

On the quantum theory of resonance scattering of atoms by light

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(Submitted March 20, 1980)

Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 9, 542–545 (5 May 1980)

The quantum structure in the scattering of resonance atoms by the field of a standing light wave was investigated. It is shown that the odd Bragg peaks depend more strongly on the field frequency than the even peaks. An agreement with the classical scattering theory is established.

PACS numbers: 32.80. — t

The scattering of Na atoms by the field of a resonance standing light wave was recently observed.¹ The experimental results agree with the theory^{2,3} in which the atom motion is assumed to be classical. The classical description is applicable when the momentum transfer to the atom is much greater than the photon momentum. The scattering diagram obtained in this case is the envelope of the Bragg peaks.

However, the Bragg structure can also be detected in an experiment. Thus, for example, the deflection of Na atoms due to absorption of one photon in the field of a traveling wave was observed in Ref. 4.

A quantum approach, which was previously used⁵ only for a rigorous resonance, must be used to describe the Bragg structure. The scattering pattern in this case is identical to the nonresonance scattering of electrons in the field of a standing wave.⁶ To determine the characteristics of resonance scattering, we must examine the finite resonance detuning. This paper is devoted to an investigation of this problem.

When the interaction time τ with the field is short, we can ignore the spontaneous emission, $\gamma\tau \ll 1$, where γ^{-1} is the lifetime of the excited atom. In the resonance approximation the Hamiltonian of the atom has the form

$$H = - \frac{(\hbar \nabla)^2}{2M} - \begin{pmatrix} \hbar \Delta & dE(y) \cos kx \\ dE(y) \cos kx & -\hbar \Delta \end{pmatrix}. \quad (1)$$

Here, the first term is the kinetic energy operator of an atom with mass M and the second term describes the transitions in a standing wave along the x axis, with a detuning 2Δ . The field amplitude, which varies slowly with y , differs from zero in a region of dimension l , and d is the dipole moment of the transition. The wave function of the incident particles $e^{ipy(0)}$ describes a monokinetic beam of atoms in the ground state. The problem reduces to that of determining the wave function of an atom after passage through the standing wave.

Under normal conditions the longitudinal momentum of the atom $p = Mv$ is much larger than the transverse momentum. Therefore, we can look for a wave function in the form $e^{ipy} \begin{pmatrix} \psi_2(x, y) \\ \psi_1(x, y) \end{pmatrix}$, where $\psi_{1,2}(x, y)$ are slowly varying functions of the coordinates.

When the time of the interaction with the field is short, where

$$(k\tau)^2 dE/M \ll 1 \quad (2)$$

the derivatives with respect to x can be discarded in the Hamiltonian, and the wave equation assumes the form $[V(y) = dE(y)/\hbar]$

$$iv \frac{\partial \psi}{\partial y} = - \begin{pmatrix} \Delta & V(y) \cos kx \\ V(y) \cos kx & -\Delta \end{pmatrix} \psi. \quad (3)$$

Let us assume that $V(y)$ is a rectangular, nonvanishing function equal to V_0 at $0 \leq y \leq l$. Thus, for $y > l$ we have

$$\psi_1(x) = \cos(\Omega(x)\tau) + \frac{i\Delta}{\Omega(x)} \sin(\Omega(x)\tau), \quad (4)$$

$$\psi_2(x) = -\frac{iV_0 \cos kx}{\Omega(x)} \sin(\Omega(x)\tau),$$

where $\Omega(x) = [V_0^2 \cos^2 kx + \Delta^2]^{1/2}$ is the local Rabi frequency and $\tau = l/v$. Expanding $\psi_{1,2}(x)$ in a Fourier series, we determine the probability W_n that the atom has a transverse momentum $n\hbar k$. The wave function of the atom in the ground (excited) state contains only even (odd) harmonics. In particular, for $\Delta \ll V_0$, $1/\tau$ we have $W_n = j_n^2(V_0\tau)$ [$j_n(z)$ is the n th-order Bessel function], which coincides with the result of Ref. 5. At large detunings $\Delta \gg V_0$, $1/\tau$ for even harmonics $W_{2n} = J_{2n}^2(V_0^2\tau/4\Delta)$, and the odd harmonics are small in terms of the parameter $(V_0/\Delta)^2$. At $V_0\tau \gg 1$ the atom scatters a large number of quanta, of the order of $V_0\tau$. For large n the Fourier coefficients can be calculated by the method of steepest descent. In this case the saddle point is determined from the condition

$$p_{\perp}(x) = 2n\hbar k = \pm \tau\hbar \frac{d\Omega(x)}{dx}, \quad (5)$$

which describes the momentum increment of a classical particle in the potential field $\hbar\Omega(x)$. Condition (2) makes it possible to ignore the variation of the initial x coordinate during the short time of flight. Thus, we find the quasi-classical distribution

$$W_{2n, 2n+1} = \frac{1}{2\pi} \left(1 \pm \frac{|\Delta|}{\sqrt{V_0^2 + \Delta^2}} \right) (n_0^2 - n^2)^{-1/2} \quad (6)$$

$$n_0 = \frac{\tau}{2} (\sqrt{V_0^2 + \Delta^2} - |\Delta|).$$

The radical function $(n_0^2 - n^2)^{-1/2}$ corresponds to the classical momentum distribution function $W(p_{\perp})$ of the scattered particles

$$W(p_{\perp}) = \frac{1}{\lambda} \int_0^{\lambda} dx \delta(p_{\perp} - p_{\perp}(x))$$

assuming that the initial coordinates of the incident particles are distributed uniformly along the x' axis, which is usually realized in view of the small (compared to the atomic-beam aperture) wavelength λ of light. In this case $2\hbar n_0 k$ is the maximum momentum for classical motion.

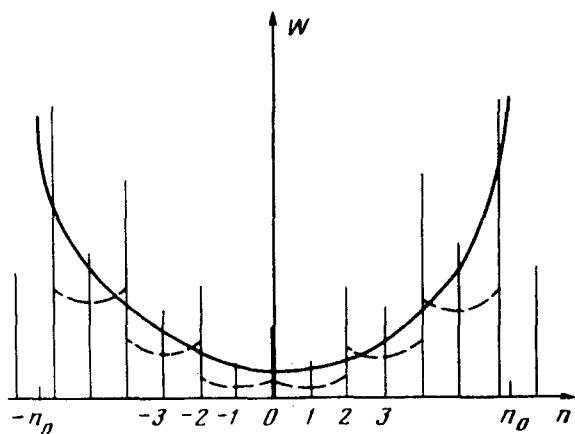


FIG. 1.

The distributions (6) for the even and odd peaks are proportional to the probabilities of finding the atom in the lower or upper quasi-energy state.

The distribution of the Bragg peaks is shown schematically in Fig. 1. For arbitrary parameters Δ , V_0 , and τ the amplitudes of these peaks oscillate about their average values given by Eq. (5). The solid line represents the classical distribution function corresponding to $\frac{1}{2}(W_{2n} + W_{2n+1})$.

When the time of flight T of a scattered particle to the detector is larger than γ^{-1} , the odd peaks are smeared out because of recoil due to spontaneous emission. The function, which describes the smearing out of the δ -shaped peak, is $f(\delta p_1) = \int d\mathbf{l} F(\mathbf{l}) \delta(\delta p_1 - l_x \hbar k)$, where $F(\mathbf{l})$ is the probability of photon emission in the \mathbf{l} direction with the transition of an atom to the ground state. For linearly polarized radiation, we have⁷ $f(\delta p_1) = (3/8\hbar k) [1 + (\delta p_1/\hbar k)^2]$. This broadening is represented by the dashed line in Fig. 1.

Thus, at $T\gamma > 1$ the diffraction pattern consists of δ -shaped peaks against a background of the distribution that was smeared by spontaneous relaxation. To determine how the field distribution along the y axis influences the particles scattering, we shall examine the model dependence $E(y)E_0 \cosh^{-1}(y/l)$. In this case Eq. (3) allows an exact solution.⁸ The smooth switching on of the field significantly affects only the odd maxima

$$W_{2n+1} = \cosh^{-2}(\pi \Delta \tau) J_{2n+1}^2(\pi V_0 \tau).$$

At $\Delta\tau \gg 1$ the odd peaks are exponentially small.

In conclusion, we note that the fine structure in the scattering of resonance particles is of interest for ultrahigh-resolution spectroscopy. Thus, in a weak field the scattering diagram contains two symmetrical first-order peaks which are described by the $F(\delta p_1)$ function. The amplitude of the peaks depends on detuning in a resonance manner, and the resonance width is determined by the transit time. This effect can be used to stabilize the frequency in molecular and atomic beams (for example, $\text{Ca}^{9,10}$) with weakly allowed transitions. Very weak fields are required for scattering of atoms in the first maximum. Thus, the required resonance field power is 10^{-4} W/cm² for Ca scattering at $\tau = 3 \times 10^{-5}$ sec.

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