Quasi-elastic nucleus-nucleus scattering cross section in the optical approximation

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A closed expression for the differential cros section of quasielastic, nucleus-nucleus scattering is obtained within the framework of optical approximation, disregarding the real part of the elastic *NN*-scattering amplituded.

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Pak *et al.*[1] obtained an expression for the differential cross section of quasi-elastic scattering of a nucleus by a nucleus A_2 (A_1 remains in the ground state and A_2 undergoes all kinds of excitations, including a disintegration) in the from of a number of incoherent collisions in degrees of multiplicity

$$\left(\frac{d\sigma}{d\Omega}(q)\right)^{q.el.} = \sum_n \left(\frac{d\sigma}{d\Omega}(q)\right)^{(n)}, \tag{1}$$

$$\left(\frac{d\sigma}{d\Omega}(q)\right)^{(n)} = \frac{1}{n!}\int \prod_{i=1}^{n} \left(T(s_i)\,ds_i\right) \left| F^{(n)}_{q.el.}(q, s_1, \ldots s_n)\right|^2, \tag{2}$$

$$F^{(n)}(q, s_1 \ldots s_n) = \frac{\delta^{(n)}}{\delta T_2(s_1)\ldots\delta T_2(s_n)}\, F^{el.}_{A_1 A_2}\left\{T_1(s), T_2(\tilde{s})\right\}, \tag{3}$$

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where $T_{1/(2)}(s) = A \int \rho(r) dz$ are the thickness functions of colliding nuclei and s and $\tilde{s}$ are the radius-vector components of nucleons in the nuclei, where $F^{el}_{A_1A_2}\, T_1(s),\ T(\tilde{s})$ is the amplitude of elastic A_1A_2 scattering whose explicit functional form is given in Refs. 1 and 2. Specific calculations according to the scheme (1)–(3) are very difficult when the sum (1) is significantly influenced by a large number of terms. It would be of interest, therefore, to obtain at least an approximate closed expression for the entire sum. It appears that this can be done in the optical approximation of the atomic numbers of colliding nuclei, and if the real part of the NN-scattering amplitude is disregarded. Since at energies of several GeV/nucleon the real part of the NN-scattering amplitude is numerically small [$\alpha = \mathrm{Re} f(0)/\mathrm{Im} f(0) \approx 0.2$–$0.3$]. and its influence on the hadron-nuclear scattering is almost immaterial, we hope that the use of $\alpha = 0$ approximation in the nucleus-nucleus scattering problem is justifiable.

Writing the amplitude of the process

$$A_1 + A_{2(i)} \to A_1 + A_{2(f)} \tag{4}$$

in the form

$$F_{if}(q) = \frac{i}{2\pi} \int d\mathbf{b}\, e^{i\mathbf{qb}}\, \psi^*_{A_2(f)}(\tilde{r}_1 \dots \tilde{r}_{A_2})\, \psi_{A_2(i)}(\tilde{r}_1 \dots \tilde{r}_{A_2}) \prod_{i=1}^{A_2} d\tilde{\mathbf{r}}_i$$
$$\times \prod_{k=1}^{A_1} \rho_1(r_k)\, d\mathbf{r}_k \left[1 - \prod_{k=1}^{A_1}\prod_{i=1}^{A_2}(1 - \gamma(b - \tilde{s}_i - s_k))\right], \tag{5}$$

where $\psi_{A_2 i(f)}$ are the wave functions of the initial (finite) state of the A_2 nucleus, $\rho_1(r)$ is the single-particle distribution density of nucleons in the A_1 nucleus, $\gamma(b)$ is the profile function of the NN-scattering amplitude (which is purely real in the $\alpha = 0$ approximation), we can clearly average over the nucleon distribution in the remaining A_1 nucleus and in the obtained result go over to the optical limit of its atomic number. As a result, we obtain

$$F_{if}(q) = \frac{i}{2\pi} \int d\mathbf{b}\, e^{i\mathbf{qb}}\, \psi^*_{A_2(f)}(\tilde{r}_1 \dots \tilde{r}_{A_2})\, \psi_{A_2(i)}(\tilde{r}_1 \dots \tilde{r}_{A_2}) \prod_{i=1}^{A_2} d\tilde{\mathbf{r}}_i$$
$$\times \left(1 - \exp\left[-\Gamma_{A_2N}(s, \{\tilde{s}\})\, T_1(b - s)\, ds\right]\right), \tag{6}$$

where

$$\Gamma_{A_2N}(s, \{\tilde{s}\}) = 1 - \prod_{i=1}^{A_2}(1 - \gamma(s - \tilde{s}_i)).$$

Using in calculating the sums of cross sections of all possible excitations of the A_2 nucleus by A_1 nucleus, the completeness conditions

$$\sum_f \psi^*_f(\tilde{r}_1 \dots \tilde{r}_{A_2})\, \psi_f(\tilde{r}'_1 \dots \tilde{r}'_{A_2}) = \prod_{i=1}^{A_2} \delta(\tilde{\mathbf{r}}_i - \tilde{\mathbf{r}}'_i) \tag{7}$$

and the factorization approximation for the nucleon distribution density in the A_2 nucleus

$$|\psi_{A_2}(\tilde{r}_1 \ldots \tilde{r}_{A_2})|^2 = \prod_{i=1}^{A_2} \rho_2(\tilde{r}_i) \tag{8}$$

we obtain for the total cross section

$$\frac{d\sigma}{dt} = \pi \sum_f |F_{if}|^2 \tag{9}$$

an expression of the form

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} \int d\mathbf{b}_1 d\mathbf{b}_2 \prod_{i=1}^{A_2} \left(\frac{T_{A_2}(\tilde{s}_i)}{A_2} d\tilde{s}_i \right) \exp i\mathbf{q}(\mathbf{b}_1 - \mathbf{b}_2)$$

$$\times [1 - \exp(-\int \Gamma_{A_2N}(s, \{\tilde{s}\}) T_1(b - s) ds) - \exp(-\int \Gamma^*_{A_2N}(s, \{\tilde{s}\}) T_1$$

$$\times (b_2 - s) ds) + \exp(-\int \Gamma_{A_2N}(s, \{\tilde{s}\})[T_1(b - s_1) + T_1(b_2 - s)] ds\}. \tag{10}$$

The common factor $\Gamma_{A_2N}(s,\{\tilde{s}\})$ can be taken out in the argument of the last exponential function in the square in Eq. (10) by assuming that the profile functions $\gamma(b)$ are real. Thus, the problem reduces to calculating monotypical expressions of the type

$$\Phi(b_1, b_2) = \int \prod_{i=1}^{A_2} \left(\frac{T_2(\tilde{s}_i) d\tilde{s}_i}{A_2} \right) \exp(-\int \Gamma_{A_2N}(s, \{\tilde{s}\}) \tau(s, b_1, b_2) ds). \tag{11}$$

As shown in Refs. 1–3, in the optical limit of the parameter A_2 and in the zero-radius approximation of the NN interaction the logarithm of expression (11) with an accuracy to numerically unimportant terms is given by the integral

$$-\ln \Phi(b_1, b_2) = \phi(b_1, b_2) = \frac{2}{\sigma} \int ds f\left(\frac{\sigma}{2} \tau(s, b_1, b_2), \frac{\sigma}{2} T_2(s)\right), \tag{12}$$

where

$$f(x, y) = z(\exp u - 1) + u(\exp z - 1) - uz,$$

$$z = x \exp(-u), \quad u = y \exp(-z).$$

Subtracting from the sum (10) the cross section for the elastic A_1A_2 scattering process, we obtain for the cross section of quasi-elastic scattering of the A_1 nucleus by A_2 nucleus

$$
\left(\frac{d\sigma}{dt}\right)^{q\,el} = \frac{1}{4\pi}\int d\mathbf{b}_1 d\mathbf{b}_2 \exp i\mathbf{q}(\mathbf{b}_1-\mathbf{b}_2)
$$

$$
\times\left\{\exp-\frac{2}{\sigma}\int d\mathbf{s}\, f\left[\frac{\sigma}{2}\left(T_1(b_1-s)+T_1(b_2-s)\right);\quad \frac{\sigma}{2}T_2(s)\right]\right.
$$

$$
-\exp-\frac{2}{\sigma}\int d\,\mathbf{s}\left[f\left(\frac{\sigma}{2}T_1(b_1-s),\frac{\sigma}{2}T_2(s)\right)\right. \tag{13}
$$

$$
\left.\left.+f\left(\frac{\sigma}{2}T_1(b_2-s),\quad \frac{\sigma}{2}T_2(s)\right)\right]\right\}.
$$

Note that from the computational point of view expression (13) for the total cross section of quasi-elastic A_1A_2 scattering is as simple as the expression for the cross section $(d\sigma/d\Omega)^{(1)}$ of single, quasi-elastic scattering and much simpler than the expressions for the $(d\sigma/\delta\Omega)^{(n)}$ cross sections for $n>1$.

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