High-frequency impedance of a pure type I superconductor in the surface superconductivity regime

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The surface impedance of a pure type I superconductor is calculated for surface superconductivity conditions. In view of low impurity concentration, the anomalous skin-effect regime is realized for the incident electromagnetic wave. It is shown that the superconducting currents, which flow along the superconductor surface perpendicularly to the constant magnetic field, provide the main contribution to the reflection of the electromagnetic wave. A comparison with experiment is made, yielding good agreement.

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The influence of surface superconductivity on the impedance of a dirty type II superconductor was investigated by Maki.[1] Because of the short mean path length, the electron motion in a magnetic field is not clearly evident, since the electrons "forget" about the bending of trajectories by the magnetic field due to frequent scattering by impurities.

The situation is much more complicated in a pure type I superconductor when the anomalous skin effect regime is realized, and it is necessary to take into account the the quantum nature of the electron motion in a magnetic field; this makes the problem highly nonlocal. The case of small temperatures compared to the order parameter was investigated earlier.[2] We shall investigate the impedance of a pure super conductor under surface superconductivity conditions at fields close to H_{c3}, which is of interest because of the available experimental data.[3]

To determine the surface impedance, it is necessary to calculate the electromagnetic response operator $Q(\omega_\nu, x, x')$, which is determined by the loop of the exact Green's functions of the system, and then to extend it analytically to real frequencies. Near H_{c3} the order parameter $\Delta\,(H,x)$ is small and it is sufficient to find the Green's functions within an accuracy of terms of the order of Δ^2. In addition, there are corrections associated with the magnetic field nonuniformity (superconducting currents in the surface layer lead to a partial displacement of the magnetic field near the surface). The corresponding correction to the vector potential is also of the order of Δ^2.[4] As a result, the correction to the electromagnetic response of a normal metal in a constant magnetic field is determined by five diagrams, two of which are associated with the change in the vector potential $\delta A\,(x)$ (their contributions are the same), while three correspond to the expansion of the Green's function to the second order in Δ. The latter diagrams turn out to be small near H_{c3}.

Thus, the problem of determining the impedance is reduced to calculating the diagram in Fig. 1. Here, the solid line corresponds to Green's function of a semi-infinite normal metal in a longitudinal magnetic field; the dashed lined corresponds to the interaction H_{int} due to the change δA in the vector potential. The vertices have $\frac{e}{mc}\hat{p}$. We shall use the Landau representation[5] for the Green's function

$$G(x, x', p_y, p_z, \epsilon_k) = \sum_n \frac{\psi_n^*(x - \lambda^2 p_y)\psi_n(x' - \lambda^2 p_y)}{i\epsilon_k - \xi_n(p_z)}, \tag{1}$$

where $\lambda = (c/eH)^{1/2}$ is the magnetic length and $\xi_n(p_z)$ is the electron energy in the nth state, which is measured from the Fermi level. We choose the boundary condition on the basis of the specular electron reflection from the surface:

$$G(x, 0, p_y, p_z, \epsilon_k) = G(0, x', p_y, p_z, \epsilon_k) = 0. \tag{2}$$

Therefore, the $\psi_n(x - \lambda^2 p_y)$ functions are solutions of the Schrödinger equation for an electron in a constant magnetic field with an infinite potential wall at $x = 0$, and the location of the minimum of the parabolic potential is determined by the momentum p_y. Depending on the value of p_y, it can lie either inside the metal or outside it.

Let us determine the impedance for longitudinal polarization of the electric field of the UHF wave, when the **E** vector is parallel to the static magnetic field **H**. Calculating the diagram in Fig. 1 by means of the Green's functions (1) and taking into account the orthogonality of the $\psi_n(x - \lambda^2 p_y)$ functions, we have for the total current

$$J_z = \int_0^\infty j_z(x)\,dx = -\int_0^\infty dx \int_0^\infty dx' A_\omega(x')\,Q^R(\omega, x, x')$$

$$= \frac{4e^2}{m^2c^2}\int_{-\infty}^{\infty}\frac{dp_y}{2\pi}\sum_n\sum_m \langle n|H_{int}|m\rangle\langle m|A_\omega|n\rangle\,\sigma^R(\omega, n, m). \tag{3}$$

The expression $\sigma^R(\omega,n,m)$ in Eq. (3) is the analytical continuation in the upper half-plane of the expression

$$\sigma(\omega_\nu, n, m) = T\sum_{\epsilon_k}\int\frac{dp_z}{2\pi}\frac{p_z^2}{(i\epsilon_{k+\nu} - \xi_n)(i\epsilon_k - \xi_n)(i\epsilon_k - \xi_m)}, \tag{4}$$

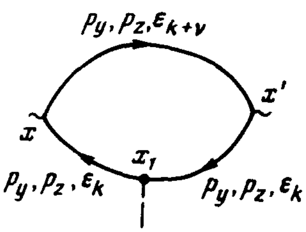

FIG. 1.

calculated for the Matsubar frequencies $\omega_\nu = 2\pi T_\nu$. The summation in Eq. (3) is carried out over the energies E_n for which the momentum $p_{zn} = [2m(\epsilon_F - E_n)]^{1/2}$ is real.

In the UHF region the skin layer depth $\delta(\omega)$ is small compared to the Larmor radius of the electrons. Therefore, we can use the ordinary expression for the vector potential in the presence of anomalous skin effect[6]:

$$A_\omega(x) = \frac{2E'(0)}{\pi i \omega} \int_0^\infty \frac{\cos kx}{k^2 + 4\pi i \omega \sigma(|k|)} dk, \qquad (5)$$

where $\sigma(|k|)$ is the Fourier component of the normal conductivity.

In the calculation of the matrix elements of $A_\omega(x)$ let us point out that the principal contribution is provided by electrons whose trajectory centers lie at large [compared to $\delta(\omega)$] distances outside the metal limits. This corresponds to the known concept of inefficiency: the major contribution to the impedance comes from the electrons that slide along the surface inside the skin layer. The quasi-classical matrix elements $\langle n|\cos kx|m\rangle$ are calculated in explicit form. Subsequent integration with respect to k gives a result different from zero only for the diagonal matrix element $(n = m)$ and only for those trajectories for which the electron, as a result of its classical motion in a magnetic field, moves away from the wall a distance less than or of the order of $\delta(\omega)$. Thus, only the term with $m = n$ remains in the summation over m in Eq. (3). Assuming that $m = n$ in (4), we integrate with respect to p_z and compute the summation over the frequencies ϵ_k, after which the analytical continuation is achieved by the replacement $i\omega_\nu \to \omega$. The matrix element $\langle n|H_{\rm int}|n\rangle$ is calculated from the known expression for the correction for the vector potential.[4] The electron spectrum $E_n(p_y)$ in the potential well for fixed p_y is determined from the Bohr-Sommerfeld quantization rule. By computing the remaining integrals in (3), we obtain the final expression for the impedance of a type I superconductor operating in the surface superconductivity regime. Isolating the real part of the relative value of the absorption coefficient of the electromagnetic wave as compared with a normal metal, we obtain:

$$\frac{R_{\parallel}}{R_0} = 1 - \frac{2.5\times 10^{-4}}{\kappa^2 - 0.156} \frac{(e^2/\lambda)}{mc^2}\left(\frac{\Omega_L}{\omega}\right)[p_F\delta(\omega)]^{7/2}\left(\frac{\xi}{\delta(\omega)}\right)\frac{H_{c3} - H}{H_{c3}}, \qquad (6)$$

where ξ is the correlation length characterizing the thickness of the superconducting layer and κ is the Ginsburg-Landau parameter.

Thus, the absorption decreases linearly with decreasing magnetic field. Equation (6), strictly speaking, is valid for those fields for which the relative correction is small. In the experiments using very pure lead (mean free path length of the order of 0.1 mm),[3] a linear dependence for the impedance was observed almost in the entire field range between H_c and H_{c3}, in agreement with Eq. (6) (the numerical coefficients for the linear dependence are also of the same order of magnitude). For transverse polarization of the UHF wave, the impedance generally has no singularity[5] near H_{c3}, but it has a smooth maximum below H_{c3}. We can assume that an increase in absorption at fields above H_{c3} is due to superconducting fluctuations.

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[1]K. Maki and G. Fisher, Phys. Rev. **184**, 472 (1969).
[2]R. G. Mints, Pis'ma Zh. Eksp. Teor. Fiz. **18**, 523 (1973) [JETP Lett. **18**, 307 (1973)].
[3]I. Ya. Krasnopolin, Radzh Rup and M. S. Khaikin, Pis'ma Zh. Eksp. Teor. Fiz. **15**, 516 (1972) [JETP Lett. **15**, 365 (1972)].
[4]A. I. Rusinov, Zh. Eksp. Teor. Fiz. **56**, 1441 (1969) [Sov. Phys. JETP **29**, 772 (1969)].
[5]A. A. Abrikosov, Pis'ma Zh. Eksp. Teor. Fiz. **56**, 1391 (1969) [Sov. Phys. JETP **29**, 746 (1969)].
[6]E. M. Lifshitz and L. P. Pitaevskii, Fizicheskaya kinetika (Physical Kinetics), Nauka, Moscow, 1979.