Possible scenario for the birth of galaxies

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A model which has galaxies forming as the result of an agglomeration of spherical clouds of neutral hydrogen is analyzed at a quantitative level. An approximate expression is derived for the time of "collapse" of the matter, accompanied by the formation of galaxies. The duration of the agglomeration process is calculated. © *1994 American Institute of Physics.*

1. The number of galaxies with masses between m and $m+dm$ in a unit volume of space can be described approximately by the empirical formula

$$dN/dm=n_m=C_0/m^2, \quad C_0=\text{const}. \tag{1}$$

In Searle and Zinn's qualitative model[1] it is assumed that galaxies form as the result of an agglomeration of a multitude of small fragments. In the present letter we take a quantitative look at this process (ignoring breakup) on the basis of the Smoluchovski equations:[2]

$$\frac{d}{dt}n_k=\tfrac{1}{2}\sum_{i+j=k} n_i n_j S_{ij}-n_k\sum_{i=1}^{\infty} n_i S_{ki}, \quad S_{ij}=\langle \sigma v\rangle_{ij}, \tag{2}$$

where σ_{ij} is the cross section for the agglomeration of two bodies with masses $m_i=im_1$ and $m_j=jm_1$. We assume that the bodies are spherical and have a uniform density ρ_0. Assuming that their relative velocities $v_{ij}=v_0$ are the same, and taking their gravitational attraction into account, we can then show that the agglomeration cross section is

$$\sigma_{i,j}=\pi p^2=\pi r_{\min}^2+\pi R r_{\min}, \quad R=2GM/v_0^2, \quad M=(i+j)m_1, \tag{3}$$

where p is the impact parameter, G is the gravitational constant (or "Newton's constant"), $r_{\min}=a_i+a_j$, and $a_k=a_1 k^{1/3}$ are the radii of the spheres. Gravitation is ignored in the case $R\ll r_{\min}$, while in the case $R\gg r_{\min}$ the second gravitational term is predominant. The two cases can be approximated by

$$\langle \sigma v\rangle_{i,j}=A_0(i^s+j^s), \quad s=\tfrac{2}{3} \text{ or } \tfrac{4}{3} \quad \left(A_0=\text{const}=2Gm_1\frac{a_1}{v_0} \text{ for } s=\tfrac{4}{3}\right), \tag{4}$$

which allows a "partial" solution of the Smoluchovski equations.

2. Adopting the initial conditions $n_1(0)\neq 0$, $n_{i\geq 2}(0)=0$, we transform to a dimensionless time $t_*=A_0 n_1(0)t$ and dimensionless variables $\nu_k=n_k(t_*)/n_1(0)$. We introduce $M_q=\Sigma k^q \nu_k$. The condition $M_1=1$ then means that the total mass of the particles is conserved; for M_0 we find the equation $dM_0/dt_*=-M_0M_s$.

Now introducing a "quasitime" $d\tau = M_0(t_*)dt_*$ and some new functions $f_k(\tau) = \nu_k/M_0$, we can rewrite the equations for them as follows:

$$\frac{d}{d\tau} f_k + k^s f_k = S_k(\tau), \quad S_k(\tau) = \sum_{j=1}^{k-1} j^s f_j f_{k-j}, \quad N(\tau) = \sum_1^{\infty} f_k = 1. \tag{5}$$

Following Ref. 3, we can then find all the functions by a recurrence procedure:

$$f_{k\geq 2}(\tau) = \exp(-k^s\tau)\int_0^{\tau} \exp(k^s\tau')S_k(\tau')d\tau'. \tag{6}$$

To find a complete solution, however, we also need to express the quasitime τ in terms of the actual time; we must resort to numerical methods to do this. Since the analytic expressions in (6) are extremely complicated for $k \geq 3$, we also determined the integral in (6) numerically. The results show that at large values of k the functions can be described by the asymptotic expression

$$f_k(\tau) \approx a_s^{-1} k^{-\beta_s} \exp(-k\gamma_s^2). \tag{7}$$

In the case $s=1$, an exact solution is known (see, in particular, Refs. 4 and 5). That solution can be found by setting $x = x(\tau) = \tau\exp(-\tau)$ and then writing the inverse dependence as a power series in x:

$$\tau = \tau(x) = \sum_{k=1}^{\infty} C_k x^k, \quad C_k = \frac{k^k}{k!k} \quad (0<\tau<1). \tag{8}$$

Direct substitution then verifies that the functions $f_k = \tau^{-1} C_k x^k$ are solutions of Eqs. (5) for the case $s=1$. At large values of k we have

$$\alpha_1 = \tau\sqrt{2\pi}, \quad \beta_1 = 3/2, \quad \gamma_1 = \sqrt{\tau - 1 - \ln\tau} \quad (0<\tau<1). \tag{9}$$

Numerical calculations show that for all $s \geq 1$ the functions $\gamma_s(\tau)$ vanish at a certain "collapse" time $\tau = \tau_s^{(\mathrm{coll})}$ (in particular, we find $\tau_1^{(\mathrm{coll})} = 1$ in the case $s=1$). At earlier times, however, we can use the relation $\langle k \rangle = \Sigma k f_k = 1/M_0$ to find the actual time.

3. For generality, we carried out numerical calculations for the five values $s = 4/6$, 5/6, 6/6, 7/6, and 8/6. We calculated the first 1000 functions $f_k^s(\tau)$ and determined the parameters α, β, and γ for each time from the values of the three functions with $k=80$, 90, and 100. From these calculations we found an approximate formula for the collapse times:

$$\tau_s^{(c)} \approx 1 - 1.32(s-1) + 0.32(s-1)^2. \tag{10}$$

Now introducing the "reduced quasitime" $\tau_* = \tau/\tau_s^{(c)}$, we see that in the interval $0.5 \leq \tau_* \leq 1$ the functions γ for the three indices $s = 6/6$, 7/6, and 8/6 fall off in an essentially linear fashion $[\gamma \approx \mathrm{const} \times (1-\tau_*)]$, while the functions α_s increase in a nearly linear fashion, with the time.

Figure 1 shows the exponents $\beta_s(\tau_*)$. We see that in the case $s=1$ the exponent quickly reaches its asymptotic value $\beta_1 = 3/2$. For the gravitational case, $s=4/3$, we can use the approximation $\beta_{4/3} \approx 1.5 + 0.41\tau_*^{-3.8}$.

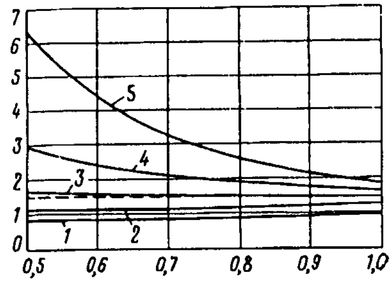

FIG. 1. The exponents $\beta_s = F(\tau_s)$.

At the time of collapse, $\tau_* \to 1$, we have $\beta_{4/3} \approx 1.9$. This is approximately the value which is observed for the mass distribution of galaxies.

The functions $M_0(\tau_*) = 1/\langle k \rangle$ found numerically are shown by the solid curves in Fig. 2. In the limit $\tau_* \to 1$ these functions should vanish, but there are deviations from this predicted behavior at the right ends of these curves. The deviations occur because the number of functions f_k taken into account was only finite, although large (1000). The dashed curves show the approximate formula which we adopted:

$$M_0^{\text{appr}} \approx (1-\tau_*)^p, \quad p = 1 + a(1-s), \quad a = 1.30. \tag{11}$$

We use this formula to find the actual (dimensionless) time.

In the case of most interest here, $s > 1$, we have

$$t_* = \int_0^\tau M_0^{-1} d\tau \approx \tau_s^{(c)} \int_0^{\tau_*} (1-x)^{-p} dx = \frac{\tau_s^{(c)}}{a(s-1)} [1 - (1-\tau_*)^{a(s-1)}]. \tag{12}$$

In this case, the critical instant $\tau_* = 1$ is reached after a finite (dimensionless) time $t_*^{(c)} = \tau_s^{(c)}/a(s-1)$. In the gravitational case, $s = 4/3$, in which we are interested, this result corresponds numerically to an actual collapse time

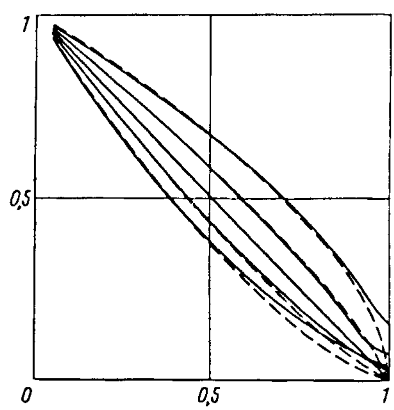

FIG. 2. The functions $M_0 = 1/\langle k \rangle = F(\tau_s)$.

$$T_{\text{coll}}=\frac{t_*^{(c)}}{A_0 n_1(0)}=0.687\frac{v_0}{G\rho_0 a_0}, \tag{13}$$

where $G=6.67\times 10^{-8}$ $cm^3/(s^2\cdot g)$ is the gravitational constant, a_0 is the radius of the original identical spheres, and v_0 is their velocity. Since the mass spectrum at the time T_{coll} obtained from these spheres is approximately the spectrum observed for galaxies ($dN/dm\sim m^{-2}$), we will attempt to apply Eq. (13) to the picture of the birth of galaxies.

4. According to the present understanding,[1] the age of the universe is $T_u=(16-18)\times 10^9$ yr, and the galaxies arose at the epoch at which the red shift $z=(\lambda_{\text{obs}}-\lambda_0)/\lambda_0$ had the value $z=4-10$ (Ref. 6). The latter number, $z=10$, corresponds to a time 300×10^6 yr after the Big Bang according to the relativistic formula $t=T_u/(1+z+z^2/2)$. We first adopt this time as the general "collapse time" T_{coll} in (13), assuming that by this time all the galaxies had become isolated from each other and subsequently evolved independently, so their mass spectrum underwent no further changes. Second, we assume that the agglomeration process began at a time t_0 after the Big Bang, and that the initial objects were spherical clouds of neutral gas (hydrogen) with a radius equal to the Jeans length $a_0=r_J=v_s(3/8\pi G\rho_0)^{1/2}$ (Ref. 6), where v_s is the sound velocity. Third, we assume that after the Big Bang the density fell off over time as $\rho\sim t^{-2}$. Assuming $\rho_0=\rho_{\text{mod}}(T_u/t_0)^2$, we rewrite (13) as an explicit expression for the time at which the agglomeration process begins:

$$t_0=\xi T_{\text{coll}} v_s/v_0, \quad \xi=(T_u/2)\sqrt{G\rho_{\text{mod}}}\,. \tag{14}$$

If we ignore the "dark mass" and take the modern density to be its observed value $\rho_{\text{mod}}=3\times 10^{-31}$ g/cm^3, we find a numerical coefficient $\xi=0.04$. The assumption $v_0\sim v_s$ would then mean that the agglomeration began at a time $t_0=12\times 10^6$ yr after the Big Bang. This result should be increased by a factor ~ 3 to take the dark mass into account, under the assumption that the latter is ten times the density of observable matter. The agglomeration process then apparently lasted $(260-290)\times 10^6$ yr and ended with a process in which the galaxies became isolated from each other, with the mass distribution observed today.

In Ref. 7 we discussed a model hydrodynamic problem of the gravitational contraction of a gas cloud having the shape of a triaxial ellipsoid. In contrast with the pancake theory of Ya. B. Zel'dovich, which ignores gravitation, we showed that a flattening of the ellipsoid may occur not in the plane with the maximum cross-sectional area but in a plane perpendicular to it. Accordingly, the disk of galaxies and its halo can rotate in different directions, in agreement with what is observed for spherical clusters in the halo of the local galaxy.[1]

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