Hybridization of collective oscillations in ^{3}He–A near the transition to the A_1 phase

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(Submitted 2 April 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 10, 593–595 (20 May 1980)

Hydrodynamic oscillations of longitudinal magnetization and entropy in a superfluid ^{3}He–A in a magnetic field are analyzed. It is shown that in the neighborhood of a transition to the A_1 phase there is a strong mixing of the entropy (and temperature) oscillations with the spin wave.

PACS numbers: 67.50.Fi, 64.70.Ja, 65.50. + m

Liquid ^{3}He-A is a mixture of two, weakly interacting superfluid components characterized by Cooper pairing in the spin states ↑↑ and ↓↓. In the presence of a strong magnetic field and in the immediate vicinity of the transition to a normal phase, the quasi-particle pairing amplitudes with a projection of the total spin $S_z = \pm 1$ are not equal to each other ($\Delta_\uparrow \neq \Delta_\downarrow$). Taking into account this fact, the order parameter describing the superfield ^{3}He-A can be represented in the form

$$A_{\mu i} = \Delta(T) d_\mu u_i = \Delta(T)(\alpha_+ \mathbf{d}_1 + i\alpha_- \mathbf{d}_2)_\mu (\mathbf{u}_1 + i\mathbf{u}_2)_i \quad , \tag{1}$$

where $(\mathbf{d}_1,\mathbf{d}_2)$ and $(\mathbf{u}_1,\mathbf{u}_2)$ are unit vectors in the spin orbital spaces, respectively, $\alpha_\pm = (\Delta_\uparrow \pm \Delta_\downarrow)/2\Delta$, and $\Delta^2 = \frac{1}{2}(\Delta_\uparrow^2 + \Delta_\downarrow^2)$.

In the hydrodynamic description of superfluid ^{3}He-A the energy density ϵ, in addition to the mass density ρ, entropy density $S=\rho\sigma$, momentum density $\mathbf{g}$, and magnetization $\mathbf{M}$, depends on two superfluid velocities

$$\mathbf{v}_s = (\hbar/2m) u_{1i} \nabla u_{2i}, \qquad \mathbf{w}_{SP} = (\hbar/2m) d_{1\mu} \nabla d_{2\mu}, \tag{2}$$

where the spin velocity w_{SP}, in contrast to v_S, is a Galilean invariant. Below we shall examine a linear, dissipation-free hydrodynamics for specified orientations of the axes $s=[\mathbf{d}_1\mathbf{d}_2]$ and $l=[\mathbf{u}_1\mathbf{u}_2]$, when

$$d\epsilon = \mu\, d\rho + T dS + \mathbf{v}_n d\mathbf{g} + \omega\, dM + \mathbf{g}_S d\mathbf{v}_S + \mathbf{g}_{SP} d\mathbf{w}_{SP}, \tag{3}$$

where M is the longitudinal magnetization and $\mathbf{g}=\rho\mathbf{v}_n+\mathbf{g}_S$ because of the Galilean covariance. The question of whether the unit vector s can be included in the hydrodynamic variables was discussed elsewhere.[1]

Using the well-known scheme of Ref. 2, we can write the system of questions

$$\rho + \nabla(\rho\mathbf{v}_n + \mathbf{g}_S) = 0, \quad \mathbf{g} + \nabla P = 0,$$
$$S + \nabla(S\mathbf{v}_n) = 0, \quad M + \nabla(M\mathbf{v}_n + M_S\mathbf{g}_{SP}/\rho) = 0, \tag{4}$$

which is supplemented by the equations of motion

$$\dot{\mathbf{v}}_S = -(1/\rho)\nabla P + \sigma\nabla T + (1/\rho) M\nabla\omega,$$
$$\mathbf{w}_{SP} = -(\gamma\hbar/2m)\nabla\omega = -(1/\rho) M_S \nabla\omega, \tag{5}$$

where $M_S=\gamma\hbar\rho/2m$ is the magnetization of a totally polarized liquid ^{3}He.

As a result of splitting the states of the Cooper pairs in a magnetic field, the spin and spatial degrees of freedom of the A phase become mixed,[2] which also occurs in the structure of superfluid fluxes of mass $\mathbf{g}_S$ and spin $\mathbf{g}_{SP}$:

$$\mathbf{g}_S = \rho_S[(\mathbf{v}_S - \mathbf{v}_n) + a\mathbf{w}_{SP}],$$
$$\mathbf{g}_{SP} = \rho_S[\mathbf{w}_{SP} + a(\mathbf{v}_S - \mathbf{v}_n)], \tag{6}$$

where the coefficient $a=2a_+a_-=(\Delta_\uparrow^2-\Delta_\downarrow^2)/(\Delta_\uparrow^2+\Delta_\downarrow^2)$ characterizes the proximity of a transition to the A_1 phase for which $\Delta_\uparrow\neq 0$ and $\Delta_\downarrow=0$ (or vice versa). We shall not introduce in explicit form the orbital anisotropy characteristic of ^{3}He-A, which can be easily taken into account, but it is not important for our purpose.

At some distance from a transition to the A_1 phase (where $|a|\ll 1$) the superfluid ^{3}He–A has weakly coupled hydrodynamic modes such as first and second sound and also a (longitudinal) spin wave with propagation rates $c_1 \gg c_S \gg c_2$. As we approach the $A\to A_1$ transition, a kinematic mixing of the spin and orbital degrees of freedom arises (see Ref. 6), which leads to curious dynamic effects. As shown in Ref. 3, a strong mixing of the density and longitudinal magnetization oscillations in the fourth sound regime ($v_n\equiv 0$) occurs in the immediate vicinity of a transition to the A_1 phase. On the other hand, if the motion of the normal component is not blocked, then we can easily obtain a system of linearized equations from Eqs. (4), (5), and (6) describing the oscillations ρ, σ, and $\xi = M/M_S$:

$$\ddot{\rho} - (\partial P/\partial \rho)\nabla^2\rho - (\partial P/\partial\sigma)\nabla^2\sigma - (\partial P/\partial\xi)\nabla^2\xi = 0,$$
$$\ddot{\sigma} - (\rho_S/\rho_n)\sigma^2\nabla^2 T - (\rho_S/\rho_n)\sigma(\xi - a)(M_S/\rho)\nabla^2\omega = 0, \qquad (7)$$
$$\ddot{\xi} - (\rho_S/\rho_n)[(\xi - a)^2 + (1 - a^2)(\rho_n/\rho)](M_S/\rho)\nabla^2\omega - (\rho_S/\rho_n)\sigma(\xi - a)\nabla^2 T = 0.$$

If the thermal expansion and magnetostriction are disregarded, then the oscillations of the entropy and of the longitudinal magnetization are not coupled to the density oscillations and, setting $\rho = \text{const}$, we obtain closed set of equations for σ and ξ:

$$\ddot{\sigma} - u_2^2\nabla^2\sigma - \sigma(\xi - a)^{-1}[u_S^2 - (1 - a^2)(\rho_S/\rho)(M_S/\rho)(\partial\omega/\partial\xi)]\nabla^2\xi = 0,$$
$$\ddot{\xi} - u_S^2\nabla^2\xi - (\xi - a)\sigma^{-1}[u_2^2 + (1 - a^2)(\rho_S/\rho)\sigma/(\xi - a)(\partial T/\partial\xi)]\nabla^2\sigma = 0, \qquad (8)$$

where

$$u_2^2(a) = (\rho_S/\rho_n)[\sigma^2(\partial T/\partial\sigma) + \sigma(\xi - a)(\partial T/\partial\xi)],$$

$$u_S^2(a) = (\rho_S/\rho_n)\{[(\xi - a)^2 + (1 - a^2)(\rho_n/\rho)](M_S/\rho)(\partial\omega/\partial\xi) + \sigma(\xi - a)(\partial T/\partial\xi)\}. \qquad (9)$$

Equations (8) describe two hydrodynamic modes of the split A phase of liquid ^{3}He, whose propagation velocity squared is given by

$$c_\pm^2(a) = \tfrac{1}{2}\{u^2(a) \pm [u^4(a) - 4(1 - a^2)v^4]^{1/2}\}, \qquad (10)$$

where

$$u^2(a) = u_S^2(a) + u_2^2(a),$$

$$v^4 = (\rho_S/\rho)(\rho_S/\rho_n)\sigma^2[(M_S/\rho)(\partial\omega/\partial\xi)(\partial T/\partial\sigma) - (\partial T/\partial\xi)^2].$$

At some distance from the A phase ($|\alpha| \ll 1$) there is only a weak coupling of the entropy and longitudinal magnetization oscillations (note that $M \ll M_S$). As we approach the temperature for transition to the A phase, $\alpha^2 \to 1$, and the longitudinal magnetization wave sets up an intensive oscillatory motion of entropy and temperature, dictating a high propagation velocity for them. The high-frequency branch in the A_1 phase links up with the hybrid, oscillatory mode examined recently.[4] However, the frequency of the lower branch vanishes together with $(1 - \alpha^2)$.

[1]H.Pleiner, J. Phys. C10, 2337 (1977).
[2]I. M. Khalatnikov, Teoriya sverkhtekuchesti (Theory of Super-fluidity), Nauka, Moscow, 1971.
[3]A. D. Gonzadze, G. E. Gurgnishvili, and G. A. Kharadze, J.E.T.P. 75, 1504 (1978) [Sov. Phys. JETP 48, 759 (1978)].
[4]M. Liu, Phys. Rev. Lett. 43, 1740 (1979).