Threshold-free, dissipative ballooning modes

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It is shown that the dissipative ballooning modes in a tokamak do not have a threshold along the pressure gradient and develop with a much larger increment than the reciprocal skin time $\gamma \sim 1/\tau_s\ (\tau_s/\tau_\theta)^{2/3}$ (τ_s) is the skin time and τ_θ is the Alfvén time for the current field, $\tau_s/\tau_\theta \gg 1$).

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The effect of finite conductivity on the flute instability of a toroidal plasma column was examined in Refs. 1 and 2.

The main result of taking into account the finite conductivity involves the loss of the stabilizing effect of shear. The instability had a threshold with respect to the pressure gradient and began to grow when the ballooning effect exceeded the stabilization due to the magnetic well. The increment of this instability $\gamma \sim 1/\tau_s\ (\tau_s/\tau_\theta)^{2/3}$ (τ_s is the skin time and τ_θ is the Alfvén time for the current field) was much larger than the reciprocal skin time.

Progress in the study of ballooning modes of the flute instability of an ideal plasma showed that the necessary stability criterion for ideal ballooning modes is more rigorous than the Mercier criterion[3] due to the new, destabilizing terms associated with shear.[4] A paradox occurred in this case: the threshold of the dissipative flute instability, which follows from Refs. 1 and 2, proved to be higher than that of ideal ballooning modes, which was obtained in Ref. 4., This paradox will be resolved below.

In this paper we investigate the dissipative ballooning modes on the basis of the equations for single-fluid, magnetic hydrodynamics with allowance for compressibility. According to Kadomtsev and Pogutse,[5] the plasma perturbations can be described by means of the electrostatic potential $\tilde{\phi}$ of the longitudinal component of the vector potential $\widetilde{A}_s$ and the perturbed pressure $\widetilde{P}$. The transformation proposed by Connor *et al.*[6] within the limit of large azimuthal numbers ($m \approx nq \gg 1$) reduces the original linearized equations to two equations for the Fourier transforms ϕ and P.

$$\frac{d}{dy}\frac{\Delta}{(1+\Delta/\Gamma)}\frac{d\phi}{dy} - \gamma^2\tau_\theta^2(1+S^2y^2)\phi$$

$$\frac{aRB^s}{2}\left(\frac{\partial}{\partial\rho}\frac{1}{B^s} - \frac{Sy}{\rho}\frac{\partial}{\partial y}\frac{1}{B^s}\right)P = 0, \qquad (1)$$

$$\left(1 - \frac{1}{\gamma^2\tau_c^2}\frac{d^2}{dy^2}\right)P - \left[1 - \frac{1}{\gamma^2\tau_c^2}\frac{d}{dy}\frac{1}{(1+\Delta/\Gamma)}\frac{d}{dy}\right]\phi = 0. \qquad (2)$$

Here $S = q'\rho/q$, $\alpha = -2P_0'Rq^2/B^2$, $\tau_\theta = Rq/C_A$, $C_A^2 = B^2/4\bar{\pi}\rho_0$, $\tau_c = Rq/C_S$, $C_S^2 = \gamma_0 P_0/\rho_0$, $\tau_s = 4\bar{\pi}\sigma a^2/c^2$, a and R are the small and large radii of the torus, q is the safety factor, $\Gamma = \gamma\tau_S/n^2q^2$, and

$$\Delta = \rho\left(\frac{g_{11}}{\sqrt{g}} + \frac{s^2y^2}{\rho^2}\frac{g_{22}}{\sqrt{g}} - 2\frac{Sy}{\rho}\frac{g_{12}}{\sqrt{g}}\right),$$

where g_{ik} are the metric coefficients of the surface coordinate system with direct lines of force.[7]

In the case of perfectly conducting plasma $\gamma\sim 1/\tau_\theta$, for which $\Delta \ll \Gamma$, and Eqs. (1) and (2) reduce to one, second-order equation for ϕ.[8]

In the case of imperfectly conducting plasma $\gamma \ll 1/\tau_\theta$, Eqs. (1) and (2) have two different scales $y\sim 1$ and $y \gg 1$, which allows us to use the Van der Pol averaging method.

The ballooning modes in the tokamak at $nq \gg 1$ are localized perturbations with respect to the radius of a plasma column, which correspond to large, characteristic y in the average equation. At $y^2 \gg M^2/\Gamma^2$ Eqs. (1) and (2) reduce to one averaged equation. For circular magnetic surfaces it has the following form:

$$\Gamma P'' - \left\{\frac{\Gamma^2}{N^2}(1+S^2y^2) + aU_0 - \frac{a^2(1+S^2y^2+M^2/\Gamma)}{2\Gamma(1+M^2/\Gamma^2)[1+\Gamma(1+S^2y^2)/N]}\right\}P = 0. \qquad (3)$$

Here $M = \tau_s/\tau_c n^2q^2$, $N = \tau_s/\tau_\theta n^2q^2$ are parameters characterizing the ratio of the skin time to the sound and Alfvén times, respectively, $M^2 = \gamma_0\beta N^2$, where γ_0 is the adiabatic exponent (in a high-temperature plasma $N \gg 1$), and v_0 is the magnetic well in the tokamak.[7]

It can be shown that in the opposite limiting case $y^2 \ll M^2/\Gamma^2$ the instability decreases appreciably. The condition $y^2 \gg M^2/\Gamma^2$, which in standard notations has the form $\gamma^2 \gg K_\parallel^2 C_s^2$, means that the average pressure perturbation does not have time to equalize along the lines of force.

An analysis of Eq. (3) shows that its potential depends essentially on the ratios S/S_k ($S_k = a^{1/2}/Nv_0^{1/2}$), a/a_k (a_k is determined from the condition $a/2 = v_0$), and β/β_k ($\beta_k = a^{4/3}/N^{2/3}$). At very small shears $S' \lesssim S_k$ and pressures $\beta \ll \beta_k$ we can see from Eq. (3), like in Ref. 9, that the increment is small, of the order of the reciprocal skin time $\Gamma = a/2v_0$.

In this paper we examine systems with a shear of practical interest $S \sim 1$ (of course, $S \gg S_k$). We solve Eq. (3) by the variational method. We write the functional corresponding to this equation and substitute as the trial function a function of the form: $P = 1/(\lambda^2 + y^2)$, where λ is a variational parameter.

After variation we obtain expressions for determination of the increment and the parameter λ:

$$(\Gamma^3 + \gamma_0 \beta N^2 \Gamma)\left[1 - N^2\left(\frac{a^2}{2} - \nu v_0\right)/2S^2\Gamma^2\lambda^2\right] = a^2N^2/2, \tag{4}$$

$$N^2\left(\frac{a^2}{2} - a v_0\right)/\Gamma^2\lambda^2 + S^3\Gamma^{1/2}\lambda/N = N^2/\Gamma\lambda^4. \tag{5}$$

In the case of large pressures at $\beta \gg \beta_k$, when the magnetic field perturbation is considerable, the expression for the increment has the form

$$\Gamma = \frac{N^{2/3}}{S^{2/3}}\left(\frac{a^2}{2} - a v_0\right)^{2/3} \quad a \gtrsim a_k; \quad \Gamma = 0 \quad a \lesssim a_k. \tag{6}$$

This increment corresponds to a nonpotential, gravitational-dissipative instability[1,2] which has a threshold nature along the pressure gradient.

In the case of small plasma pressures when $\beta \lesssim \beta_k$ and $a \lesssim a_k$ we can obtain from Eqs. (4) and (5) the relation

$$\Gamma^3 + \gamma_0 \beta N^2 \Gamma = a^2 N^2/2. \tag{7}$$

We obtain a qualitatively new result from this expression: the increment of the dissipative ballooning modes does not have a threshold along the pressure gradient. An instability begins to develop at any arbitrarily small gradient with the increment $\Gamma \sim a^{2/3}N^{2/3}[\gamma \sim 1/\tau_s(\tau_s/\tau_\theta)^{2/3}]$. Thus, the aforemention paradox vanishes.

The general case is shown in Fig. 1, in which the increment of the dissipative ballooning modes is plotted as a function of the pressure gradient of plasma a for different values of pressure. Curve 1 corresponds to β equal to zero or to a fully compressible liquid $\gamma_0 = 0$, curve 2 was constructed for $\tilde{\beta} = \beta\gamma_0 N^{2/3} = 1$, and curve 3 was constructed for $\tilde{\beta} = 10$.

The region corresponding to a high-temperature plasma lies between curves 2 and 3. The dashed curve represents the threshold increment (6), in obtaining it we assumed that the velocity of sound is infinite. The ion sound in this case equalized the pressure perturbations along the lines of force and at $a < a_k$ the instability did not develop. The picture is basically different when the finite velocity of the ion sound is taken into

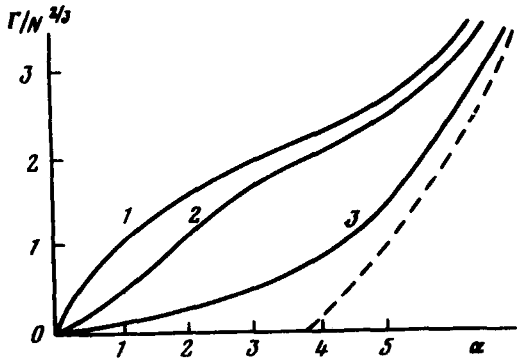

FIG. 1. Dependence of the increment $\Gamma/N^{2/3}$ on α for different values of $\tilde{\beta}=\beta\gamma_0 N^{2/3}$: 1, $\tilde{\beta}=0$; 2, $\tilde{\beta}=1$; 3, $\tilde{\beta}=10$. The dashed curve represents the threshold increment (6).

account: the pressure perturbations do not have time to equalize along the lines of force and the ballooning instability, which develops due to the Alfven oscillations, can exist at any pressure gradient. The increment of these dissipative ballooning modes decreases with increasing plasma pressure, as seen in Fig. 1.

An estimate of the transport coefficients according to the relation $\chi \sim \gamma/K_\perp^2$ shows that they are of the order of $\alpha a^2/\tau_s$, i.e., of the order of the pseudoclassical coefficients.

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