the atoms. This width has reached 4.5 μm in the technically most advanced experiment which has been carried out to date.[1] Such deep cooling of atoms makes possible substantial progress in fields such as atomic interferometry, the development of frequency standards, and (potentially) research on quantum-statistics effects, e.g., Bose condensation.

The cooling of atoms by the method of velocity-selective coherent population trapping is based on a combination of two effects:[1-3] At a certain velocity of the atom, there exists a coherent quantum state in which an atom is not excited by the laser light. Atoms in other states, on the other hand, scatter photons in random directions, and in the course of a random walk in momentum space these atoms become trapped in a nonabsorbing state. As time elapses, they accumulate there.

Let us consider an atom with a Λ level scheme, e.g., ^{4}He (the $2^3S_1 - 2^3P_1$ transition). The excited state $|e\rangle$ decays at identical rates γ to the sublevels $|g1\rangle$ and $|g2\rangle$ of a long-lived degenerate metastable state. Other channels for $|e\rangle$ decay are forbidden. We write the operator representing the interaction of the atom with the field of the laser light as

$$\hat{V}(t) = \hbar g[(|e\rangle\langle g1|\hat{T}_{+\hbar k} + |e\rangle\langle g2|\hat{T}_{-\hbar k})\exp(-i\omega t) + \text{H.a.}], \tag{1}$$

where the operator $\hat{T}_g$ represents a displacement (along the z axis) in momentum space.

The effect of this operator on the eigenfunction $|p_z\rangle$ of the operator representing the z projection of the atomic momentum, corresponding to the eigenvalue p_z, is determined by the equation $\hat{T}_q|p_z\rangle=|p_z+q\rangle$. In addition, g is the Rabi frequency, which is the same for the two light–excited transitions. For simplicity we assume that the laser-light frequency $\omega=kc$ is tuned to resonance with the frequency (ω_0) of the electron transition in the Λ scheme when the recoil effect is taken into account:

$$\omega=\omega_0-\omega_R. \tag{2}$$

According to Refs. 3 and 4, after a long duration (t) of the interaction of the atoms with a field of this type, the distribution of atoms with respect to the z projection of the momentum has two narrow peaks, at $p_z=\pm\hbar k$. Their typical width δp falls off in proportion to $t^{-1/2}$, so the effective temperature of the atoms tends toward zero. It was also concluded in Refs. 3 and 4 that the efficiency ϵ, i.e., the relative number of atoms which become involved in the cooling process, tends toward a constant value.

In a real experiment there is always a nonzero rate (Γ_{12}) of the relaxation of the low-frequency atomic coherent (of the off-diagonal element $\langle g1|\hat{\rho}|g2\rangle$ of the atomic density matrix), caused by various factors: fluctuations in the laser light, atomic collisions, a nonuniform magnetic field, etc. When we incorporate Γ_{12}, we find that the width of the peaks tends toward a finite value[5,6]

$$\delta p_\infty=\frac{\hbar k}{2\omega^R}\left(\frac{g^2\Gamma_{12}}{2\gamma}\right)^{1/2} \tag{3}$$

as time elapses. The efficiency at $t\gg\Gamma_{12}^{-1}$ was estimated in Refs. 5 and 6 to be const $\times t^{-1/2}$.

Bardou *et al.*[7] have recently pointed out that the asymptotic method of Ref. 4, which is valid at small p_z, does not yield the correct time dependence $\epsilon(t)$. The approximations adopted in Ref. 4 essentially correspond to the picture of a rate of optical excitation of the atoms which is tending toward a constant value as $p_z\to\pm\infty$. Actually, it falls off as p_z^{-2}, because of the Doppler effect. This point was dealt with in Ref. 7 by using the Levi–Gnedenko statistical theory; the results $\delta p\propto t^{-1/2}$ and $\epsilon\propto t^{-1/4}$ were found.

For the same reasons as in Ref. 4, the method used in Refs. 5 and 6 overestimates the atomic cooling efficiency at $t\gg\Gamma_{12}^{-1}$.

In the present letter we find a long-time solution of the kinetic equation for an ensemble of atoms being cooled by velocity–selective coherent population trapping. This solution is valid over the entire range of projections of the atomic momentum: $-\infty<p_z<+\infty$.

From the elements of the atomic density matrix, $\rho_{ii}(p_z)=\langle i;p_z|\hat{\rho}|i;p_z\rangle$, where $i=g1,g2,e$, which are diagonal in the indices of the electronic state and the translational state, we construct the function

$$w(p_z)=\rho_{g1g1}(p_z-\hbar k)+\rho_{g2g2}(p_z+\hbar k)+\rho_{ee}(p_z), \tag{4}$$

for which the following equation holds:[3]

$$\frac{\partial w(p_z)}{\partial t} = -2\gamma\rho_{ee}(p_z) + \gamma\int_{-\hbar k}^{\hbar k} \Phi(u)[\rho_{ee}(p_z-\hbar k+u)+\rho_{ee}(p_z+\hbar k+u)]du. \quad (5)$$

The kernel $\Phi(u)$ of the integral operation describing the random recoil process is a normalized even function. An explicit expression for this kernel is given in Refs. 3 and 8, among other places. Equation (5) of course preserves the normalization of the density matrix: $\int_{-\infty}^{\infty} w(p_z,t)dp_z=1$ (here and below, we are explicitly writing the time argument of the density matrix). We write the population of the upper level in the form[9,10]

$$\rho_{ee}(p_z,t)=R(p_z)w(p_z,t),$$
$$R(p_z)=\frac{g^2[(kv_z)^2+g^2\Gamma_{12}/2\gamma]}{(kv_z)^4+(g^2+\gamma^2)(kv_z)^2+g^4}, \quad v_z=p_z/M. \quad (6)$$

We first consider the asymptotic behavior of the solution of Eq. (5) as $p_z\to\pm\infty$. In this region (region I), the semiclassical approximation is valid, and Eq. (5) becomes a Fokker–Planck equation without a fourth term:

$$\frac{\partial w_{\rm I}(p_z,t)}{\partial t}=\frac{\partial^2}{\partial p_z^2}[D_{zz}w(p_z,t)],$$
$$D_{zz}=\gamma(\hbar k)^2(1+\varphi)R(p_z), \quad \varphi=\frac{1}{(\hbar k)^2}\int_{-\hbar k}^{\hbar k}\Phi(u)u^2du. \quad (7)$$

For the 2^3S_1–2^3P_1 transition of ^{4}He which was used in the experiments of Refs. 1 and 2, we would have $\varphi=2/5$. At $|p_z|\gg\hbar k\gamma/\omega_R$, the diffusion coefficient becomes

$$D_{zz}=B/p_z^2, \quad B=\gamma(\hbar gM)^2(1+\varphi). \quad (8)$$

Equation (7) with diffusion coefficient (8) has the following solution for the case of an even initial condition $w_0(p_z)=w_0(-p_z)$:

$$w_{\rm I}(p_z,t)=\int_0^\infty d\sigma\,\exp(-4B\sigma^2t)p_z^{5/2}J_{-1/4}(p_z^2\sigma)\sigma$$
$$\times\int_0^\infty d\xi w_0(\xi^{1/2})\xi^{-1/4}J_{-1/4}(\xi\sigma). \quad (9)$$

Here $J_\nu(x)$ is the Bessel function of the first kind. At small values of σ (corresponding to the limit under consideration here, of a long interaction time), we retain only the first time of the series expansion of the Bessel function: $J_\nu(x)\approx(x/2)^\nu/\Gamma(\nu+1)$, where $\Gamma(x)$ is the gamma function. We then find a limitation on the range of applicability of this asymptotic solution:

$$t\gg p_0^4/B, \quad (10)$$

where p_0 is the characteristic width of the initial distribution $\omega_0(p_z)$. We can now easily evaluate the integral over σ in (9); we find

$$w_{\rm I}(p_z,t)\approx\frac{2^{5/4}Np_z^2\exp(-p_z^4/16Bt)}{\Gamma(3/4)(8Bt)^{3/4}}, \quad (11)$$

where $2N$ is the relative number of atoms which have gone off to the region of large momenta. This fraction tends toward one as $t\to\infty$. The function $w_{I}(p_z,t)$ has two, symmetrically positioned local maxima, at $p_z=\pm(8Bt)^{1/4}$; it falls off sharply at larger values of $|p_z|$. The condition for the applicability of expression (8) imposes yet another restriction [along with (10)] on the applicability of expression (11):

$$t\gg\frac{1}{B}(\hbar k\gamma/\omega_R)^4. \tag{12}$$

The asymptotic behavior in region I must be joined with the asymptotic behavior in the inner region, II (the union of these two regions constitutes the entire set of values $-\infty< p_z<+\infty$). Under condition (12), there is an overlap of the regions in which the asymptotic behavior is found by the different methods. The joining of these results in the common region [at $|p_z|$ on the order of $(Bt)^{1/4}$], in which Eq. (7) (of the diffusion type) holds, is carried out on the basis of both the value of the function $w(p_z,t)$ and the flux

$$j_z=-\frac{\partial}{\partial p_z}(D_{zz}w).$$

The fact that we have $(j_z/w)\to 0$ as $t\to\infty$ in the joining region indicates that region II may also contain a small interval $|p_z|\ll\hbar k$, for which the asymptotic behavior should be of the type const $\times t^{-3/4}/R(p_z)$.

Throughout region II, including the interval $|p_z|\ll\hbar k$, a semiclassical description must be replaced by the use of quantum equation (5). Let us assume $t\gg g^2/\omega_R^2\gamma$ or $\tau\gg\Gamma_{12}^{-1}$. Under these conditions, the integral term on the right side of (5) is obviously[3,4,6] a weak function of p_z. On the other hand, its time dependence is determined by the behavior of the asymptotic results in (11), according to the joining condition. Equation (5) becomes

$$\begin{gathered}\frac{\partial w_{II}(p_z,t)}{\partial t}=-2\gamma R(p_z)w_{II}(p_z,t)+Bt^{-3/4},\\ \beta=\frac{NB^{1/4}}{\Gamma(3/4)(\hbar k)^2(1+\varphi)}.\end{gathered} \tag{13}$$

Since the particular form of the initial distribution $w_0(p_z)$ does not affect the asymptotic behavior in region II as $t\to\infty$, we can write a solution of Eq. (13) in the form

$$w_{II}(p_z,t)=\beta\int_0^t\exp[-2\gamma R(p_z)(t-\tau)]\tau^{-3/4}d\tau. \tag{14}$$

The integral in (14) can be evaluated easily:

$$w_{II}(p_z,t)=4\beta t^{1/4}\exp[-2\gamma R(p_z)t]F[1/4,5/4,2\gamma R(p_z)t], \tag{15}$$

where $F(\alpha,\gamma;x)$ is the confluent hypergeometric function. Using the asymptotic representations of this function at large and small values of x (Ref. 11), we find

$$w_{II}(p_z,t)\simeq\beta t^{-3/4}/2\gamma R(p_z),\quad t\gg[2\gamma R(p_z)]^{-1}. \tag{16}$$

At $p_z=0$, the function $w_{\rm II}(p_z,t)$ has a sharp peak, whose width is determined by expression (3):

$$w_{\rm II}(p_z,t)\simeq \frac{\beta t^{-3/4}}{\Gamma_{12}[1+(\delta p/\delta p_\infty)^2]},$$
$$t\gg\Gamma_{12}^{-1},\quad k|v_z|\ll g^2/\gamma. \tag{17}$$

At short times, the peak is in a process of formation:

$$w_{\rm II}(p_z,t)\simeq 4\beta^{1/4}[1-\tfrac{8}{5}\gamma R(p_z)t],$$
$$t\leqslant[2\gamma R(p_z)]^{-1}. \tag{18}$$

The single peak in the function $W_{\rm II}(p_z,t)$ at $p_z=0$ corresponds to two peaks, at $p_z=\pm\hbar k$, on the real distribution $f(p_z,t)=\rho_{g1g1}(p_z)+\rho_{g2g2}(p_z)+\rho_{ee}(p_z)$ (Refs. 3–6).

The cooling efficiency at $t\gg\Gamma_{12}^{-1}$ and $t\gg(\beta\delta p_\infty/\Gamma_{12})^{4/3}$ is

$$\epsilon=\pi\beta\delta p_\infty t^{-3/4}\Gamma_{12}^{-1}, \tag{19}$$

instead of the overestimate in Refs. 5 and 6.

In the case $\Gamma_{12}=0$, we find from (18) that the width of the peak in the distribution function can be estimated to be

$$\delta p=\alpha_0\frac{\hbar k}{2\omega_R}\left(\frac{g^2}{2\gamma t}\right)^{1/2},$$

and the efficiency can be estimated to be

$$\epsilon=\epsilon_0\beta t^{-1/4}\frac{\hbar k}{2\omega_R}\left(\frac{g^2}{2\gamma}\right)^{1/2},$$

where the coefficients α_0 and ϵ_0 are on the order of one. A direct solution of the kinetic equation for the case $\Gamma_{12}=0$ has thus confirmed the results found from the statistical calculations.[7]

If we adopt a model which is not realized in practice, $R(p_z)\to R_m\neq 0$, $D_{zz}\to D_m$ as $p_z\to\pm\infty$ (this is model II of Ref. 7), we find the asymptotic form of the solution at large momentum to be

$$w_{\rm I}(p_z,t)\approx\frac{2N\exp(-p_z^2/4D_mt)}{(4\pi D_mt)^{1/2}}.$$

From the joining conditions, we find the time evolution of the efficiency to be $\epsilon\propto t^{-1/2}$ at $t\gg\Gamma_{12}^{-1}$ (Refs. 5 and 6) or $\epsilon\approx$ const at $\Gamma_{12}=0$, as $t\to\infty$ (Ref. 4).

The general formula for the solution of Eq. (5) as $t\to\infty$—a formula which holds for $-\infty<p_z<+\infty$ and which combines all the cases discussed above—is

$$w(p_z,t)=4\beta t^{1/4}F[1/4,5/4;2\gamma R(p_z)t]\exp[-(2\gamma R(p_z)t+p_z^4/16Bt)]. \tag{20}$$

The analytic solution in (20) gives a good description of the results of a semiclassical numerical calculation.[9,10]

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