Elimination of divergences in the integral formulation of the Yang–Mills theory

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It is shown that in the four-dimensional Yang–Mills theory the divergences in the expression $z\langle P\ \exp\{g\int_\Gamma A_\mu dx^\mu\}\rangle$ for a smooth non-self-intersecting contour Γ can be eliminated by renormalizing the coupling constant g and the factor z. Anomalies arise in the three-dimensional case.

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An integral formulation of the gauge theory, based on the concept of the phase factor $g(\Gamma) = P\exp\{g\int_\Gamma A_\mu dx^\mu\}$, was proposed by Yang.[1] Analysis of the dynamics of $g(\Gamma)$ introduces the concept of a chiral field at the contour.[2] The higher conservation laws in the three-dimensional Yang–Mills theory[2,3] were formulated in these terms, and thre are reasons to assume that in perturbation theory for contour variables it is possible to obtain string-type excitations,[2–6] and hence determine the Wilson criterion for quark containment.[7]

The perturbation theory for contour variables, like for the usual Green's functions, can be constructed by integration of the corresponding Schwinger equations, and the iteration method, which gives qualitatively new answers, must apparently be different from perturbation theory with respect to the coupling constant. The equations, however, must satisfy the ordinary perturbation theory. To write such equations, we must show that

$$z < \mathcal{g}(\Gamma) > = z < P\exp\{g\int_0^1 A_\mu(x(s))x'_\mu(s)ds\}>$$

is finite at least within the framework of the standard perturbation theory. This paper is devoted to solving this problem, which was analyzed in Refs. 8 and 9 in the first orders of perturbation theory.

The main difficulty in renormalization of the expression $\langle g(\Gamma)\rangle$ lies in the fact that this quantity is nonlocal, whereas the usual R-operation theory is tailored to work with local objects. It was suggested[8] that $\langle g(\Gamma)\rangle$ can be written by using a local interaction of the Yang–Mills field with one-dimensional fermions, which "live on the contour" Γ:

$$< \mathcal{g}_{\alpha\beta}(\Gamma) > = N\frac{\delta^2}{\delta\bar\lambda_\beta(1)\delta\lambda_\alpha(0)}\int\exp\left\{\int d^Dx\left[\frac{1}{8}\operatorname{tr}F_{\mu\nu}^2 + \frac{1}{4b}\operatorname{tr}(\partial_\mu A_\mu)^2\right.\right.$$

$$\left.\left. + \int_0^1 ds[\bar\psi(s)\partial_s\psi(s) + g\bar\psi(s)A_\mu(x(s))x'_\mu(s)\psi(s) + i\bar\lambda\psi + i\bar\psi\lambda\right]\right\}\Delta(A)$$

$$\times\prod_x dA\prod_s d\bar\psi d\psi\,|_{\lambda=\bar\lambda=0},\quad F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu + g[A_\mu, A_\nu],\quad \bar\lambda\psi = \bar\lambda_i\psi_i, \tag{1}$$

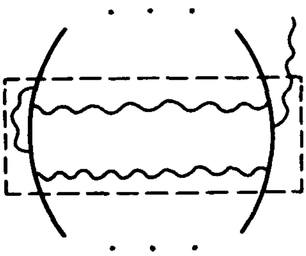

FIG. 1.

see Ref. 10 for the notations. The validity of Eq. (1) can be proved by performing an integration over the fields $\bar{\psi}$, ψ and assuming that $\partial_s^{-1} = \theta(s)$, and $\theta(0) = 0$.

The diagram technique of perturbation theory with respect to g for the functional (1) is comprised of the lines of the vector field A_μ (wavy lines), the lines of the hosts $\bar{c}$, c (dashed lines), as well as the liens of the one-dimensional fermion propagators $G(s-s') = \theta(s-s')$ (solid lines). In the described diagram technique, in addition to the ordinary divergences in the Green's functions of the local vertices, which are eliminated by ordinary renormalization, there are divergences in the diagrams with internal lines that terminate at the contour. First, we shall examine diagrams containing no local vertices (like those in Fig. 1). To study them, it is convenient to use in the gluon lines $D_{\mu\nu}$ the effective α representation:

$$\mathcal{D}_{\mu\nu}(q,s) = \delta_{\mu\nu}\int d\eta \int_0^\infty \frac{da}{(a)^{D/2}} \exp\left\{ iq\eta - \frac{|x(s) - x(s+\eta)|^2}{4a} \right\} \text{ for } b = 1,$$

and to assume that $D = n - 2\epsilon (n = 3, 4)$. Assuming that the $x_\mu(s)$ contour is non-self-intersecting and infinitely smooth, $D_{\mu\nu}(q,s)$ can be represented in the form

$$\mathcal{D}_{\mu\nu}(q,s) = \delta_{\mu\nu}\int_0^\infty da \frac{a^{-\frac{D-1}{2}}}{|x'(s)|}\left\{ 1 + \sum_{n\geq 1} \frac{1}{n!}\left[\frac{1}{a}\sum_{k\geq 4} a_k\left(\frac{\partial}{\partial q}\right)^k\right]^n \right\} \times \exp\left\{ -\frac{q^2 a}{|x'(s)|^2} \right\} \quad (2)$$

for $|x'| = \text{cosnt}$.

To investigate the ultraviolet behavior of the graphs, it is convenient to replace the Fourier transform $G(q)$ of the Green's function $G(s)$ by its asymptotic form

$$G_{as}(q) = \int_0^\infty a^{-1/2} e^{-a q^2} da \, .$$

The general structure of the asymptotic form of any subgraph like that in Fig. 1 has the form

$$\int \prod_l dq_l d\alpha_l \, e^{-\sum_l q_l^2 \alpha_l} \prod_{l\in\{l_\psi^{int}\}} (\alpha_l)^{-1/2} \prod_{l\in\{l_A^{int}\}} (\alpha_l)^{-\frac{D-1}{2}} \prod_{v_{int}} \delta(\;)\,[1+\ldots]. \tag{3}$$

Here $l_\psi^{\rm int}$ are the internal fermion lines, $1_A^{\rm int}$ are the internal boson lines, $v_{\rm int}$ are the internal vertices, and δ is a function describing the one-dimensional momentum conservation law. Calculating the total pole λ after replacing the variables $\alpha_i = \lambda\,\xi_i$, $\Sigma\,\xi_i = 1$, and $\overline{\lambda q} = Q$, we obtain $(L_A^{\rm ext} - 3)/2$ for $D = 4$ and $(L_A^{\rm int} + L_A^{\rm ext} - 3)/2$ for $D = 3$. The terms ... in (3) contribute nothing to the whole negative λ powers. Thus, at $D = 4$ the diagrams without external A_μ lines and with one external A_μ line diverge, i.e., the necessary counterterms have the form $z_{\psi k+1}\ \bar\psi\partial_s\psi(\bar\psi\psi)^k$ and $z_{Ak+1}\ \bar\psi A_\mu x'_\mu \psi (\bar\psi\psi)^k$. The counterterms for $k > 0$ are anomalous [they are missing in the original expression (1)], but they contribute nothing, since each diagram has a loop comprised of ψ lines.

The graphs, containing both local vertices on the contour, are examined analogously.

According to the logic[10] of the finite local Green's functions, we can show that the expression

$$\int \exp\Big\{\int d^D x\Big[\frac{1}{8}\mathrm{tr}\, z_2(\partial_\mu A_\nu - \partial_\nu A_\mu + z_1 z_2^{-1} g\,[A_\mu A_\nu])^2 + \frac{1}{4b}\mathrm{tr}(f(\Box)\,\partial_\mu A_\mu)^2 - \tilde z_2(\bar c\,\Box c - \tilde z_1 \tilde z_2^{-1} g\bar c\,\partial_\mu(A_\mu c))\Big] + \int_0^1 ds\,[z_{2\psi}(\bar\psi\partial_s\psi + z_{2\psi}^{-1} z_{1\psi} g\bar\psi(s)A_\mu \times x'_\mu(s)\psi(s)) + i\bar\lambda\psi + i\bar\psi\lambda\Big]\Big\}\prod_x dA_\mu d\bar c\, dc \prod_s d\bar\psi d\psi$$

for the conditions $\tilde z_2^{-1}\tilde z_1 = z_{2\psi}^{-1} z_{1\psi} = z_2^{-1} z_1$ is finite after removal of regularization. For this it is sufficient to use the Ward identity of the form

$$\frac{1}{b} f^2(\Box)\,\partial_\mu^x \frac{\delta Z_R}{i\delta J_\mu^a(x)} = \int dy\,\mathcal{D}_0(x-y)\,\partial_\mu J_\mu^a(y)\, Z_R + \int dy\, J_\rho^{tr\,b}(y)\, t^{bcd} \times g\tilde z_1 \frac{\delta}{i\delta J_\rho^c(y)} G_R^{da}(y, x, J, \lambda) + \int ds\, \tilde z_1 g\Big[\bar\lambda_i(s) T_{ij}^b \frac{\delta}{i\delta\bar\lambda_j(s)} - \lambda_i(s) T_{ij}^b \frac{\delta}{i\delta\lambda_j(s)}\Big] G_R^{ab}(y, x, J, \bar\lambda, \lambda) = 0$$

and to show the finiteness of the diagram in Fig. 2.

Thus, we showed that the quantity $z(\epsilon,g)\langle P\exp\{g_0(\epsilon,g)\int_\Gamma A_\mu dx^\mu\}\rangle$ is finite as $\epsilon\to 0$, $n = 4$.

At $n = 3$ a counterterm, which appears only in the second order, has the form $m(\epsilon)g^2|x'|\bar\psi(s)\psi(s)$. This leads to a replacement of the θ function by the Green's function $G_m(s,s') = \theta(s-s')\exp\{-m(\epsilon)g^2\int_s^{s'}|x'|d\tau\}$ of the operator $\partial_s + m(\epsilon)g^2|x'(s)|$, i.e., the final quantity as $\epsilon\to 0$, and $n = 3$ is

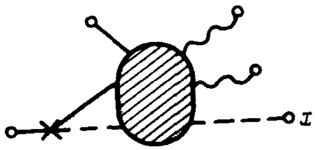

FIG. 2

$$\exp\left\{ -m(\epsilon)\, g^2 \int_0^1 |x'(s)|\, ds \right\} \langle P\exp\left\{ g\int_0^1 A_\mu dx^\mu \right\} \rangle .$$

The appearance of these "mass" fermion anomalies can lead to a modification of the variational derivative quantum equation for the renormalized contour variables.

An investigation of the renormalized functional of the surface, suggested in Ref. 11, is an interesting problem.

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