Nonohmic hopping conductivity in the exponential band tail

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The variable range hopping conductivity in a strong electric field E at a temperature $T=0$ is investigated. An exponential increase of the density of states $g(\varepsilon)$ with energy ε is assumed. It is shown that there is an energy level $\varepsilon=\varepsilon_B(E)$ ("the transport level"), along which an electron hops without rising or lowering in average. The dependence $\sigma(\varepsilon)$ has the form: $-\ln\sigma\sim 1/E$. © *1994 American Institute of Physics.*

In the present paper we discuss the hopping conductivity (σ) along localized states in a strong electric field (E). At present, this problem is being intensively investigated, both theoretically and experimentally, in application to disordered semiconductors.[1–4] We consider the case in which the density of unoccupied states, $g(\varepsilon)$, increases from the Fermi level $\varepsilon=\varepsilon_F$ with the increase of energy ε; the temperature is $T=0$.

In the field E the Fermi level ε_F inclines. An electron can jump from under the Fermi level by making an activationless hop against the field (Fig. 1). This hop and subsequent hops along unoccupied states give rise to an electric current.

Such a mechanism of conductivity at $T=0$, with $g(\varepsilon)=$const, was considered by Shklovskiĭ.[5] It turned out that

$$-\ln\sigma\sim E^{-1/4}. \tag{1}$$

The hops take place in the energy range $\sim(e^3E^3/g)^{1/4}$ near $\varepsilon=\varepsilon_F$.

We will first examine the one-dimensional case. In this case a rather simple and complete investigation can be carried out. Let us assume $\varepsilon_F=0$. An electron with energy ε [the charge $(-e)$], which is localized at the point $x=0$, can make a hop to any unoccupied state with the energy ε' and the coordinate x, if $\varepsilon'+eEx<e$. The concentration $N(\varepsilon,x)$ of the final states, which is accessible for an electron with initial energy $\varepsilon>0$ that hops from the point $x=0$ to the point $x>0$ (to the right), is

$$N(\varepsilon,x)=\int_0^{\varepsilon+eEx} g(\varepsilon')d\varepsilon'. \tag{2}$$

The typical hop length to the right, $x_t(\varepsilon)$, is determined from the condition

$$\int_0^{x_t} N(\varepsilon,x)dx=1 \tag{3}$$

[here and below $g(\varepsilon)$ is the one-dimensional density of states].

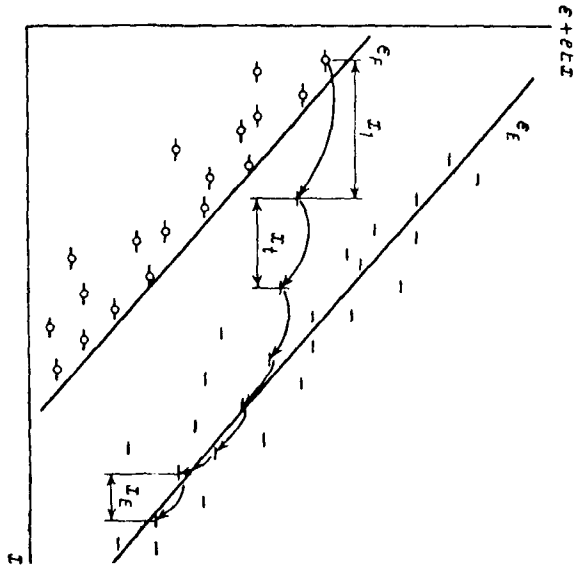

FIG. 1. Schematic diagram of the electron transitions in a disordered system of localized states in an electric field.

Let us determine the typical final-state energy ε_t'. The hopping frequency ν is given by the expression

$$\nu = \nu_0 \exp\left(-\frac{2|x|}{a}\right), \tag{4}$$

where ν is the characteristic frequency, a is the state radius, and $|x|$ is the hopping length. The frequency ν does not depend on ε and ε'. This means that an electron can reach any accessible state with equal probability. Following Ref. 6, ε_t' can be found from the condition that the concentration of unoccupied states, which lie below ε_t', is equal to half of the accessible state concentration ("the typical hop approximation"):

$$\int_0^{\varepsilon_t'} g(\varepsilon)d\varepsilon = \frac{1}{2} N(\varepsilon, x_t). \tag{5}$$

Equations (3) and (5) determine ε_t' and x_t as functions of ε and E. We investigated Eqs. (3) and (5) for the case in which $g(\varepsilon)$ has the form

$$g(\varepsilon) = g_F \exp\left(-\frac{\varepsilon}{\varepsilon_0}\right) \tag{6}$$

(g_F is the density of states at the Fermi level, and ε_0 is a constant). The integrals in (3) and (5) can then be easily calculated. The results of this investigation are given below.

The basic result is that the system (3), (5) has a solution $\varepsilon_t' = \varepsilon$—the final energy is equal to the initial energy. This solution is for the value $\varepsilon = \varepsilon_E$. In other words, there is an energy level ε_E—the transport level—along which a typical electron hops without lowering or rising. After excluding x_t from (3) and (5) we obtain the following equation for $\varepsilon_E (\alpha = eE/g_F\varepsilon_0^2)$:

$$\exp\left(\frac{\varepsilon_E}{\varepsilon_0}\right) - 1 - \ln\left[2 - \exp\left(\frac{\varepsilon_E}{\varepsilon_0}\right)\right] = \alpha. \tag{7}$$

In a weak E, there is a similar level ε_T, whose position is determined by the temperature.[7] Assuming the existence of an effective temperature in a strong E, Shklovskiĭ *et al.*[8] suggested that there exists a transport level $\varepsilon_E(E)$. Our results show that such a level actually exists. (The generalization to the three-dimensional case is discussed below.)

It can be shown that for any hop the final energy ε' is always located between ε and ε_E. This means that an electron with an energy $\varepsilon<\varepsilon_E(\varepsilon>\varepsilon_E)$ gradually approaches, by means of hopping, the transport level $\varepsilon=\varepsilon_E$ from below (from above).

The conductivity mechanism can be considered now in the following way. An electron jumps from under the Fermi level into the region of unoccupied states, $\varepsilon>0$ (the field generation of electrons). This is the first, the longest, and consequently the hardest hop. Then an electron makes a series of consecutively shorter hops, approaching the level $\varepsilon=\varepsilon_E$. The electron density n in the region $\varepsilon>0$ increases. This increase is limited by the electron–hole recombination. If the electron lifetime is long enough, the electrons will accumulate at the level ε_E. In our approximation there are no electrons in the region $\varepsilon>\varepsilon_E$.

When $\alpha\ll 1$, the hops take place in a small $(\varepsilon_0\sqrt{\alpha})$ vicinity of the Fermi level (the one-dimensional analogy of the case investigated in Ref. 5). We do not consider this case here. When $\alpha\gg 1$, we find $[x_E\equiv x_t(\varepsilon_E)]$:

$$\varepsilon_E=\varepsilon_0 \ln \alpha, \quad \frac{eEx_E}{\varepsilon_0}\simeq \ln 2. \tag{8}$$

We see that the ε_E level is situated sufficiently far (at a distance of several ε_0) from the Fermi level.

It can be shown that the existence of the transport level is attributable to the increasing profile of $g(\varepsilon)$, rather than to the specific form of (6), which is used here to obtain the analytical expressions.

Let us consider now the field generation of electrons. An electron with energy $\varepsilon<0$ makes a hop to the right from the point $x=0$ to another point $x>0$ with $\varepsilon'>0$ (Fig. 1). The least distance at which an unoccupied state can exist is $(-\varepsilon/eE)$. Therefore, a typical first hopping length x_1 is determined from Eq. (3), with $(-eE/\varepsilon)$ as a lower limit in the integral. A typical final energy ε_1' is determined from Eq. (5), where ε_t' should be substituted for ε_1' and x_t should be substituted for x_1. When $\alpha\gg 1$, we find that

$$\frac{eEx_1}{\varepsilon_0}=\ln(\alpha)-\frac{\varepsilon}{\varepsilon_0}, \quad \frac{\varepsilon_1}{\varepsilon_0}=\ln\left(\frac{\alpha}{2}\right). \tag{9}$$

If $x_1(\varepsilon)$ is known, the field generation rate $G(E)$ per unit length can be calculated as follows:

$$G(E)=\int_{-\infty}^{0} g^*(\varepsilon)\nu[x_1(\varepsilon)]d\varepsilon.$$

Here $g^*(\varepsilon)$ is the density of the occupied states [the specific form of $g^*(\varepsilon)$ turns out to be unsignificant].

To calculate the electron linear density n in the states with $\varepsilon>0$, we must know the lifetime $\tau(\varepsilon)$. When τ is large enough, most of the electrons reach the level $\varepsilon=\varepsilon_E$ and have time to make many hops along the level. We shall restrict the discussion to this case and find n from the relation $n=G(E)\tau_E$ $[\tau\equiv\tau(\varepsilon_E)]$. Using the electron drift velocity $v_E=x\nu(\varepsilon_E)=\nu_0 x_E \exp(-2x_E/a)$, we determine the current $j=env_E$.

The time τ_E can depend on E. We assume that this dependence is not exponential. For $\alpha\gg1$ we find that

$$\ln\sigma\sim\left(-\frac{2\varepsilon_0}{eEa}\right)\ln(2\alpha). \tag{10}$$

We have observed the dependence $-\ln\sigma\sim E^{-1}$ in a noncompensated crystalline silicon.[9] The conductivity was determined by hops along the impurity D^--band tail states. (A detailed report on these results will be published later.) It is now believed that the D^--band tail density of states of a noncompensated, doped, crystalline semiconductor rapidly increases with increasing ε, as was assumed above.

It should be noted that the dependence $\ln\sigma\sim(-E^{-1})$ was observed in three-dimensional samples. However, it is easy to show that for electrons with $\varepsilon=\varepsilon_E$ the hopping frequency opposite the field is $\exp[\varepsilon_0(1-\ln 2)/eE\alpha]$ times greater than across E if $\alpha\gg1$. (The typical hop lengths are compatible.) When $\varepsilon<\varepsilon_E$, the difference is still great. Therefore, for $\alpha\gg1$ and $eEa/\varepsilon_0\ll1$ (the latter inequality was found in all experiments we know) the three-dimensional case becomes a one-dimensional case. The main result of this work must therefore be true for three-dimensional samples.

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[1] D. I. Aladashvili *et al.*, *Hopping and Related Phenomena*, edited by Fritzsche and M. Pollak, North-Holland, 1990, p. 283.
[2] H. Bottger and D. Wegener, *ibid.*, p. 317.
[3] R. Stachowitz *et al.*, Phil. Mag. B **62**, 1362 (1990).
[4] D. Monroe, *Hopping Transport in Solids*, edited by M. Pollak and B. Shklovskiĭ, North-Holland, 1990, p. 49.
[5] B. I. Shklovskiĭ, Fiz. Tekh. Poluprovodn. **6**, 2335 (1972).
[6] B. I. Shklovskiĭ *et al.*, Phys. Rev. Lett. **62**, 2989 (1989).
[7] F. Shapiro and D. Adler, J. Non-Cryst. Solids **77&78**, 139 (1988).
[8] B. I. Shklovskiĭ *et al.*, *Transport, Correlation, and Structural Defects*, edited by H. Fritzsche (World Scientific, Singapore, 1990).
[9] E. M. Gershenzon *et al.*, Proc. XII Vsesojuzn. Conf. Phys. Semicond. Abstracts, Kiev, Part 2, 1990, p. 13.
[10] R. N. Bhatt and T. M. Rize, Phil. Mag. **42**, 859 (1980).

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