Effect of a random voltage on equilibrium current fluctuations in a quantum conductor

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(Submitted 8 November 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 11, 801–805 (10 December 1994)

Source fluctuations in the current (Langevin fluctuations) in a quantum conductor depend on the resistance of the external circuit. The possibility that the spectral density of the voltage fluctuations could be a nonmonotonic function of the magnitudes of the quantum and classical resistances is discussed.

Research on electron transport has been concerned primarily with two current-flow regimes: Either the voltage or the current in the circuit is fixed. The first situation is simpler to analyze. In particular, for a quantum conductor with dimensions $L \ll L_{\text{ph}}$ (L_{ph} is the phase relaxation length), the statistics of the charge transport over long time intervals can be described completely by means of a matrix for the elastic scattering of electrons.[1] When the current is fixed, the situation is considerably more complicated. The simplest way to study it is to invoke the concept of Langevin forces.[2] Along that approach, in (for example) a closed circuit containing a series connection of a quantum resistance and a classical resistance (r and R), one represents the fluctuations of the total current, δI, as the sum of "source" current fluctuations j in the quantum resistor and a contribution from fluctuations of the voltage drop V across the classical resistance:

$$\delta I = \delta j + \delta V_g, \tag{1}$$

where $g = 1/r$.

In the limit $r/R \to 0$, in which the low-frequency fluctuations of the total current tend toward zero ($\delta I_{\omega \to 0} \to 0$), the source fluctuations j should cancel out, and we should have

$$\delta V_{\omega=0} g = -\delta j_{\omega=0}. \tag{2}$$

Along this approach, which was taken (in particular) in Refs. 3 and 4, it is usually assumed that the source current fluctuations $j_{(v)}$ are determined exclusively by the expectation value of the voltage drop across resistance r (or the expectation value of the current), and it is assumed that the correlation functions $\langle jj \rangle$ can be calculated as in the problem with a fixed voltage $V = \langle I \rangle r$. In the present letter we are interested in the effect of fluctuations of the voltage V on the source current fluctuations $j_{\langle v \rangle}\{V\}$ at equilibrium. This effect can be substantial. It can lead to a significant change in the resultant magnitude of the fluctuations in the total current and in the voltage drop across the quantum resistance. The effect of voltage fluctuations on electron transport was studied by Hekking *et al.*,[5] who reached the conclusion that the probability for the passage of electrons from one reservoir to another is greatly modified by the emission of photons (voltage oscillations), with the result that the current–voltage characteristic becomes nonlinear at low currents, while the linear conductance is suppressed. We believe that the probability for the passage of electrons in a semiclassical external field set up by voltage fluctuations changes only slightly. In this letter we concentrate on the effect which stems from the phase shift (of the real part of the action describing the motion of the electron), which was studied previously in Ref. 6. When the phase shift is taken into account, the nonsimultaneous correlation function of the currents depends on the voltage fluctuations in the following way:[6]

$$\langle\langle j(t_1)j(t_2)\rangle\rangle_v=\frac{2e^2}{h^2}\int dEdE'\exp\left(i\,\frac{E-E'}{\hbar}(t_1-t_2)\right)n(E)[1-n(E')]$$

$$\times\sum_m\{2T_m^2+T(E)[1-T(E)][\Phi(t_2-t_1)+\Phi^*(t_2-t_1)]\},\qquad(3)$$

where

$$\Phi(t_2-t_1)=\exp\left(i\,\frac{e}{\hbar}\int_{t_1}^{t_2}\delta V(\tau)d\tau\right).\qquad(4)$$

Here T_m is an eigenvalue of the matrix tt^+ (t is the matrix of transmission amplitudes through the quantum resistor; we ignore the energy dependence of these amplitudes), and $n(E)=[\exp(E/\Theta)+1]^{-1}$ (Θ is the temperature in energy units). The angle brackets $\langle\ \rangle_V$ mean an expectation value over voltage fluctuations. We find this expectation value by a procedure which is independent of finding the expectation value over the electron degrees of freedom, which is denoted by $\langle\ \rangle$. Assuming that the resistor R is classical, we write the voltage drop across it as

$$-V(t)=I(t)R+V_R(t),\qquad(5)$$

where $V_R(t)$ are "source" voltage fluctuations, which we assume to be Gaussian. Their spectral density is given by the Nyquist formula[7]

$$\langle V_R^2(\omega)\rangle=2R(\omega)\hbar\omega\left(\frac{1}{2}+\frac{1}{\exp(\hbar\omega/\Theta)-1}\right).\qquad(6)$$

For the voltage across the quantum resistance we find the following expression from Eqs. (1) and (4):

$$V=\frac{r}{r+R}(-jR-V_R).\qquad(7)$$

The result of taking the expectation value of the exponential function,

$$\left\langle\exp\left[i\,\frac{e}{\hbar}\int_{t_1}^{t_2}V(t')dt'\right]\right\rangle_V=\Phi(t_2-t_1),$$

can be written in terms of irreducible correlation functions:

$$\Phi(t_2-t_1)=\exp\left(-\frac{e^2r^2}{2\hbar^2(r+R)^2}\int\int_{t_1}^{t_2}dt'dt''\langle V_R(t')V_R(t'')\rangle\right)\chi\left(-\frac{erR}{\hbar(r+R)}\right).\qquad(8)$$

Here

$$\chi(\lambda)=\left\langle \exp\left(i\lambda \int_{t_1}^{t_2} j(t')dt' \right)\right\rangle$$

is a characteristic function for the probability that a certain charge

$$Q=\int_{t_1}^{t_2} j(t')dt'$$

will be transported by fluctuation currents j over a time $|t_2-t_1|$.

A method for ordering the operators j at different times was described in Ref. 8 in an analysis of the phase shift of a quantum galvanometer (spin) in the field of tunneling electrons. The situation in which we are interested in the present paper is analogous: The role of the "spin" is played by an electron, and we are analyzing the phase shift of this electron. Expression (8) can be simplified in two limiting cases; that of short times, $|t_2-t_1|\ll\tau_{\text{corr}}$, where τ_{corr} is the correlation time of the random process $V(t)$, and that of long times, $|t_2-t_1|\gg\tau_{\text{corr}}$. If $\Delta t=|t_2-t_1|\ll\tau_{\text{corr}}$, expression (8) is dominated by the binary voltage correlation function at coincident times:

$$\Phi(t_2-t_1)=\exp[-\alpha(t_2-t_1)^2], \tag{9}$$

$$\alpha=\frac{1}{2}\frac{e^2}{\hbar^2}\left[\frac{r^2}{(r+R)^2}\langle V_R^2(0)\rangle_V+\frac{r^2R^2}{(r+R)^2}\langle\langle j^2(0)\rangle\rangle_V\right]. \tag{10}$$

Expressions (9) and (10) give the correct time dependence of Φ under the condition $\alpha\tau_{\text{corr}}^2\gg 1$, i.e., under the condition that Φ become vanishingly small at times $\Delta t\sim\tau_{\text{corr}}$. If the conditions $\alpha\tau_{\text{corr}}^2\ll 1$ and $\Phi(\tau_{\text{corr}})\sim 1$ hold instead, the asymptotic behavior at $\Delta t\gg\tau_{\text{corr}}$ is important, with $\Phi(\Delta t)=\exp(-\gamma\Delta t)$, where

$$\gamma=\frac{1}{2}\frac{e^2}{\hbar^2}\frac{r^2 2R\Theta}{(r+R)^2}-\lim_{\Delta t\to\infty}\frac{1}{\Delta t}\ln\chi\left(-\frac{erR}{\hbar(r+R)}\right)$$

$$=\frac{1}{2}\frac{e^2}{\hbar^2}\frac{r^2 2R\Theta}{(r+R)^2}+\frac{1}{2}\frac{e^2}{\hbar^2}\frac{r^2R^2}{(r+R)^2}\langle j_{\omega=0}^2\rangle+\dots . \tag{11}$$

It follows from (10) and (11) that in the limit $r/R\to 0$ the decay of $\Phi(\Delta t)$ is governed exclusively by the source current fluctuations j. We write the results for the spectral density of the fluctuations in the voltage across the quantum resistor at a low frequency:

$$S_V(\omega=0)=\int dt\langle V(0)V(t)\rangle=\frac{r^2 2\Theta R}{(r+R)^2}+\frac{(rR)^2}{(r+R)^2}\langle j_{\omega=0}^2\rangle, \tag{12}$$

where, in the case $\alpha\ll\Omega^2$ (Ω is a cutoff frequency which determines the dispersion of the resistance), we have

$$\langle j_{\omega=0}^2\rangle=2\frac{e^2}{\hbar\pi}\sum_n [T_n^2\Theta+T_n(1-T_n)G(\gamma)]. \tag{13}$$

In the case $\gamma\ll\Omega$ we have

$$G(\gamma)=\Theta(1+C\gamma), \tag{14}$$

where C is a constant on the order of one. In the case $\gamma \gg \Omega$ we instead have

$$G(\gamma)=\frac{\hbar\gamma}{\pi}\ln\frac{\Omega}{\gamma}. \tag{15}$$

In the case $\alpha \gg \Omega^2$ we have

$$\langle j^2_{\omega=0}\rangle = 2\,\frac{e^2}{\hbar\pi}\sum_n\,[T_n^2\Theta + T_n(1-T_n)\Theta F(\alpha)], \tag{16}$$

where, in the case $\hbar\sqrt{\alpha} \ll \Theta$, we have

$$F(\alpha)=1+\frac{1}{6}\,\frac{\alpha\hbar^2}{\Theta^2}$$

and, in the case $\hbar\sqrt{\alpha} \gg \Theta$, we have

$$F(\alpha)=\frac{4}{\sqrt{\pi}\hbar}\,\frac{\sqrt{\alpha}}{\Theta}.$$

In this approximation, the linear conductance of the quantum resistance and thus of the entire circuit is insensitive to fluctuations. A direct averaging of the current operator will obviously yield the same result, since we are considering only the phase shift, not the change in probabilities. We intend to discuss whether this situation contradicts the Nyquist theorem in a separate paper.

We turn now to an analysis of the behavior of the parameters γ and α as a function of the resistances r and R. In the limit of a small external resistance $R/r \ll 1$ we find from (11)

$$\gamma=\frac{e^2}{\hbar^2}R\Theta,$$

and the difference between the spectral density calculated for the case with quantum effects and the classical value S_{cl} is

$$\frac{S-S_{\mathrm{cl}}}{S_{\mathrm{cl}}} \approx \frac{1}{12}\,\frac{e^2R}{\hbar}\,\frac{R}{r}.$$

We turn now to the regime which seems the most interesting to us: that with $R/r \gg 1$, with negligible fluctuations in V_R, and with a linear decay, so we have $\Phi=\exp(-\gamma\Delta t)$. If there is only a slightly open "channel" in the quantum resistance ($T \ll 1$), the statistics of the charge transport can be represented as two independent Poisson charge-transport processes, in which charge is transported in opposite directions with an average electron-transport velocity $2\Theta(t/h)T$. The characteristic function in this case is

$$\chi(\lambda)=\exp\left(-8\Theta\,\frac{tT}{h}\sin^2\frac{\lambda}{2}\right).$$

From (11) we then find

$$\gamma=8\Theta \frac{T}{h} \sin^2\left(\frac{e^2}{\hbar 2} r\right). \tag{17}$$

In the case $e^2 r/(\hbar 2)=\pi m$, where m is an integer expression, expression (17) vanishes. The physical reason for this result is that the phase shift of an electron during the tunneling of another electron is

$$\frac{er}{\hbar}\int j dt=\frac{e^2 r}{\hbar}=\phi,$$

and the phase does not undergo relaxation under the condition $\phi=2\pi m$. In this situation we should of course consider dynamic fluctuations in the phase shift, $\delta\phi=\phi-\langle\phi\rangle$. Under the condition $\sqrt{\delta\phi^2}\ll\langle\phi\rangle=e^2 r/h$, this deviation of the spectral density of the fluctuations in the voltage across the quantum resistor from a monotonic behavior may be observed. The scale of the deviations of the spectral density from the spectral density expected classically is

$$\frac{S-S_{\rm cl}}{S_{\rm cl}}\approx T(1-T)\left(\frac{4}{\pi^2}\ln\frac{\hbar\Omega\pi}{4\Theta T}-1\right).$$

We wish to repeat that the primary assertion of this letter is that there is a nontrivial dependence of the source current fluctuations on the voltage fluctuations.

We wish to thank L. S. Levitov, D. E. Khmel'nitskiĭ , C. Glattli, and M. Sanquer for useful discussions. This study was supported by the Russian Fund for Fundamental Research (Grant 93-02-2113) and the International Science Foundation (Grant M9M000).

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Translated by D. Parsons