Collapse of the electric field in double layers

N. G. Belova, A. A. Galeev, R. Z. Sagdeev, and Yu. S. Sigov
Institute of Space Research, USSR Academy of Sciences

(Submitted April 1, 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 9, 551–555 (5 May 1980)

By means of a numerical modeling of the Buneman instability it is shown that short-lived, nonstationary double layers with a potential differential of $\Delta\phi \leqslant 270\, T/e$ (T is the plasma temperature and e is the electron charge) are produced spontaneously in a quiescent plasma with an electron current. Self-similar solutions of the hydrodynamic equations are found. These equations describe the development of the plasma cavity in which a strong acceleration of the electron and ion components of the plasma occurs, and also the explosive buildup of the electric field with a characteristic distribution of a double layer.

PACS numbers: 52.35.Fp

An electrical double layer is usually understood to mean a plasma layer with comparable dimensions to the Debye radius, inside which there is a strong separation of charges and monotonic finite variation of the electric field potential. The simplest example is the cathode double layer which was first observed by Langmuir.[1] The appearance of a double layer is associated with the passage of a strong current through a plasma. Bohm[2] discovered that for a stable existence of the cathode layer, the ions must flow into the layer with a velocity

$$u_i \geqslant \sqrt{T_e/m_i}\ , \qquad (1)$$

where u_j, T_j, and m_j are the hydrodynamic velocity, temperature, and mass of the j-type particles. This condition follows from the requirement that the ion density should decrease slower in the direction toward the center of the layer than the density of the blocked electrons and, therefore, an excess of positively charged space charge should appear. In a completely analogous fashion the condition

$$u_e \gtrsim \sqrt{T_i/m_e}\,. \tag{2}$$

must be satisfied on the cathode side. Thus, a stationary double layer can be formed if the electrons and ions enter it from the opposite sides with velocities in excess of the critical velocity. In the case of strong double layers (i.e., at $e\Delta\phi \gg T_j$, where $\Delta\phi$ is the potential jump inside the double layer) the electron and ion fluxes are related to the Langmuir condition[3]

$$I_i = \sqrt{\mu}\, I_e\,, \tag{3}$$

where $I_j = n_j u_j$ and $\mu = m_e/m_i$. We can see that the ion flux must be at least sonic in order for a stationary double layer to exist in a plasma with a current. A very important question is how a quiescent plasma with a current can evolve to a state with a strong ion flux. This question has been ignored almost entirely in both theoretical and experimental studies of the double layer (see Refs. 4 and 5, respectively).

In this paper we show that nonstationary double layers are a consequence of the nonlinear evolution of the Buneman instability of a plasma with a current[6] and they can provide a brief acceleration of the ion streams to the amount predicted by the Langmuir relation (3). The appearance of electric double layers is illustrated by the results of a numerical modeling of the plasma behavior in a plane, one-dimensional layer under the condition of continuous particle injection at the boundary.[7,8] Figure 1 shows the behavior of the electrons and ions in the phase plane (x, v_x), which is characteristic of a nonstationary double layer. It applies to the case when the initial flow velocity of the electrons exceeds the Buneman instability threshold $[u_e = 1.8(T_e/m_e)^{1/2}]$. The plasma was assumed to be isothermal in order to avoid the

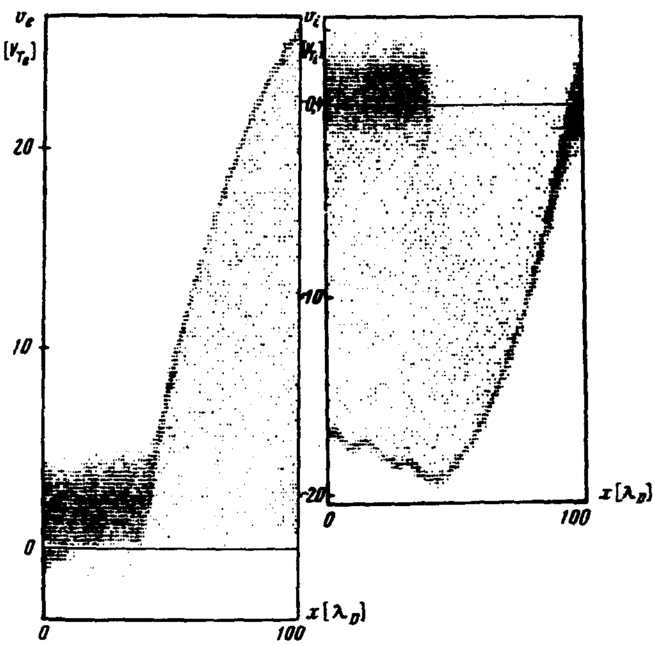

FIG. 1.

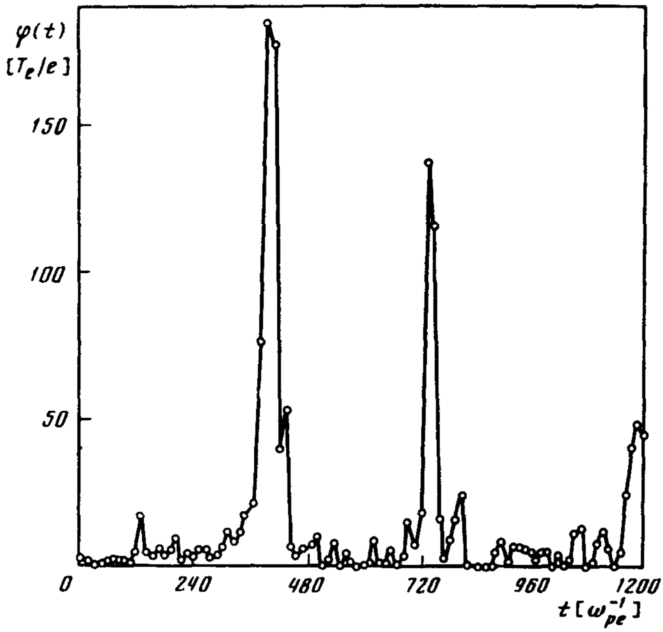

FIG. 2.

weaker effects of ion-sound instability development, which can lead to the formation of weak double layers.[9] The length of the calculated interval amounted to $100\lambda_D$ [($\lambda_D = (T_e/4\pi e^2 n)^{1/2}$) is the Debye length]. The particles with a specified velocity distribution $f_{0j}(v)$ were injected from the particle reservoirs from the boundaries of this interval; only particles with positive velocities entered the volume from the left boundary, and with negative velocities from the right. The $f_{0j}(v)$ were chosen in the form of Maxwellian distributions that were shifted by the magnitude of the flow velocity of particles of a given kind. We see that at some stage in the development of the Buneman instability a potential jump is formed in the plasma that accelerates the electrons in the direction of the current velocity, and the ions in the opposite direction. In the phase plane these accelerated particles have the form of thin jets. Figure 2, in which the potential difference at the ends of the interval is depicted as a function of time, shows that such nonstationary double layers appear during the brief time interval that is necessary for development of the Buneman instability. The buildup of the potential difference with time is explosive in nature [$\Delta\phi \sim (t_0 - t)^{-2}$].

Although the initial stage of Buneman instability in a cold plasma allows an exact solution, the complete description of the entire preparatory stage of double-layer development is nonetheless a complicated problem at the present time. Here, therefore, we shall restrict ourselves to a description of its explosive stage on the basis of an analysis of the equations of motion and the continuity equations of both components in a quasi-neutral plasma:

$$m_e I_e^2/2n^2 + T_e \ln(n/n_0) = m_e I_e^2/2n_0^2 + e\phi, \tag{4}$$

$$nu_e = I_e = \text{const}, \tag{5}$$

$$m_i(\partial u_i/\partial t + u_i \partial u_i/\partial x) = -T_i\, \partial \ln n/\partial x - e\partial\phi/\partial x\,, \tag{6}$$

$$\partial n/\partial t + \partial(nu_i)/\partial x = 0\,. \tag{7}$$

In the stationary case it is easy to determine from this the dependence of the particle densities on the potential and to show that the Bohm condition, i.e., the requirement that a negative space charge must appear at the left edge of the double layer $\{d\,[n_e(\phi) - n_i(\phi)]/d\phi \geqslant 0 \text{ as } \phi \to 0\}$, is identical to the Buneman instability criterion. Since its characteristic development time is determined by the inertia of the ions, the electron flux is assumed to be stationary. Using Eq. (4), we write the equation of motion for ions in the form

$$(\partial u_i/\partial t + u_i\,\partial u_i/\partial x) = -(\partial/\partial x)\,[(T_e + T_i)n + m_e I_e^2/n\,]\,. \tag{8}$$

Here, the second term on the right-hand side can be interpreted as a negative pressure: the pressure increases as a result of rarefaction of the plasma, which leads to a further ejection of the plasma. The buildup of the finite-amplitude rarefaction perturbations occurs explosively when the last term dominates in the total pressure. We shall seek the solution of Eqs. (7) and (8) in the self-similar form:

$$n = (t_0 - t \qquad f_1[\quad /(t_0 - t)^{\alpha}\,], \tag{9}$$

$$u_i = (t_0 - t)^{\alpha - 1} f_2[x/(t_0 - t)^{\alpha}]\,. \tag{10}$$

The solution of Eqs. (7) and (8) for $\alpha = 0$ is the closest to the results of the numerical experiment:

$$\begin{aligned} n &= n(0)(1 - t/t_0)\cos^{-2}(x/L), \\ u_i &= [L/2(t_0 - t)]\sin(2x/L), \end{aligned} \tag{11}$$

where $n(0) = (\sqrt{\mu})I_e t_0/L$; $0 \geqslant x \geqslant -\pi L/2$.

In this case the energy for the particle acceleration in the double layer is drawn from the external current generator. The particle reservoirs play the role of the latter in the numerical modeling. As a result of imposing the energy conservation requirement in the system, we must set $\alpha = \frac{1}{2}$. Thus, the solution has the form

$$\begin{aligned} n &= 2n(0)(1 - t/t_0)^{1/2}\, \frac{1 + 2x^2/L^2(1 - t/t_0)}{\sqrt{1 + x^2/L^2(1 - t/t_0)}}\,, \\ u_i &= [\quad /2(t_0 - t)]\,/\,[1 + 2x^2/L^2(1 - t/t_0)\,]\,. \end{aligned} \tag{12}$$

In this case the formation process of a strong, nonstationary double layer looks like a collapse of the quasi-static electric field in the plama cavity, which is stopped by the increasing pressure of the electric field. The maximum potential jump can be estimated from Eqs. (11) and (12) using Eq. (4).

[1]I. Langmuir, Phys. Rev. **22**, 450 (1913).
[2]D. Bohm, In: The Characteristics of Electrical Discharges in Magnetic Fields, ed. by A. Guthrie and R. Wakerling, McGraw-Hill, N.Y., 1949, p. 77.
[3]I. Langmuir, Phys. Rev. **33**, 954 (1929).
[4]P. Carlqvist, In: Wave Instabilities in Space Plasmas, ed. by P. Palmadesso and K. Papadopoulos, D. Reidel, Dordrecht, 1979, p. 83.
[5]S. Torven, ibid., p. 109.
[6]L. A. Artsimovich and R. Z. Sagdeev, Fizika plazmy dlya fizikov (Plasma Physics for Physicists), Atomizdat, Moscow, 1979, p. 287.
[7]Yu. S. Sigov and Yu. V. Khodyrev, Chislennye Metody Sploshnoi Sredy **7**, 109 (1976).
[8]G. Joyce and R. H. Hubbard, J. Plasma Phys. **20**, 391 (1978).
[9]T. Sato, Ion Acoustic Double Layers, lecture SMR/61-32, Autumn College on Plasma Physics, October 16—November 23, 1979.