Influence of the $n\bar{n}$ channel on the properties of a proton-antiproton atom

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The influence of the second channel ($n\bar{n}$) on nuclear level shifts and the probabilities of radiative transitions in the $p\bar{p}$ atom are determined. The obtained formulas can also be used for other hadronic atoms ($\Sigma^- p$, $K^- p$, etc.).

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A formula, which relates the nuclear shifts of the s levels of a hadronic atom to the length of scattering by the strong-interaction potential, was recently obtained.[1] It shows[1)] that the $p\bar{p}$ system can have a bound state with a binding energy $\epsilon \approx 1$ MeV, a width $\Gamma \lesssim 200$ keV, an orbital moment $l = 0$, and spin $S = 0$ or 1 (the Qs level, or quasi-nuclear meson, according to Ref. 3). This result, however, was obtained within the framework of one-channel approximation.

The need to take the second ($n\bar{n}$) channel into account in this problem was pointed out by Kerbikov[4] who concluded that the binding energy of the Qs level is not specified by the shift of the $1s$ atomic level and can vary within very wide limits. In view of the importance of this conclusion, this problem must be examined in more detail in subsequent experiments. We give a formula that determines the nuclear shifts of the levels in the two-channel problem, show that for a shift $\Delta E(1s) \approx 3$ keV (see Ref. 2) the location of the Qs level can be predicted reliably, and estimate the influence of $n\bar{n}$ channel on the probabilty of radiative transitions in the $p\bar{p}$ atom.

Extending Bethe's method[5] to the two-channel case, we obtain an equation for the energy of s levels of a hardonic atom:

$$2[\psi(1-\lambda^{-1}) + \ln\lambda + \lambda/2] = \frac{1}{a_{cs}} + \frac{1}{2}R\lambda^2 + O(\lambda^4)\,, \tag{1}$$

where $\hbar = m = e = 1$ (m is the reduced mass), $\psi(z) = \Gamma'(z)/\Gamma(z)$, $E = -\lambda^2/2$ is the energy of the level, which was measured from the $p\bar{p}$-channel threshold, $\Delta = 2(m_n - m_p) = 2.59$ MeV, and a_{cs} is the $p\bar{p}$-scattering length:

$$a_{cs} = (-a_{11} + \rho a_{12})^{-1}, \quad \rho = a_{12}/(a_{22} + \sqrt{2\Delta})\,,$$

$$R \equiv R(\lambda^2) = r_{11} - a_{12}\frac{2r_{12} + \rho\left[2(\sqrt{2\Delta} + \sqrt{2\Delta + \lambda^2})^{-1} - r_{22}\right]}{a_{22} + \sqrt{2\Delta + \lambda^2} - (1/2)\, r_{22}\lambda^2}\,. \tag{2}$$

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Here, $\alpha_{ij} = \lim_{E\to 0}\{C_{ii}(\eta)M_{ij}(E)C_{jj}(\eta)\}$, $M_{ij}(E)$ is related to the T matrix by the relation $T=(M-ip)^{-1}$, $\eta = 1/k = -i/\lambda$, $V_{11}(\eta) = [2\pi\eta/(1-\exp(-2\pi\eta))]^{1/2}$, $C_{12}=0$, $C_{22}=1$, and r_{ij} is the matrix of effective radii.[6] In the one-channel case $\alpha_{11} = -1/a_{cs}$, and R does not depend on λ and is identical to the effective radius. The value ρ^2 gives the relative probability of finding the system in the $n\bar{n}$ state within the range of nuclear forces. The applicability condition of the expansion (1) has the form: $\lambda r_{ij} \ll 1$.

Expanding (2) in terms of the small parameter $\lambda^2/4\Delta \lesssim 0.1$ and assuming, in accordance with Ref. 6, that $r_{11}=r_{22}=r_e=r_{12}\approx 0$, we obtain

$$R = r_e - \rho^2 \frac{(2\Delta)^{-1/2} - r_e}{1-\lambda^2/\lambda_0^2}\,, \qquad \lambda_0^2 = -\Lambda^2\left(1+\frac{a_{22}}{\sqrt{2\Delta}}\right), \tag{3}$$

where $\Lambda^2 = 4\Delta\,[1-(2\Delta)^{1/2}r_0]^{-1}$. For the $p\bar{p}$ system $(2\Delta)^{-1/2} = 4.0\,F$ and for $r_e \approx 2F$ the parameter $\Lambda^2 = 20.7$ MeV.

The pole term in $R(\lambda^2)$ appears as a result of taking into account the $n\bar{n}$ channel. The special case, in which $(\alpha_{22}+\sqrt{2\Delta})\to 0$ and λ_0^2 lies in the atomic spectrum region, does not seem too probable. Excluding this possibility, we have

$$R(\lambda^2) \approx R(0) = (1+\rho^2)\,r_e - \rho^2(2\Delta)^{-1/2}\,. \tag{4}$$

In this case Eq. (1) is essentially identical to the equation for one-channel problem,[1] differing from it only in that the effective radius $r_{cs} = R(0)$ decreases with increasing channel binding, becomes zero at $\rho^2 \sim 1$, and then becomes negative. It follows from Fig. 1 that for physically reasonable parameters r_e and ρ the binding energy $\epsilon(Qs)$ lies in the vicinity of 1 MeV, in agreement with the predictions of Ref. 1.

We shall now discuss the case when the pole $\lambda^2 = \lambda_0^2$ in Eq. (3) is located in the

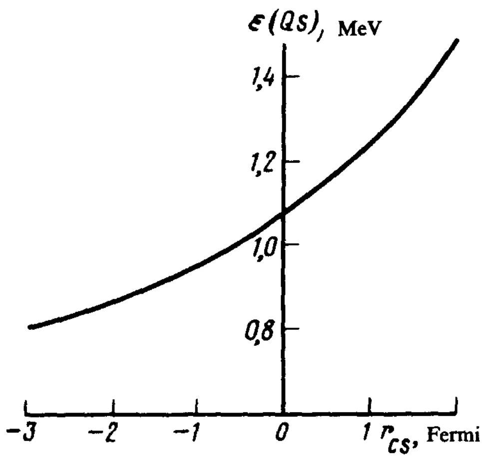

FIG. 1. Dependence of the binding energy of the Qs quasi-nuclear level on the effective radius r_{cs} (for the experimental[2] value of the 1s level shift).

energy region of the quasi-nuclear level. If $\lambda \gg 1$ and the channel binding is weak ($\alpha_{12} \to 0$), then Eq. (1) has an approximate solution

$$\lambda^{-1} = a_s \left[1 - \sigma \frac{\operatorname{sgn} y}{|y| + \sqrt{1 + y^2}} + O(\rho^2) \right], \tag{5}$$

where $\sigma = (\rho^2/4a_s\sqrt{2\Delta})^{1/2}$, $y = 1/4\sigma[(\lambda_0 a_s)^{-2} - 1]$, and a_s is the scattering length in a strong potential. It is important that the solution (5) for any λ_0^2 values is close to the solution of the one-channel problem ($\lambda = 1/a_s$). A numerical solution of Eq. (1) shows that this property remains even at $\rho \sim 1$: one of the roots remains close to that of the one-channel equation,[1] while the second root advances together with λ_0^2.

Figure 2 shows the location of the pole $E_0 = -\lambda_0^2/2$ as a function of the $n\bar{n}$-scattering length $a_s = -1/\alpha_{22}$. As a rule, the pole $E = E_0$ lies far from the energy region described by the expansion of the effective radius (1). In particular, a shift[2] $\Delta E(1s) = 3.0$ keV corresponds to $a_s \approx 7$ F (see Ref. 1) and, therefore, $E_0 \approx 5$ MeV. In this case the pole in $R(\lambda^2)$ does not appear, and the location of the the Qs level, which is stable with respect to the second channel,[2)] almost coincides with the result of the one-channel approximation.[1] The pole from the second channel has a binding energy $\epsilon < 1.5$ MeV only in a narrow range of values $a_s = 3.5$–4.0 F[3)] (see shaded region in Fig. 2). In contrast to the Qs level discussed above, this level becomes a pure $n\bar{n}$ state as $\rho \to 0$ (B. O. Kerbikov brought the possibility of its existence to our attention).

A measurement of the γ rays, spectrum of emitted during transitions between the levels of the $p\bar{p}$ atom, is a convenient method of checking the theoretical calculations

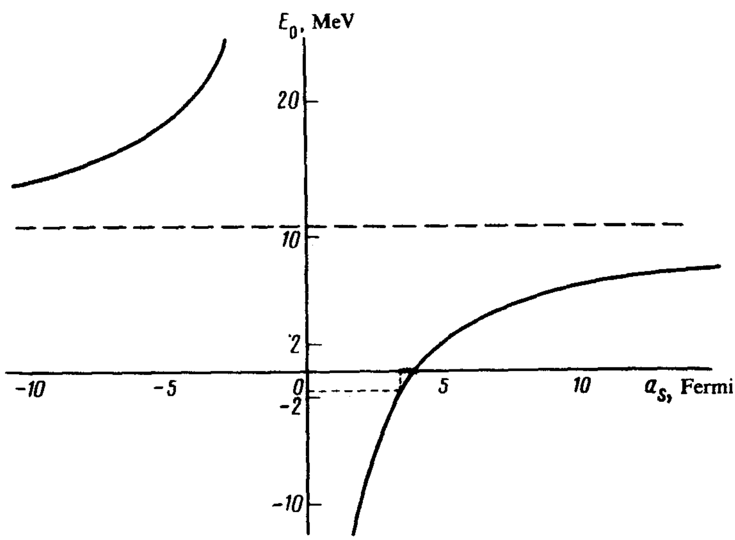

FIG. 2. Energy $E_0 = -\lambda_0^2/2$ corresponding to the pole in $R(\lambda^2)$.

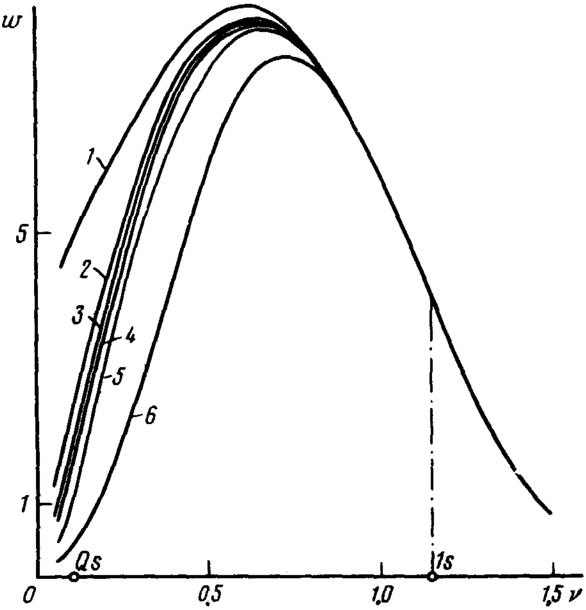

FIG. 3. Probability of radiative transition from the $2p$ level to the νs level in units of 10^{11} sec^{-1}. Curves 1-6 correspond to the following values of the parameters r_e (in Fermis) and ρ^2: $1 - r_e = 2, \rho = 0$; $2 - r_e = 2$, $\rho^2 = 1$ or $r_e = \rho = 0$; $3 - r_e = \rho^2 = 2$; $4 - r_e = 0, \rho^2 = 1$; $5 - r_e = 0, \rho^2 = 2$; $6 - r_e = 0, \rho^2 = 10$. There is no channel binding when $\rho = 0$.

of the level shift. The probabilities of radiative transitions $np \rightarrow \nu s$ were calculated in Ref. ($\nu = \lambda^{-1}$ is the analog of the principal quantum number for the s level). The influence of the $n\bar{n}$ channel decreases the weight of the $p\bar{p}$ component of the wave function, and the transition probability is reduced:

$$\omega(np \rightarrow \nu s) = w_0\{1 + \beta_1[\rho^2(2\Delta)^{-1/2} - (1+\rho^2)r_e]\}^{-1}. \tag{6}$$

Here w_0 is the porbability of the $np \rightarrow \nu s$ transition in the zero-radius nuclear-force approximation, and $\beta_1 = \beta_1(\nu)$; these values can be determined by Eq. (6.1) in Ref. 1. In the atomic spectrum region ($\lambda \lesssim 1$) the correction for w_0, because of allowance for the $n\bar{n}$ channel, does not exceed $\rho^2/2\pi^2\sqrt{2\Delta} \lesssim 1\%$. On the other hand, at $\epsilon \sim \epsilon(Qs) \approx 1$ MeV it reaches a value of $\rho^2/\nu\sqrt{2\Delta} \sim 1$. The results of numerical calculation of the probability of $E\,1$ transition are shown in Fig. 3. At $\nu = 0.112$ (this corresponds to the binding energy $\epsilon = 1$ MeV) a change in the parameter ρ form 0 to 1.5 reduces the probability w by a factor of two.

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[1)]The experimental value[2] of the shift of the $1s$ level of the $p\bar{p}$-atom was taken into account.
[2)]Note that the other branch, obtained in Ref. 4 (which has a binding energy $\epsilon \sim 10$ keV as $\rho \rightarrow 0$ and

corresponds to the $n\bar{n}$ level), does not have stability with respect to variation of the parameter ρ^2.

[3] This corresponds to an atomic-level shift $\Delta E(1s) = 2.0–2.2$ keV. In this case the Qs level has a binding energy of ~ 3 MeV, i.e., it is outside the applicability region of Eq. (1).

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