Quantum magnetic-size effects in metals with open Fermi surfaces

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Giant quantum magnetization oscillations of metallic plates, which are attributable to the conduction electrons on the open sections of the Fermi surface, were observed. An inhomogeneous state (Shoenberg effect) can occur in magnetic fields in which there is a degeneration of the quantum levels of electron energy.

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The quantum thermodynamic oscillations of conduction electrons in a layer of metal in a parallel magnetic field **H**, which had been predicted by Kosevich and Lifshitz,[1] were observed fairly recently in the investigation of electrical conductivity of filamentary antimony crystals (whiskers),[2] and also in measuring the magnetization of whiskers of a number of metals.[3] Quantum oscillations in weak magnetic fields can occur[4] in metal films. Similar oscillations were observed in the electrical conductivity of bismuth films.[5]

The effects mentioned above pertain to metals with closed electron orbits in a magnetic field, if there is a specular reflection of electrons from the surface of the sample. The quantum oscillation periods, depending on H or the layer thickness L, are determined by the area of extremal electron orbits that are truncated by the sample's size.[1] In the neighborhood of the field H_c which cuts off the extremal orbits by the sample's size, there is a sharp transition from oscillations that are characteristic for a massive metal to magnetic-size oscillations. The quantum oscillations in the region of fields $H \sim H_c$ were analyzed in Ref. 6. In the case of diffusive electron scattering from the sample's surface, the oscillations corresponding to the given extremal orbit drop out[7] and quantum oscillations occur along the nonextremal sections of the Fermi surface.[8]

Since the extremal investigation of quantum magnetic-size effects has become realistic,[2,3,5] we focus attention on the possibility of the occurrence of quantum effects produced by electrons with open orbits. As is known, the electrons in a massive metal do not participate in the production of quantum oscillations in the open sections of the Fermi surface. In a thin sample whose thickness L is smaller than the length of the free path of the charge carriers l, the finite motion of electrons along the normal to the sample's surface leads to quantization of the energy of the charge carriers that belong to open sections of the Fermi surface. These electrons contribute substantially to the thermodynamic and kinetic characteristics of thin plates. This gives rise to the appearance of new oscillation effects which yield detailed information on the shape of the open electron orbits.

If there is a specular reflection of the charge carriers at the sample's surface, then the quantum energy levels of the conduction electrons in the plate in a parallel magnetic field are determined by the quasi-classical quantization condition[1]

$$S\left(\epsilon,\, p_z,\, p_x;\, \frac{LeH}{c}\right) = \frac{2\pi\hbar eH}{c}\,(n+\gamma), \tag{1}$$

where S is the cross-sectional area of the intersection of the isoenergetic surface $\epsilon(\mathbf{p}) = \epsilon$ with one plane $p_z = \text{const}$. This area is bounded by the straight lines p_x and $p_x + (LeH/c)$ (see Fig. 1a). The magnetic field is directed along the z axis and the normal to the plate is directed along the y axis. The value of γ in Eq. (1) is smaller than or equal to unity, which is an approximation of the surface potential by an infinitely high potential barrier. Here e is the electron charge, $\hbar$ is the Planck constant, and c is the velocity of light.

In the case of open electron orbits the area S in Eq. (1) is a periodic function depending on p_x with a period equal to $\hbar G$, where G is the period of the reciprocal lattice in the direction of the open orbits. For magnetic fields,

$$H_j = j\frac{c\hbar G}{eL}, \qquad j = 1, 2, 3, \ldots \tag{2}$$

the area S, which is independent of p_x, is

$$S\left(\epsilon,\, p_z,\, p_x;\, \frac{LeH_j}{c}\right) = j\sigma(\epsilon,\, p_z), \tag{3}$$

where σ is the area of the open section of the isoenergetic surface within the limits of the reciprocal lattice cell. Hence, it follows from Eq. (1) that at $H = H_j$ the quantum energy levels are independent of the quasi-momentum component p_x. In the fields $H \neq H_j$ the degeneracy of levels is removed. This results in the appearance of characteristic properties in the density of states of electrons with open orbits and an oscillatory dependence of the thermodynamic and kinetic values on H and L.

Let us examine the quantum magnetization oscillations of conduction electrons. In the fields $H \neq H_j$ satisfying the condition

$$\frac{a}{L} << \frac{|H - H_j|}{H_1} << 1, \tag{4}$$

for the oscillating part of magnetization,we obtain

$$M_{osc} = \frac{G}{\pi^2 L^{3/2}} \sqrt{\frac{e}{c}} \left(p_e - \frac{\sigma}{\hbar G}\right)\left(\frac{\partial \sigma}{\partial \zeta}\right)^{-1} \left|\frac{\partial^2 \sigma}{\partial p_{ze}^2} \sum_{s=1}^{2} \frac{1}{R_s}\right|^{-1/2} \sqrt{\cos \alpha}$$

$$\frac{1}{\sqrt{j}} |H - H_j|^{-1/2} \sum_{k=1}^{\infty} \frac{1}{k^2} \sin\left\{k\left[\frac{L\sigma}{\hbar^2 G} + \frac{eL^2}{jc\hbar^2 G}\left(p_e - \frac{\sigma}{\hbar G}\right) H - H_j)\right] \pm \frac{\pi}{4} \pm \frac{\pi}{4}\right\} \times$$

$$\cos\left(\frac{kj}{2m_0} \frac{\partial \sigma}{\partial \zeta}\right) \Psi\left(\pi k \frac{TL}{\hbar^2 G} \frac{\partial \sigma}{\partial \zeta}\right), \tag{5}$$

where a is the lattice constant, the mass m_0 determines the spin splitting of the electron energy levels in a magnetic field, $\Psi(z) = z/\sinh(z)$, ρ_e is the extremal chord of the Fermi surface $\mathscr{E}(\mathbf{p}) = \zeta$ in the direction of the normal $\mathbf{N}$ to the plate, α is the angle between $\mathbf{N}$ and the normal to the Fermi surface at the points it intersects the extremal chord, and R_s are the radii of curvature at these points (see Fig. 1a); the signs of $\pi/4$ in the argument of the sine are determined by the signs of $(H - H_j)$ and $\partial^2\sigma/\partial p_{ze}^2$, respectively, at the point p_{ze} of the extremum of the area $\sigma, \gamma = 1$.

At $H = H_j$ for M_{osc} we have the following expression:

$$M_{osc} = M_j \equiv -\frac{1}{\pi^2\sqrt{2\pi}} \frac{e}{\hbar c} \sqrt{\frac{G}{L}} \left(\frac{\partial \sigma}{\partial \zeta}\right)^{-1} \left|\frac{\partial^2 \sigma}{\partial p_{ze}^2}\right|^{-1/2} \frac{\sigma}{j} \sqrt{\cos \alpha}$$

$$\times \sum_{k=1}^{\infty} \frac{1}{k^{3/2}} \sin\left(k \frac{L\sigma}{\hbar^2 G} \pm \frac{\pi}{4}\right) \cos\left(\frac{kj}{2m_0} \frac{\partial \sigma}{\partial \zeta}\right) \Psi\left(\pi k \frac{TL}{\hbar^2 G} \frac{\partial \sigma}{\partial \zeta}\right), \tag{6}$$

where the upper sign in the argument of the sine is chosen for $\partial^2\sigma/\partial p_{ze}^2 > 0$ and the lower sign is chosen for $\partial^2\sigma/\partial p_{ze}^2 < 0$. In the fields $|H - H_j| \sim H_1 a/L$ Eq. (5) and (6) are matched.

It follows from the results for M_{osc} that as the magnetic field changes the magnetization near H_j produces sharp spikes whose amplitude increases as $|H - H_j|^{-1/2}$ when $H \to H_j$, and reaches the values of M_j (see Fig. 1b).

The oscillation period

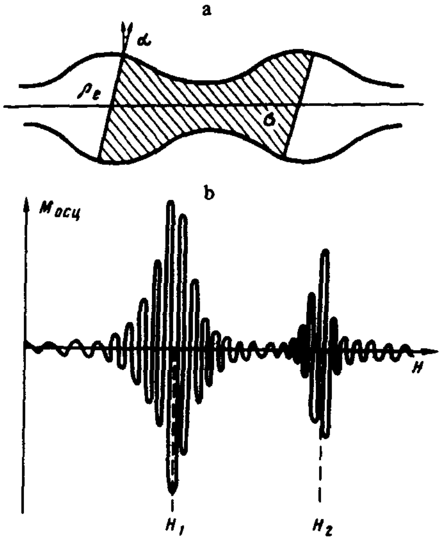

FIG. 1.

$$\Delta H = 2\pi j \frac{c\hbar^2 G}{eL^2}\left|\rho_e - \frac{\sigma}{\hbar G}\right|^{-1} \tag{7}$$

can be expressed in terms of the extremal values of the chord and the area σ corresponding to the given section of the Fermi surface (Fig. 1a). In the intermediate fields $H \neq H_j$ the magnetization oscillations have a more complex field dependence than that in Eq. (5), which is determined by the truncated areas of the extremal electron orbits.[1] In these fields the oscillation amplitude is

$$\left|\frac{M_{osc}}{M_j}\right| \sim \left(\frac{a}{L}\right)^{1/2} << 1. \tag{8}$$

We can easily see that at $H \approx H_j$ the amplitude of the examined oscillations is comparable with the contribution to M_{osc} from the electrons in the extremal closed sections of the Fermi surface with orbits that do not touch the sample's boundaries. In metals in which the dispersion of charge carriers is complex, the magnetization oscillations are caused by electrons from different sections of the Fermi surface. The contribution of the open sections to M_{osc} can be easily isolated because of the significant modulation of the magnetic-size oscillations described above (see Fig. 1b) and the dependence of M_j on L.

The oscillations, which decay exponentially with increasing temperature T, are significant at $T \lesssim T_0$, where $T_0 = \hbar^2 G/(\pi L \partial\sigma/\partial\zeta)$. Assuming for estimates that $\partial\sigma/\partial\zeta \sim 2\pi m$ (m is the mass of the free electron) for a plate of thickness $L \approx 10^{-3}$ cm we have $T_0 \approx 0.3$ K, $H_1 \approx 40$ kOe, and $\Delta H \approx 0.4$ Oe. At temperatures $T \lesssim T_0$ and $H \to H_j$ the magnetic susceptibility χ may reach high values, which renders the homogeneous state unstable, i.e., the Shoenberg effect is produced.[9] Like in the massive metals (see Ref. 10), the nonuniform structure is realized under the conditions $|M| \ll H$ and $|\chi| > 1$.

The mentioned magnetization anomalies of the conduction electrons, which are attributable to the corresponding density peculiarities of the electronic states, appear in the quantum oscillations of the thermodynamic and kinetic quantities. To observe the indicated effects, the samples ($l > L$) must be sufficiently pure and satisfy the condition for a specular reflection of electrons from the sample's surface.

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