Electron energy transport in a weakly collisional plasma

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It is shown analytically that the energy flux is not collinear with the temperature gradient during the heating of a low-density plasma, because of inverse bremsstrahlung. The analysis is carried out by dividing the electrons into groups of thermal collisionless electrons, which determine the energy transfer, and cold (subthermal) collisional electrons, which determine the increase in the effective temperature.

Research in laser controlled fusion has revealed several paradoxical properties of heat transfer in a weakly collisional, fully ionized plasma when the electron mean free path l_e is much longer than the length scale of the spatial variations, L. One of these paradoxical properties is a limitation on electron heat transfer (see, for example, a review[1]). We showed in Ref. 2 that this experimental requirement, which is paradoxical from the standpoint of the kinetic theory of ordinary gases, stems from the Coulomb interaction between plasma particles. In the plasma there are always particles with low velocities $v<v_{\rm Te}(L/l_e)^{1/4}$, for which collisions are a governing factor, despite satisfaction of the condition for a collisionless nature of the thermal electrons: $L \ll l_e$. Here $v_{\rm Te}=(\kappa_B T_e/m_e)^{1/2}$, $l_e=v_{\rm Te}/\nu_{ei}$, $\nu_{ei}=4(2\pi)^{1/2}e^2e_i^2n_i\Lambda(3m_e^2v_{\rm Te}^3)^{-1}$, e is the charge of an electron, $e_i=Z|e|$ is the charge of an ion, n_i is the ion number density, and Λ is the Coulomb logarithm.

In the present letter we show that the same factor leads to the result that the electron energy flux density (a vector) is not exclusively collinear with the temperature gradient. We should stress that numerical simulations of heat transfer have previously yielded indications that the energy flux is not collinear with the temperature gradient.[3] Those indications did not attract the attention they deserved. Below we discuss the reason for this property, and we offer an analytic description of it.

In the standard formulation for laser controlled fusion, we assume that the plasma is heated by virtue of inverse bremsstrahlung of electromagnetic radiation, whose electric field is $\mathscr{E}(\mathbf{r},t)=1/2\mathbf{E}(\mathbf{r},t)\exp(-i\omega_0 t)+\text{c.c.}$ The frequency of this radiation is high: $\omega_0 \gg \nu_{ei}$ and $\omega_0 \gg (v_{\rm Te}/L_E)$, where L_E is the length scale of the spatial variation in the field of the electromagnetic radiation. Dividing the electron distribution function into a high-frequency component and a steady-state component f_0 (Ref. 4), we can then use f_0 to determine both the energy flux and the temperature increment. In accordance with Ref. 5, we assume that f_0 differs only slightly from a Maxwellian distribution f_M:

$$f_0(\mathbf{v})=f_M\left\{1+\left(\frac{v^2}{3v_{\rm Te}^2}-2\right)I-\frac{e\,\delta\varphi}{\kappa_B T_e}+(v_iv_j-\tfrac{1}{3}\delta_{ij}v^2)J_{ij}\right\}+\delta f_c, \tag{1}$$

where $\delta\varphi$ is the electrostatic potential, and

$$I=\frac{e^2|\mathbf{E}|^2}{4m_e^2\omega_0^2 v_{\mathrm{Te}}^2},\ J_{ij}=\frac{e^2}{4m_e^2\omega_0^2 v_{\mathrm{Te}}^4}(E_iE_j^*+E_i^*E_j-\tfrac{2}{3}\delta_{ij}|\mathbf{E}|^2). \tag{2}$$

Here δf_c satisfies the equation[5]

$$i\mathbf{k}\cdot\mathbf{v}\delta f_c-I_{st}^{ei}[\delta f_c]-I_{st}^{ee}[\delta f_c]=Y_0+Y_a, \tag{3}$$

where I_{st}^{ei} and I_{st}^{ee} are collision integrals, and

$$Y_0=-(2\pi)^{1/2}\nu_{ei}v_{\mathrm{Te}}^3 I\frac{\partial}{\partial v_i}\left(\frac{v_i}{v^3}f_M\right), \tag{4}$$

$$Y_a=-3\left(\frac{\pi}{2}\right)^{1/2}\nu_{ei}\frac{v_{\mathrm{Te}}^5}{v^5}J_{ij}(v_iv_j-\tfrac{1}{3}\delta_{ij}v^2)\left(\frac{v^2}{2v_{\mathrm{Te}}^2}-3\right)f_M. \tag{5}$$

In writing Eq. (3) we assumed that I and J_{ij} have a coordinate dependence $\exp(i\mathbf{k}\cdot\mathbf{r})$. By virtue of the linearity of Eq. (3), we can thus formulate several general results.

In discussing corollaries of Eq. (3) we consider velocities that are not small, and for which the inequality $v\gg v_k=v_{\mathrm{Te}}(kl_e)^{-1/4}$ holds. Under this condition, we can assume that the particles with such velocities are collisionless. By virtue of the condition $kl_e\gg 1$, such electrons can be called "thermal." For them we find from (3)

$$\delta f_{c,T}(\mathbf{v})=-i(Y_0+Y_a)\left(\frac{P}{\mathbf{k}\cdot\mathbf{v}}+i\pi\delta(\mathbf{k}\cdot\mathbf{v})\right), \tag{6}$$

where P means the Cauchy principal value. Equation (6) can be used to determine the contribution of thermal electrons to the electron energy flux; we write it as a sum of two terms:

$$\mathbf{q}_T=\int d\mathbf{v}\times\tfrac{1}{2}m_e v^2\mathbf{v}\delta f_{c,T}(\mathbf{v})=\mathbf{q}_0+\mathbf{q}_a. \tag{7}$$

The "potential" part of the energy flux in this case is

$$\mathbf{q}_0=-\frac{i\mathbf{k}}{k^2}\frac{e^2|\mathbf{E}|^2 n_e}{2m_e\omega_0^2}\nu_{ei}. \tag{8}$$

In calculating (8) we ignored small quantities on the order of $(kl_e)^{-1}$. At this particular accuracy level, the divergence of expression (8) in the steady state agrees with the power of electromagnetic radiation absorbed by electrons through inverse bremsstrahlung.[6] The second term, the "solenoidal" term in Eq. (7), is unimportant for this balance. It is on the same order of magnitude as (8) and has the following form:

$$\mathbf{q}_a=-i\left\{\frac{\mathbf{E}(\mathbf{E}^*\cdot\mathbf{k})}{k^2}+\frac{\mathbf{E}^*(\mathbf{E}\cdot\mathbf{k})}{k^2}-2\frac{\mathbf{k}|\mathbf{E}\cdot\mathbf{k}|^2}{k^4}\right\}\frac{e^2n_e\nu_{ei}}{4m_e\omega_0^2}. \tag{9}$$

This expression is nonzero if the electric vector of the heating electromagnetic radiation has both a component parallel to the vector **k** and components perpendicular to it. We need to stress that the energy flux in (7) results from ballistic transport. Since the temperature is a scalar, its gradient is parallel to **k**. Expression (9) is accordingly the part of the electron energy flux which is perpendicular to the temperature gradient.

Below we discuss the case in which the temperature increment is related to the subthermal collisional electrons $[v<v_k=v_{\mathrm{Te}}(kl_e)^{-1/4}]$, for which we have, according to Ref. 5,

$$\delta f_c(\mathbf{v})=\delta f_0\left\{1-\frac{i\mathbf{k}\cdot\mathbf{v}}{2\nu(v)}\right\}+\frac{1}{6\nu(v)}Y_a, \tag{10}$$

where $\nu(v)=3(\pi/8)^{1/2}\nu_{ei}(v_{\mathrm{Te}}/v)^3$. For the discussion below it is sufficient to use the expression

$$\delta f_0=-\frac{9I}{2k^2l_e^2}f_M(v)\Gamma(\tfrac{6}{7})(\tfrac{2}{7})^{1/7}\sin\frac{\pi}{7}\frac{N^{8/7}}{\xi^{1/4}}K_{1/7}(\tfrac{4}{7}\xi^{7/4}), \tag{11}$$

where $K_{1/7}$ is the modified Bessel function, $N=\pi^{1/2}(4/9)Zk^2l_e^2$, $\xi=N^{2/7}(v^2/2v_{\mathrm{Te}}^2)$ and we are assuming $Z\gg 1$.

According to Eqs. (10) and (11), the increment in the thermal energy of the subthermal cold electrons is

$$\delta T_e=T_eI\frac{1.73Z^{5/7}}{(kl_e)^{4/7}}. \tag{12}$$

This quantity is substantially greater than the increment in the thermal energy of the collisionless electrons,[2] $\delta T_{e,T}\sim T_e(kl_e)^{-1}$. At the same time, under the condition

$$Z^{5/4}>kl_e>1 \tag{13}$$

expression (12) is greater than the electron oscillation energy in the field of the electromagnetic radiation, so we can say that there is an actual heating of the subthermal electrons.

The cold, collisional electrons make a relatively small contribution to the electron energy flux.[2] At the same time, under the condition $kl_e>Z^{2/3}$, Eqs. (10) and (11) lead to the following expression for the energy flux density transported by collisional electrons:

$$\mathbf{q}_c(\mathbf{k})=i\mathbf{k}\delta T_e\frac{0.16\kappa_{\mathrm{SH}}}{k^2l_e^2Z}\ \text{or}\ \mathbf{q}_c(\mathbf{r})=\frac{0.16\kappa_{\mathrm{SH}}}{4\pi l_e^2Z}\int\frac{d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial\delta T_e(\mathbf{r}')}{\partial\mathbf{r}'}. \tag{14}$$

Here $\kappa_{\mathrm{SH}}=(128/3\pi)n_ev_{\mathrm{Te}}\kappa_Bl_e$ is the thermal conductivity of the strongly collisional, fully ionized plasma. A distinctive feature of the energy flux in (14) is that its direction is opposite that which would correspond to ordinary Fick's law for the heat flux. At the same time, expression (14) differs only slightly from the collisionless flux in (8). Expression (14) was derived by expressing I in terms of δT_e in accordance with (12). If we also express I in terms of δT_e in Eq. (8), we find[2] the usual effective nonlocal thermal conductivity of a weakly collisional plasma. In the 1D case, that expression leads to the coefficient describing the limitation on heat transfer. In contrast, if the "solenoidal" part of the energy flux in (9) can be written formally in terms of the gradient δT_e, then the expression preserves the vector dependence on the vector **E**. From the discussion above we can conclude that the deviation from a collinear arrangement of the electron energy flux and the temperature gradient which has been seen in previous numerical calculations finds an explanation on the basis of ideas of the analytic theory of weakly collisional plasmas.

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