Electromagnetic (Schwinger) scattering of fast neutrons in crystals

A. N. Dyumin, I. Ya. Korenblim, V. A. Ruban, and B. B. Tokarev
B. P. Konstantinov Institute of Nuclear Physics, USSR Academy of Sciences

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It is shown that for fast neutrons incident at a small angle ($\sim 1'$) to the crystallographic axis of a perfect single crystal, the contribution to the scattering cross section due to electromagnetic interaction increases by several factors of ten, because of interference effects.

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When fast particles (whose wavelength λ is much smaller than the lattice period d) pass through a crystal, an interference enhancement of the electromagnetic interaction processes can occur.

Such phenomena have been widely studied theoretically and experimentally in connection with the investigation of the photon bremsstrahlung and electron-positron pair creation processes as well as the elastic scattering of ultra-relativistic electrons in crystalline targets (see Ref. 1, where a complete bibliography on these problems is given).

The scattering of fast ($\lambda \ll d$) charged particles with a momentum $\mathbf{p} = \hbar\mathbf{k}$ by a heavy atom of size $a = a_0 Z^{-1/3}$ ($a_0 = \hbar^2/me^2$ is the Bohr radius, and Z is the nuclear charge), which has a highly pronounced forward directionality, occurs primarily in the region of small angles $\theta \sim (ka)^{-1} \sim \lambda/a \ll 1$. In this case the characteristic transferred momentum $q \sim q_\perp \sim k\theta \ll d^{-1}$ is almost at right angles to k, while its longitudinal component $q_\parallel k\theta^2 \sim (ka^2)^{-1} \ll d^{-1}$ decreases with increasing energy. Consequently, the atoms along the direction of particle motion scatter coherently within the limits of the effective length $l \sim q_\parallel^{-1} \gg d$. As a result, the total scattering cross section increases considerably and depends on the direction of particle entry into the crystal relative to its axes.

Here, we are going to shown that analogous interference effects characteristic of charged particles should also appear when fast neutrons (with an energy ~MeV) pass through a crystal. The slowly decaying electromagnetic interaction of the magnetic moment of a neutron $\boldsymbol{\mu}_n = \frac{\gamma_n}{2}(e\hbar/2M_n c)\boldsymbol{\sigma}$ with the electric field of the atom $\mathbf{E} = -\frac{\mathbf{R}}{R}\frac{d\phi}{dR}$ plays the key role in this case[2]:

$$U = -\gamma_n\left(\frac{e\hbar}{2M_n^2c^2}\right)\vec{\sigma}[\mathbf{E}\,\mathbf{p}] = \gamma_n\frac{\mu_n}{M_n c}\frac{1}{R}\frac{d\phi}{dR}(\vec{\sigma}\mathbf{L}), \tag{1}$$

where $\gamma_n = 1.91$, and M_n, $\boldsymbol{\sigma}$, and $\mathbf{L} = \mathbf{r}\times\mathbf{p}$ are the neutron mass, spin, and orbital moment. This weaker-than-nuclear electromagnetic interaction of the neutron leads to Schwinger scattering, which predominates in the small-angle region and is described in the Born approximation by the differential cross section[2]:

$$d\sigma_s = |f(q)|^2 d\Omega = \frac{\gamma_n^2}{4}\left(\frac{e^2}{M_n c^2}\right)^2[Z - F(q)]^2\cot^2\frac{\theta}{2}\,d\Omega\,. \tag{2}$$

Here $q = |\mathbf{k}' - \mathbf{k}| = 2k\sin(\theta/2)$ is the scattering vector and $F(q)$ is the atomic form factor which describes the screening of the nuclear Coulomb field by the electrons in the atom.

When the neutrons are incident almost parallel to the crystallographic axis (at angles $\theta < d/l \sim \frac{1}{ka}\frac{d}{a} \ll 1$), the scattering by the individual atom chains can be analyzed separately and the interference only along an individual chain must be taken into account.[3] Thus, the interference part of Schwinger scattering per crystal atom can be written in the form[1)]

$$\frac{d\tilde{\sigma}_s}{d\Omega} = e^{-2W}|f(q)|^2\frac{2\pi}{d}\sum_n \delta\left(\mathbf{q}\mathbf{e} - \frac{2\pi}{d}n\right), \tag{3}$$

where $W(q) = \frac{1}{2}q^2\bar{u}^2$ is the Debye-Waller factor, $\bar{u}^2$ is the mean square of the thermal fluctuations of the atoms in the crystal, $\mathbf{e}$ is a unit vector in the chain direction, $n = 0$, ± 1,.... In the calculation of the total cross section $\tilde{\sigma}_s$ we can restrict ourselves to the principal term with $n = 0$ in Eq. (3). The contribution of the remaining terms to the total cross section is small in terms of the parameter $(d/a)(1/4\pi ka) \ll 1$. Thus, for the total cross section we obtain

$$\tilde{\sigma}_s = \frac{8\pi}{kd} \int_0^{2k\theta} dq \, | f(q) |^2 \frac{e^{-q^2 \overline{u^2}}}{\sqrt{4k^2\theta^2 - q^2}} . \tag{4}$$

This cross section corresponds to the scattering of neutrons by the uniform average potential of the atom chain $U(\rho) = \frac{1}{d}\int_{-\infty}^{\infty} U(\sqrt{\rho^2 + z^2})dz$, since the momentum transfer along the chain is equal to zero.

A screening of the nuclear Coulomb field by the electron cloud and the thermal vibrations of the lattice atoms limit the effective region of transferred momenta, which a significant coherent enhancement of the scattering occurs:

$$a^{-1} < q < (\overline{u^2})^{-1/2} .$$

Therefore, to estimate the cross section $\tilde{\sigma}_s$, we can integrate only in this interval in (4) after setting $e^{-q^2\overline{u^2}} \approx 1$, and $F(q) \approx 0$. Thus, we have

$$\frac{\tilde{\sigma}_s}{\sigma_s} = \frac{a}{d}\left(\ln\frac{a}{R_{\text{я}}}\right)^{-1} \frac{\sqrt{4k^2a^2\theta^2 - 1}}{ak\theta^2} , \tag{5}$$

where σ_s is the Schwinger scattering cross section by an individual atom and R_{nuc} is the nuclear radius at which the potential (1) is truncated from below in the calculation of σ_s. The coherent enhancement reaches a maximum for a beam incidence angle of $\theta_{max} \sim (\sqrt{2k}\, a)^{-1}$ relative to the chain:

$$\left(\frac{\tilde{\sigma}_s}{\sigma_s}\right)_{max} \approx \frac{a}{d} \frac{2ka}{\ln(a/R_{nuc})} . \tag{6}$$

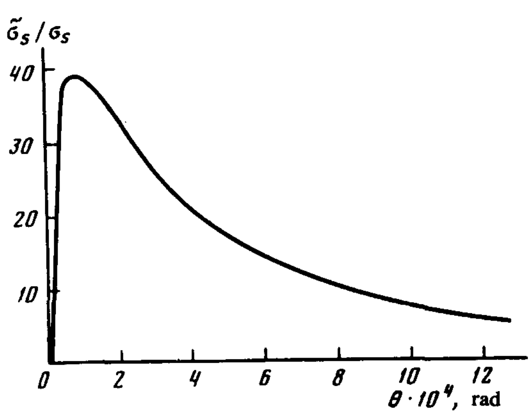

FIG. 1. Dependence of the total Schwinger scattering cross section of neutrons with an energy $E = 1$ MeV on the entrance angle θ relative to the crystallographic axis for tungsten ($T_D = 384$ K) at a temperature of $T = 300$ K. Screening of the nuclear field was determined by using the Thomas-Fermi method $\sigma_s = 1.6 \times 10^{-26}$ cm^2.

This formula has an obvious physical meaning. The cross section (6) corresponds to summation of the scattering amplitudes along the coherence length $l \sim q_{\parallel}^{-1} \sim ka^2$. The logarithmic factor is due to faster than Coulomb decay of the Schwinger potential (1), so that a wide range of angles contributes to σ_s, rather than just $\theta \sim (ka)^{-1}$.

For neutrons with an energy $E = 1$ MeV ($k \approx 2 \times 10^{12}$ cm^{-1}) the Schwinger scattering in heavy-element crystals such as tungsten ($d/a \approx 25$, $a/R_{\rm nuc} \approx 10^3$) should increase by a factor of 20–30 for $\theta_{\max} \approx 1'$.

The result of a numerical calculation from the exact formula (4) for tungsten is shown in Fig. 1, in which it can be seen that the total Schwinger scattering cross section in the crystal at the maximum, which is 40 times greater than that for an individual atom, is comparable to the nuclear scattering cross section of neutrons.[2)] Therefore, the discussed effect can be detected from the orientation dependence of the intensity of a neutron beam that passes through a single crystal. To do this, we need rather perfect crystals (with a small mosaic defect of $< 1'$) of heavy elements, which have a comparatively high Debye temperature, so that $\sqrt{\overline{u^2}} \ll a$.

[1)]The screening effect[4] can be ignored because of the weak electromagnetic interaction of the neutrons.
[2)]We note that the entire neutron beam scattered by the single crystal acquires a slight polarization ($\sim 10^{-5}$) parallel to the chain axis.

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