

Coherent transport of single cooper pairs in Josephson arrays

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One-dimensional arrays of small Josephson junctions have been studied. The Josephson energy E_J was assumed to be small compared to the Coulomb energy E_C . The dynamics of this system can be described in terms of quantum particles (Cooper pairs) that propagate coherently through the array. By applying the voltages to the gates coupled capacitively to the electrodes of the junctions one can add Cooper pairs into the array one by one. By changing the gate voltages in time one can inject single Cooper pairs into the array. An injected Cooper pair propagates through the array as a wave packet. For a ring-shaped array pierced by a magnetic flux the critical current can be carried by one Cooper pair. The critical current may decrease with increasing number of Cooper pairs in the array. © 1994 American Institute of Physics.

The investigation of low-dimensional Josephson junction arrays currently attracts interest of theoreticians and experimentalists (see Refs. 1 and 2 for reviews). Considerable theoretical efforts have been directed toward the study of the equilibrium phenomena in infinite arrays. In particular, it has been shown that the point of a superconductor–insulator transition depends on the electrochemical potential μ of the array³ (the latter can be controlled by applying the voltage to the gate underneath the array).

The insulating phase occurs due to the Coulomb gap for mobile excitations. We consider arrays with a small Josephson energy, $E_J \ll E_C$ and $E_C = (2e)^2/2C$, where these excitations are extra Cooper pairs at the electrodes of the junctions. We investigate first a ring-shaped Josephson array pierced by a magnetic flux. This system begins showing superconducting properties when the first mobile excitation appears. Even in the absence of mobile excitations at the equilibrium (in the insulating phase), we can inject a Cooper pair into a linear array by applying time-dependent voltages (V_1 and V_2 in Fig. 1) to the gates near its edge. The coherent propagation of an injected Cooper pair through the array is another subject of our investigation.

We write the Hamiltonian of the array in the charge representation:

$$H = -\frac{E_J}{2} \sum_{i=1}^N (|m_i - 1, m_{i+1} + 1\rangle e^{2\pi i \Phi / N \Phi_0} \langle m_i, m_{i+1}| + \text{H.c.}) + U(m_1, \dots, m_N), \quad (1)$$

An excess number m_i of Cooper pairs on the electrode i determines its charge $q_i = 2em_i$.

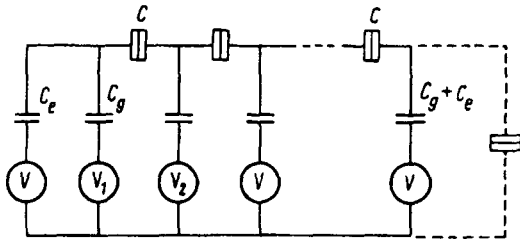


FIG. 1. Linear array of Josephson junctions. $C_e = (C_0 - C_g)/2$ is an equivalent capacitance of a semiinfinite array. "Dashed" tunnel junction has very small conductance and capacitance.

The Josephson term contains the magnetic flux Φ that pierces a circular array ($\Phi_0 = h/2e$). The second term describes the Coulomb energy² of a given charge configuration (m_1, \dots, m_N),

$$U = \frac{1}{4e} \sum_{i,j=1}^N q_i u(i-j) q_j - V \sum_{i=1}^N q_i. \quad (2)$$

Here $u(i-j) = e(CC_g)^{-1/2} \exp(-|i-j|/\lambda)$ (for $C \gg C_g$, see Fig. 1) is the potential of the electrode i created by a Cooper pair sitting on the electrode j . The gate voltage V (see Fig. 1) plays a role of the electrochemical potential of the array.

1. Ring-shaped array. For simplicity we assume that the screening radius of the Coulomb interaction $\lambda = (C/C_g)^{1/2}$ is large, $\lambda \gg N$. The number of extra Cooper pairs, $M = \sum_i m_i$, in the ground state can be controlled by a gate voltage, $M \approx [C_g V N / 2e]$, where $[x]$ denotes the integer number nearest to x .

Even if there are no extra Cooper pairs in the array, $M = 0$, an exponentially small Josephson current,

$$I_J(\Phi) = \frac{2e\delta}{\hbar} \sin\left(2\pi \frac{\Phi}{\Phi_0}\right), \quad \delta = \frac{N^N E_J}{(N-1)!} \left(\frac{E_J}{2E_C}\right)^{N-1}, \quad (3)$$

still flows as a result of the virtual tunneling of the "background" Cooper pairs around the ring.

A single extra Cooper pair, $M = 1$, behaves like a free particle in an empty band, $E^{(0)}(k, \Phi) = -E_J \cos(k - 2\pi\Phi/N\Phi_0)$, $|k| < \pi$, for $E_J \ll E_C/N$. The virtual tunneling of the "background" Cooper pairs renormalizes (increases) its tunneling amplitude, $E_J \rightarrow E_J [1 + NE_J/E_C + O((NE_J/E_C)^2)]$. Taking into account the virtual tunneling around the ring, we find that the ground state energy and the Josephson current are Φ_0 periodic functions of the magnetic flux,

$$I_J(\Phi) = \frac{4\pi e E_J}{\hbar N^2} \left\{ \frac{\Phi}{\Phi_0} \right\}, \quad (4)$$

where $\{x\} \equiv x - [x]$ denotes a fractional part of x . The ground state is separated from the first excited state by a gap, which reaches its minimum value $\approx \delta(3)$ near the degeneracy point $\Phi = \Phi_0/2$. In a nonideal system the potential backscattering of an extra Cooper pair increases the gap which stabilizes the Φ_0 -periodicity of the Josephson current.

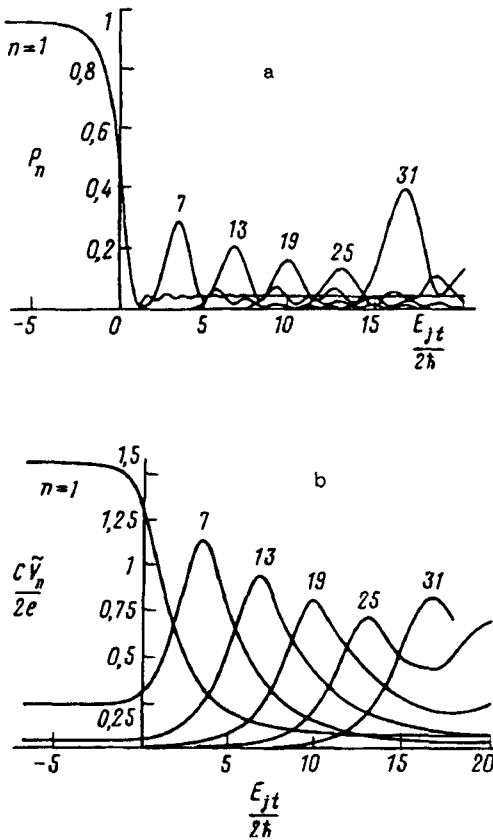


FIG. 2. *a*—The probabilities P_n for a Cooper pair (*a*) and *b*—the time-dependent components V_n of the voltages (b) on the electrodes for the array with 30 junctions (\tilde{V}_1 includes only a net voltage due to an extra Cooper pair). $C/C_g=10$, $d\tilde{U}/dt=0.5E_J^2/\hbar$.

If there are two extra Cooper pairs, $M=2$, a strong Coulomb interaction keeps them at a distance $N/2$. In the limit $E_J \ll E_C/N$, they move like a rigid object with a tunneling amplitude NE_J^2/E_C (for even N). Therefore, the Josephson current in the ground state is smaller than that found from (4) by a factor $4NE_J/E_C \ll 1$.

The Josephson current I_J decreases further with an increase in the number M of extra Cooper pairs until M reaches $N/2$. Then I_J begins to increase, obeying an “electron–hole” symmetry, $I_J(M) = I_J(N - M)$. Accordingly, the Josephson current depends *stepwise* on the gate voltage. The steps correspond to an addition of new extra Cooper pairs to the array.

2. Linear array (Fig. 1). For small gate voltages $|V| < e/C_0$ there are no extra Cooper pairs deep in the array in the equilibrium [here $C_0 = 2(CC_g + C_g^2/4)^{1/2}$ is the self-capacitance of the electrodes]. Applying the voltages $V_{1(2)} = V + V_{1(2)}$ to gates 1 and 2, we can control the number of Cooper pairs m_1 on the leftmost electrode. If $\tilde{V}_1/\tilde{V}_2 = -(C_0 + C_g)/2C - 1$, the voltages $V_{1(2)}$ do not polarize charges inside the array, and $m_1 = [(\tilde{V}_2 C_g / C - V) C_0 / 2e]$ in the equilibrium.

We assume that the initial state is the equilibrium ground state $(m_1, \dots, m_N) = (1, 0, \dots, 0)$. Its Coulomb energy (see Ref. 4) U_1 differs from the energies

U_i of the other states ($0, \dots, m_i=1, 0, \dots$) by $\bar{U} = U_1 - U_i = -2e\bar{V}_2 C_g / C$. Decreasing V_2 (and increasing V_1) in time, we can inject a Cooper pair into the array. The injection occurs at $|\bar{U}| \leq E_J \ll E_C$. In general, the initial state ($1, 0, \dots, 0$) fails to be a true ground state of the system before the injection starts. Our line of reasoning is therefore valid if the relaxation of the charge of the array (e.g., due to the tunneling of electrons through the “dashed” junction in Fig. 1) is slow on a time scale $(d\bar{U}/dt)^{-1} e^2 / C_0$ of the voltage sweeping.

The injection is analogous to the Zener tunneling from a localized level into an energy band. It occurs with the probability close to unity, if one sweeps the voltages slow enough, $d\bar{U}/dt \leq E_J^2 / \hbar$. An injected Cooper pair propagates through the array as a wave packet with the group velocity E_J / \hbar (Fig. 2a). Coherent propagation of a Cooper pair through the array can be detected (in principle) by measuring the time-dependent components of the voltages on the electrodes (Fig. 2b) or their correlations.

The observation of the effect in real systems can be hindered by a disorder in the Josephson and Coulomb energies (the latter may be due to polarization of random offset charges on the electrodes). Another factor—the presence of quasiparticles—can be excluded (for $e^2 / 2C_0 < \Delta$, where Δ is the superconducting gap), as was demonstrated in recent experiments.⁵

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¹J. E. Mooij and G. Schön, in *Single Charge Tunneling*, eds. H. Grabert and M. H. Devoret, NATO ASI Series, Vol. 294, Plenum Press, New York, 1992.

²D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. L. Altshuler, P. A. Lee, and R. Webb (North-Holland, Amsterdam, 1991).

³M. P. A. Fisher *et al.*, Phys. Rev. B **40**, 546 (1989); M. Wallin *et al.*, Phys. Rev. B **49**, 12115 (1994).

⁴For nonuniform gate voltages the charges q_i in Eq. 2 should be defined as $q_i = 2em_i + C_g \bar{V}_i$.

⁵P. Lafarge *et al.*, Nature (London) **365**, 422 (1993).

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