Non-Gaussian fluctuations of the current and voltage in quantum conductors

G. B. Lesovik

Institute of Solid State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Russia

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The fluctuations of the current and the voltage in a quantum conductor are non-Gaussian. Various reasons for this situation are discussed. The frequency shift of Josephson generation is analyzed as an example of a phenomenon in which this deviation from a Gaussian distribution is important. © *1994 American Institute of Physics.*

In a nonequilibrium state, the statistics of charge transport through a conductor is generally not Gaussian. A well-known example of this case is the electron current in an electron tube under saturation conditions. As Schottky showed,[1] the statistics of the transport in this case can be assumed Poissonian with a characteristic function

$$\chi_p(\Lambda)=\langle\exp(i\Lambda N)\rangle=\exp[\exp(i\Lambda)-1]\langle N\rangle. \tag{1}$$

Here $\langle N\rangle$ is the average number of electrons transmitted through the tube, and the angle brackets $\langle\ \rangle$ mean an average. The reason for the fluctuations in this case is the discrete nature of the electron charge, as can be seen clearly from the expression for the mean square fluctuation of the transported charge Q (Ref. 1): $\langle\delta Q^2\rangle=te\langle I\rangle$. Fixing the current and letting the charge tend toward zero, we find $\langle\delta Q^2\rangle=0$. Another example is a binomial distribution of the charge transported through a coherent quasi-1D conductor with a transmission T, which was studied in Refs. 2. Under the condition $V\gg\Theta$, where V is the applied voltage, and Θ the temperature, the characteristic function of this distribution is

$$\chi_b(\Lambda)=[1+T(\exp(i\Lambda)-1)]^{\langle N\rangle/T}. \tag{2}$$

The square of the charge fluctuations, $\langle\delta Q^2\rangle=te\langle I\rangle(1-T)$, again vanishes in the case $e=0$. Quantum shot noise stems from the discrete nature of the charge, as does the classical noise, and from the probabilistic nature of the tunneling of electrons through the barrier. An important point for the discussion below is that for these two random processes the cubic (actually, all odd) irreducible correlation functions are nonzero, $\langle\langle\delta Q^3\rangle\rangle=e^2t\langle I\rangle$, for a Poisson process, while we have $\langle\langle\delta Q^3\rangle\rangle=e^2t\langle I\rangle(1-T)\times(1-2T)$ for a Bernoulli process. Here the double angle brackets $\langle\langle\ \rangle\rangle$ mean an irreducible correlation function. We now consider our final example. In a mesoscopic coherent conductor, a change in the position of a single impurity leads to a change in the total conductance $G=R^{-1}$ by an amount on the order of[3]

$$\delta G\sim\frac{e^4}{\hbar^2}R.$$

The fluctuations of the current due to jumps of an impurity can be written $\delta I(t)=\delta G(t)V$, if we ignore other sources of noise. The characteristic function

$$\chi_{\rm im}(\Lambda)=\exp\left(\left[i\Lambda\int_0^{t_0}\delta I(t)dt\right)\right]$$

takes the following form for the case in which there is one mobile impurity, at times $t_0 \gg \Gamma^{-1}$:

$$\chi_{\rm im}(\Lambda)=\exp\left[-t_0\Lambda^2(\Delta GV)^2\frac{\gamma_1\gamma_2}{\Gamma^3}\left(1-i\Lambda\Delta GV\frac{\Delta\gamma}{\Gamma^2}\right)^{-1}\right]. \tag{3}$$

Here γ_1 is the reciprocal time of the transition from state 1, with conductance G_1, to state 2, with conductance G_2; γ_2 is the rate of the inverse transition; $\Delta G=G_1-G_2$; $\Delta\gamma=\gamma_1-\gamma_2$; and $\Gamma=\gamma_1+\gamma_2$. Clearly, the fluctuations of the voltage in a circuit which has a stabilized current and which contains sources of non-Gaussian fluctuating currents will also have a nontrivial statistics with nonzero irreducible correlation functions of higher orders. Let us examine the consequences of this situation in the example of Josephson generation. The effect of voltage fluctuations on the linewidth was analyzed in Ref. 4 under the assumption that the fluctuations were Gaussian. Taking an average of the expression

$$\Phi(t)=\left\langle I(t')I(t'+t)=\left\langle \text{Re}\exp\left[i\frac{2e}{\hbar}\int_0^t[V_0+\delta V(t')]dt'\right]\right\rangle\right.$$

(which characterizes generation with decay) over fluctuations, Larkin and Ovchinnikov[4] found

$$\Phi(t)\sim \text{Re}\exp[i\omega_J t-\lambda(t)t], \tag{4}$$

where

$$\lambda(t)=t\,\frac{1}{2}\left(\frac{2e}{\hbar}\right)^2\langle\delta V^2(0)\rangle$$

in the limit $t\to 0$ and

$$\lambda(t)=\frac{1}{2}\left(\frac{2e}{\hbar}\right)^2\langle\delta V^2_{\omega=0}\rangle$$

in the limit $t\to\infty$. Here $\langle\delta V^2_{\omega=0}\rangle$ is the Fourier transform of the binary correlation function at a low frequency, and we have $\omega_J=(2e/\hbar)V_0$. For non-Gaussian fluctuations we find, in place of expression (4),

$$\Phi(t)\sim \text{Re}\exp([i\omega_J t-\lambda(t)t+i\Delta\omega_J(t)t]).$$

Here

$$-\lambda(t)t=\sum_{k=1}^{\infty}\left(\frac{i2e}{\hbar}\right)^{2k}\frac{1}{(2k)!}\int\dots\int^t dt_1\dots dt_{2k}\langle\langle\delta V(t_1)\dots\delta V(t_{2k})\rangle\rangle, \tag{5}$$

$$i\Delta\omega_J t=\sum_{k=1}^{\infty}\left(\frac{i2e}{\hbar}\right)^{2k+1}\frac{1}{(2k+1)!}\int\dots\int_0^t dt_1\dots dt_{2k+1}\langle\langle\delta V(t_1)\dots\delta V(t_{2k+1})\rangle\rangle, \qquad (6)$$

In the limit $t\gg\tau_{\rm corr}$, where $\tau_{\rm corr}$ is the correlation time of the random process $\delta V(t)$, we find from (5) and (6)

$$\Delta\omega_J=\sum_{k=1}^{\infty}\left(\frac{2e}{\hbar}\right)^{2k+1}\frac{(-1)^k}{(2k+1)!}\langle\langle\delta V^{2k+1}_{\omega=0}\rangle\rangle, \qquad (7)$$

$$\lambda=\sum_{k=1}^{\infty}\left(\frac{2e}{\hbar}\right)^{2k}\frac{(-1)^{k+1}}{(2k)!}\langle\langle\delta V^{2k}_{\omega=0}\rangle\rangle. \qquad (8)$$

It can be seen from (7) that the frequency of the Josephson generation depends on not only the average voltage on the contact, $\langle V\rangle=V_0$, but also the statistics in general. Expression (8) shows that the linewidth is determined not only by the second cumulant of the fluctuations but also by all higher-order ones. In the case $t\ll\tau_{\rm corr}$ we have $\Delta\omega_J\sim t^3$. In this case it would be more appropriate to say that the frequency "floats away" rather than shifting. Let us estimate the magnitude of the effect, using the examples of non-Gaussian processes discussed above. We consider a circuit which has a stabilized current I and which contains a Josephson junction with a normal resistance r in series with a mesoscopic resistor R. For the time being we ignore all fluctuations expect those of the resistance R. This is a reasonable approximation if many impurities change position. We then find

$$\delta V=-I\,\delta R\approx -I\,\frac{\delta G}{G^2}\,.$$

The average in which we are interested,

$$\Phi(t)=\left\langle \mathrm{Re}\,\exp\left[i\,\frac{2e}{\hbar}\int_0^t \delta V(t)'dt'\right]\right\rangle=\left\langle \mathrm{Re}\,\exp\left[-i\,\frac{2e}{\hbar}\,\frac{I}{G^2}\int^t \delta G(t)'dt'\right]\right\rangle,$$

can be rewritten as follows with the help of characteristic function (3):

$$\Phi(t)=\mathrm{Re}\,\chi_{\rm im}\left(-\frac{2e}{\hbar}\,R\right).$$

The frequency shift under the condition $(e/\hbar)R^2\Delta GI(\Delta\gamma/\Gamma^2)\gg 1$ is then

$$\Delta\omega_J\simeq\frac{2e}{\hbar}\,R^2I\sum_{\rm im}\Delta G\,\frac{\gamma_1\gamma_2}{\Gamma\Delta\gamma}\,.$$

In this expression we are assuming that the contributions from all impurities are summed independently. This is a good approximation under the condition $e^2R/\hbar \ll 1$. An estimate of the frequency shift yields $\Delta\omega_J\sim I/e(e^2R/\hbar)^3N_{\rm im}$, where $N_{\rm im}$ is the number of mobile impurities.

We turn now to the fluctuations which stem from the discrete nature of the charge. If the one-particle fluctuation currents δj through the junction are much greater than the critical current, we can ignore the nonlinearity of the Josephson junction and set $\delta V = -\delta j r$. We then find

$$\Phi(t) = \mathrm{Re}\chi\left(-\frac{2er}{\hbar}\right),$$

where

$$\chi(\Lambda) = \left\langle \exp\left[i\Lambda \int^t \delta J dt' \right] \right\rangle.$$

Assuming that the charge-transport process is a Poisson process, we find

$$\Phi(t) = \exp\left(\left[\exp\left(-i\,\frac{2e^2r}{\hbar} \right) - 1 \right] \frac{It}{e} + i\,\frac{It}{e}\,\frac{2e^2r}{\hbar} \right).$$

We thus find

$$\Delta\omega_J = \mathrm{Im}\frac{I}{e}\left[\exp\left(-i\,\frac{2e^2r}{\hbar} \right) - 1 + i\,\frac{2e^2r}{\hbar} \right]$$

and that the frequency shift is oscillatory:

$$\Delta\omega_J = \frac{I}{e}\left(\frac{2e^2r}{\hbar} - 1 - \sin\frac{2e^2r}{\hbar} \right).$$

The oscillation results from the oscillation of the phase shift $[\phi]_{\mathrm{mod}\,\pi} = 2e/\hbar \int \delta V dt$ during the tunneling of one electron. This conclusion is apparently not completely correct. The problem is that in the case of a fluctuating voltage the charge-transport statistics itself changes, becoming non-Poissonian. There is the possibility, however, that the oscillations of the frequency shift as a function of the resistance are not completely smoothed out. We intend to examine this question in detail in a future paper.

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