# Relativistic spin-quadrupole gravitational effect

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The general theory of relativity predicts that oscillations can be induced in a quadrupole mechanical oscillator moving near a rotating gravitational body. For an oscillator in an orbit around the earth the relative oscillation amplitude may exceed $10^{-10}$. The feasibility of observing this effect is discussed.

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The feasibility of observing new, relativistic, gravitational effects in the earth's field was the subject of a number of investigations. Papapetrou[1] and Schiff[2] predicted the existence of spin-orbit and spin-spin gravitational effects in the slow precession of a gyroscope axis placed in a near-earth orbit. Zel'dovich predicted the Zeeman gravitational effect[3] (rotation of the polarization plane of an electromagnetic wave in the field of rotating the earth). Observation of the first two effects is the main goal of an experiment, presently under preparation, arbitrarily named "relativistic gyroscope," which is now being prepared (see, for example, Ref. 4). The goal of this paper is to examine the new, relativistic, gravitational effect which follows from the general theory of relativity (GTR) and to discuss the possibility of observing it.

  

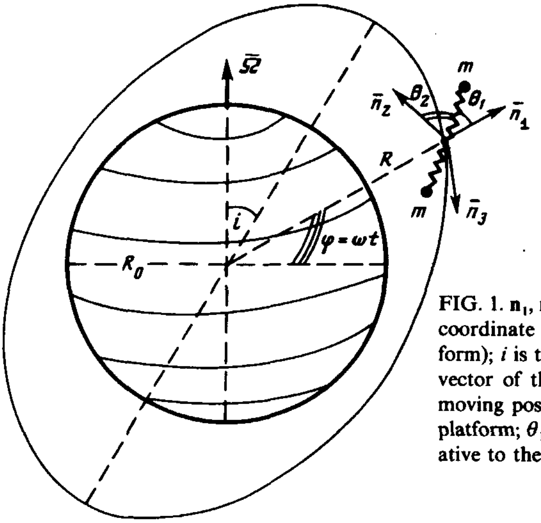


FIG. 1. $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{n}_3$ are the direction vectors in the orbital coordinate system (rigidly bound with the orbital platform); $i$ is the slope angle of the orbit plane relative to the vector of the angular velocity $\vec{\Omega}$ of the sphere; $\phi$ is the moving position angle of the center of mass of the orbital platform; $\theta_1$ and $\theta_2$ determine the oscillator orientation relative to the orbital platform.

A sphere with mass $M$ and radius $R_0$, which rotates with an angular velocity $\Omega$, produces around itself a nonuniform, quasi-magnetic field that distorts the purely Newtonian trajectories of the test bodies in the neighborhood of $M$. This field is responsible for the spin-spin effect and for the gravitational Zeeman effect. Let us assume that a platform with a revolution period $2\pi/\omega$, on which a mechanical, linear, quadrupole oscillator comprised of two masses $m$ bound by a rigidity $k = m\omega_m^2$ and separated by a distance $l$, is placed in an orbit around the sphere $M$. Let us also assume that the rotation period is equal to the revolution period. A calculation based on the GTR shows that the difference in accelerations $F/m$ acting on the oscillator masses is

$$\left(\frac{F}{m}\right)_{\mathrm{SQGE}} = A \times \Phi(\theta_1, \theta_2, i, \phi, t); \qquad A = \frac{aGM\Omega\,\omega\,l}{c^2}\,\frac{R_0^2}{R^3}, \tag{1}$$

where $G$ is the gravitational constant, $R$ is the distance between the platform and the center of the sphere, and $\Phi$ is a dimensionless, generally periodic function which depends on the angles $\theta_1$, $\theta_2$, $i$, and $\phi$ (the notations are self-evident from Fig. 1) and which reaches $\Phi_{\max} = 7$ for the most favorable amplitude values of $\theta_1$, $\theta_2$, and $i$; and $\alpha$ is equal to 0.4 for a homogeneous sphere and to $\approx 0.33$ for the earth. It is clear that this difference in the accelerations induces oscillations in the mechanical oscillator. As seen in expression (1), the quantity $(F/m)_{\mathrm{SQGE}}$ depends on the rotational velocity $\Omega$ of the sphere and on the quadrupole dimensions $l$; therefore, it would be appropriate to call the oscillations induced in the oscillator due to the difference in accelerations the spin-quadrupole gravitational effect (SQGE). If $M = M_{\oplus}$, $\Omega = \Omega_{\oplus}$, $R \approx R_0 = R_{\oplus}$, and $l = 1\times 10^2$ cm, then at $\Phi = \Phi_{\max} = 7$ the amplitude is $(F/m)_{\mathrm{SQGE}} \approx 1.2\times 10^{-14}$ cm/sec$^2$. Such difference in accelerations at $\omega_m^2 \geqslant 1.5\,\omega^2$ can be measured comparatively easily, since the oscillator response $\Delta l_{\mathrm{SQGE}}$ is of the order of $1\times 10^{-8}$ cm (measurements can be performed using a capacitive probe). To record $(F/m)_{\mathrm{SQGE}}$ against a background of thermal motion, we would require $\tau \approx 10^5$ sec at $T = 300$ K, oscillator's $Q = 10^4$, and $m = 10^3$ g. For larger $\tau$, $Q$, or $m$ the quantity $\Delta l_{\rm SQGE}$ can be measured precisely.

An important difference between the SQGE and the other relativistic, nonwave effects is the monotonic energy dissipation in the oscillator if $Q$ is finite. The energy in this case is "drawn out" of the platform's energy whose orbit monotonically contracts due to the SQGE.

The most serious obstacles preventing an easy realization of the experiment on observation of SQGE are the purely Newtonian tidal accelerations (for nonvanishing eccentricity $e$ of the platform's orbit) and accelerations due to the earth's oblateness—$\epsilon$, if the orbit differs from a purely polar orbit by the angle $i$. The oscillator will respond to the sum of all these accelerations. However, since the dependences of $(F/m)_{\rm SQGE}$ and of the purely Newtonian accelerations on $\theta_1$, $\theta_2$, and $i$ are different, we can subtract the Newtonian accelerations after performing three measurements for three orientations of the oscillator. It should be emphasized that in the case we need not know *a priori* the orbit parameters: they are measured incidentally with the necessary accuracy. We give one of the possible schemes of such "subtraction."

In the first measurement the oscillator is oriented in such a way that $\theta_1 = 0$ and $\theta_2 = \pi/2$. We denoted by $\Delta l_1$ the oscillator's response. In the second measurement $\theta_1 = \theta_2 = \pi/4$, the oscillator's response is $\Delta l_2$. Finally, in the third measurement $\theta_1 = \pi/2$ and $\theta_2 = \pi/4$, the oscillator's response is $\Delta l_3$. By measuring the oscillator's responses for the three described orientations, we obtain the relativistic effect $A$:

$$A = \omega_m^2\left(\frac{7}{4}\,\Delta l_1 + 4\,\Delta l_2 + \Delta l_3\right). \tag{2}$$

To ensure that the described subtracting procedure is correct for the large effects, the angles $\theta_1$ and $\theta_2$ must be measured with sufficiently high accuracy. If we want to measure the relativistic effect with up to 10% accuracy, then at $e \approx 10^{-6}$ and $i \approx 10^{-5}$ rad the permissible errors $\Delta\theta_1$ and $\Delta\theta_2$ must not exceed $10^{-3}$ rad, which is easily attainable. The choice of the orbit parameters $e$ and $i$ is determined by the condition that the requirements for the dynamic range of the equipment for recording the response $\Delta l$ should not be set too high.

We do not give here the subtraction methods for finer parasitic effects and limit ourselves to summarizing the main effects with relative acceleration amplitudes of the order of $10^{-12}$–$10^{-13}$ cm/sec$^2$, such as, for example, the linear effects of order $\epsilon e(GM_\oplus/R^3)l$; in addition, there are linear effects with amplitudes of the same order of magnitude: $e\epsilon(G^2M_\oplus^2/R^6)\,(l/\omega_m^2)$ or $\epsilon^2 i(G^2M_\oplus^2/R^6\omega_m^2)l$. All these effects have a phase "coloration" and hence can be "subtracted" by using the same three measurements (for three measurements we obtain six values: three amplitudes and three phases).

The tidal accelerations in the oscillator, caused by the moon or the sun, can also be subtracted without additional measurements, if the orbit parameters are known with a relative accuracy of $\sim 10^{-3}$.

In order for $(F/m)_{SQGE}$ to exceed the difference in accelerations due to the Lorentz force in the earth's magnetic field, the potential difference between the masses $m$ must not be greater than $\approx 0.1$ mV.

In recommending the measurement procedure outlined above, we assumed that $\omega_m \neq \omega$, i.e., the oscillations induced in the oscillator are nonresonant. By finely adjusting the value $k = m\omega_m^2$ in such a way that $\omega_m = \omega$, we can obtain a much larger response $\Delta l_{SQGE}$; however, the higher-order, nonlinear, parasitic effects in these case must be taken into account (and subtracted).

In conclusion, we note that $\Delta l_{SQGE}/l \simeq 1 \times 10^{-10}$ does not differ greatly from the precession of the gyroscope axis $\Delta\theta_{SQGE}$ for the spin-spin gravitation effect ($\Delta q_{SQGE} \approx 8 \times 10^{-10}$ rad after $\tau \approx 10^5$ sec); however, there is no need to used highly sensitive techniques for the SQGE.[4]

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