On the numerical values of the fine-structure constant and the gravitational constant

I. L. Rozental'
Institute of Space Research, USSR Academy of Sciences

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The numerical values of the constants α_e and α_g are interpreted on the basis of the large combination and quasi stability of the proton.

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The criticality of the existence of basic, stable bound states (atoms, nuclei, stars and galaxies) to the numerical values of fundamental physical constants was pointed out in Refs. 1, 2. This criticality is evident in the following fact: if one of the fundamental constants, on which a given stable state depends, changes within the limits

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from ten percent to an order of magnitude, then it will not exist. The analysis performed in Refs. 3, 4 demonstrated that the known cosomological coincidences of combinations of fundamental constants, which were pointed out many years ago by Dirac, are also due to existence of the basic stable states.

These facts indicate that there is a close relationship between the numerical values of the fundamental constants and the coexistence of basic, stable bound states.

Some interpretations of this relationship were proposed in Refs. 1–4. The most graphic interpretation reduces to the assumption that the universe will pass through numerous cycles; at the beginning of each cycle the numerical constants, which do not necessarily coincide with the constants that have existed in the previous evolution cycle of the universe, are formed.[1] In the present cycle, which began about 10^{10} years ago, a set of constants, sufficient for the existence of the basic, stable bound states, was "formed."

We note that the stability in this context is understood to mean the fulfillment of the inequality $t > t_u$ (t is the lifetime of the state, $t_u \sim 1/H_0$ is the life of the universe, and $H_0 \sim 10^{-17}$ sec^{-1} is the Hubble constant).

The purpose of this paper is to relate the numerical values of the fine structure constant $\alpha_e = e^2$ and the dimensionless gravitational constant $\alpha_g = Gm_p^2$ [2)] to the proton stability, which is understood in the same sense as the stability of the basic bound states

$$t_p > t_u \tag{1}$$

or

$$t_p > t_s , \tag{2}$$

where t_p is the proton lifetime and t_s is the life of a star of the main sequence. The subsequent analysis is performed within the framework of a theory (grand unification) that combines the weak, electromagnetic, and strong interaction (large combination. The subsequence specification reduces to two comparatively general assumptions: 1) the decay of a proton into a hadron and lepton is allowed within the framework of the unified theory and 2) strong interaction is represented asymptotically by the free theory. In such a case for sufficiently large 4-momentum transfer q^2 the strong interaction constant α_s can be approximated by the following dependence:

$$a_s = \frac{a}{\ln(q^2/q_0^2)} \tag{3}$$

a and q_0^2 are certain constants; without significantly restricting the generality, we can set $q_0^2 \sim m_p^2$. Since the effective constant α_e depends weakly on q^2, the unification occurs for the following characteristic mass m_{wes}:

$$a_s \sim a_e \sim \frac{a}{2 \ln \frac{m_{wes}}{m_p}} \tag{4}$$

If the proton is unstable, then t_p[6-8]:

$$t_p \sim a_e^{-2} \frac{m_{wes}^4}{m_p^5} \sim (a_e^2 m_p)^{-1} \exp(2a/a_e). \tag{5}$$

Comparing (5) and (1), we obtain the constraint

$$a_e \lesssim -2a \ln \frac{H_0}{a_e^2 m_p} \tag{6}$$

Within the framework of the most popular unification model, which is based on the SU(5) group,[6-8] the effective constant $a \sim \frac{1}{2}$[3]. Thus, universe in which the condition (1) is satisfied, has the constraint

$$a_e \lesssim 1/80. \tag{7}$$

The constraint on the gravitational constant can be determined from two different assumptions which lead to the same result.

1) Let us assume that the universe is closed. Then the semiempirical relation[2,3,9]

$$M_u \sim a_g^{-2} m_p. \tag{8}$$

exists for its mass.

Using the inequality (1) and the obvious relation for the radius of the universe: $R_u \sim GM_u \sim t_u$, we obtain the constraint

$$a_e \lesssim -(\ln a_g)^{-1}. \tag{9}$$

2) This relation can be obtained on the basis of the inequality (2) without assuming that the universe is closed. In fact, the average mass of a star is $M_s \sim \alpha_g^{-3/2} m_p$ and the average luminosity is $L \sim m_e^2 \alpha_g^{-1/2}$ (m_e is the electron mass), see Refs. 2, 3, 9. Thus, by using the obvious relation $t_s \sim \eta M_s/L$ ($\eta \sim 10^{-3}$ is the coefficient of conversion of the rest mass of a star into radiation[4]), we again obtain the constraint (9) which is satisfied in the present of the universe evolution cycle in the limit.

The lower limit of the value of α_e can be estimated by constructing a grand unification. Some time ago[10] it was assumed that the Planck mass $m_{pl} \sim G^{-1/2}$ is the maximum allowable mass of elementary particles. These arguments received support in the evaporation theory of black holes.[11] Thus, a grand unification is possible if $m_{wes} \lesssim G^{-1/2}$. Using this condition and the relation (4), we obtain:

$$a_e \gtrsim \frac{a}{2 \ln \frac{m_{pl}}{m_p}}. \tag{10}$$

Specifically, the model based on the SU(5) group has the restriction

$$a_e \gtrsim 1/170. \quad (11)$$

Thus, it can be assumed that the numerical value of the constant $\alpha_e \sim 1/100$ and the relation $\alpha_e \sim -(\ln\alpha_g)^{-1}$ are attributable to two facts: 1) the stability of the proton in the universe in terms of the inequalities (1) or (2) and 2) the possibility of constructing a closed unified field theory.

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[1]For a specific scheme for the formation of the quality G at the start of an expansion cycle of the universe, see Ref. 5.

[2]$\hbar = c = 1$.

[3]The parameter α depends on the number of quarks. However, if it is less than 10, then this dependence is weak.

[4]Since the quantity η is included in the final expression (9) under the sign at the logarithm, its value is not important.

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