Free energy of superconducting islands with even land odd numbers of electrons

V. P. Mineev

L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117334 Moscow, Russia

(Submitted 22 November 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **60**, No. 12, 861–864 (25 December 1994)

The difference between the thermodynamic potentials with odd and even numbers of electrons is calculated for unusual superconductors which have zeros in the spectrum of elementary excitations. The current through a superconducting electrometer with a Coulomb blockade made from such a superconductor is independent of the parity of the number of electrons in the electrometer. There can be a dependence of this sort only if the spectrum of the superconducting material is free of zeros.

The current through a transistor (electrometer) with a Coulomb blockade changes periodically as the control voltage on the gate is varied. The recent observation of a change in the period of the current (from e to $2e$) through an electrometer in a superconducting state[1] has demonstrated directly that there is a difference between the free energies of small superconductors with even and odd numbers of electrons at low temperatures. The transistor in Ref. 1 was made of aluminum. In the present letter, we calculate this difference between free energies for superconductors whose energy spectrum has lines or points of zeros on the Fermi surface, in connection with the familiar problem of identifying superconducting states of high-T_c superconductors and superconducting compounds with heavy fermions. The results show that the current through an electrometer made from such a superconductor with a Coulomb blockade does not depend on the parity of the number of electrons in the electrometer. A dependence of this sort prevails only in superconductors whose spectrum of elementary excitations has a finite gap, as in ordinary superconductors. The observation of a doubling of the period of the current through a superconducting electrometer at low temperatures thus probably indicates that the superconductivity of the material from which the electrometer is made is of an unusual nature.

The transistor (electrometer) with a Coulomb blockade is a rectangular aluminum film (an island) which is part of an electric circuit between two tunnel junctions with capacitances C_1 and C_2. The film used in the experiments of Ref. 1 had a thickness of 200 Å, a width of 600 Å, and a length of 2×10^4 Å. The island was coupled by a capacitance C_g to a gate, which made it possible to vary the electrostatic potential of the island through the application of a voltage V_g (Fig. 1). The quantum-mechanical states of the island are well determined if (a) the coupling of the island with the external circuit is weak, i.e., if the resistance of one of the junctions satisfies $R > h/e^2$, and (b) that the total capacitance of all the junctions, $C_\Sigma = C_1 + C_2 + C_g$, is small enough that the elementary

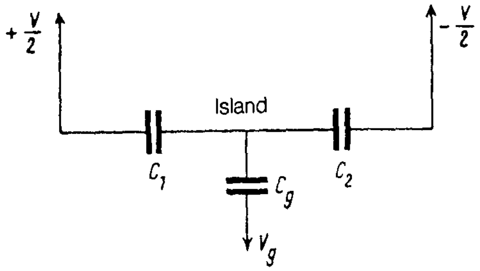

FIG. 1. Electric circuit of an electrometer with a Coulomb blockade.

electrostatic energy $E_c = e^2/2C_\Sigma$ is greater than T. Under these conditions, the total electrostatic energy of the island,

$$U = \frac{Q^2}{2C_\Sigma} = \frac{(en + C_g V_g)^2}{2C_\Sigma}, \tag{1}$$

is a periodic function of the gate voltage (Fig. 2). Accordingly, the current through the island for the given voltage in the external circuit, V , is also a periodic function of $C_g V_g$, with a period of e.

This is the situation as long as the aluminum island is in its normal state. In the superconducting state, it was observed[1] that peaks in the current corresponding to an odd number of electrons in the island begin to be suppressed at temperatures below 0.3 K (corresponding to $0.4T_C$) and that the $j = j(C_g V_g)$ dependence becomes periodic with a period of $2e$ (see also Refs. 2–5). A natural explanation for this effect, proposed in Ref. 1, is that the free energies (more precisely, the Ω potentials) of the island in the superconducting state with even and odd numbers of electrons, $\Omega_{\text{odd}} - \Omega_{\text{even}}$, differ by an amount on the order of the size of the gap in the spectrum of elementary excitations, Δ, in the limit $T \to 0$. Accordingly, under the condition $\Delta > E_c > T$, the parabolas in the

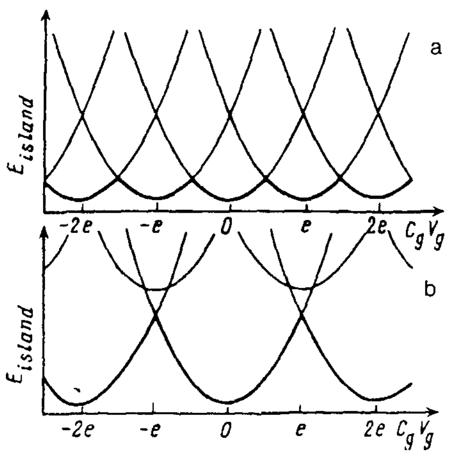

FIG. 2. Qualitative behavior of the energy of the island as a function of the charge $C_g V_g$. a—Under the conditions $e^2/2C_\Sigma > T$, $e^2/2C_\Sigma > \delta\Omega$; b—$e^2/2C_\Sigma > T$, $e^2/2C_\Sigma < \delta\Omega$.

plots of $U=U(c_gV_g)$ with odd values of n rise to a height $\Omega_{\text{odd}}-\Omega_{\text{even}}$ and become unobservable (Fig. 2). Correspondingly, the current $j=j(C_gV_g)$ becomes a periodic function of C_gV_g with a period of $2e$.

The island constitutes two Josephson junctions connected in series. A theory for the Josephson current in such an arrangement can be derived through an elementary generalization of the Feynman approach[6] to a Josephson junction, as a two-level quantum-mechanical system. Introducing in the two-level system an intermediate state through which a direct current flows at a zero value of the external potential difference, we find

$$j\sim\frac{2e}{\hbar}\frac{E_1E_2\sin\phi}{E_{\text{island}}}. \tag{2}$$

Here E_1 and E_2 are the Josephson energies of junctions 1 and 2, respectively; ϕ is the "external" phase difference (the phase difference between the junctions to which the island is connected); and E_{island} is the energy of the island, which includes both the electrostatic part U and a term whose value jumps by an amount $\delta\Omega=\Omega_{\text{odd}}-\Omega_{\text{even}}$ upon changes in C_gV_g corresponding to a change in the parity of the number of electrons on the island. A more accurate quantum-mechanical discussion of the Josephson current through an island was offered in Ref. 7 and generalized in Ref. 4.

From Ref. 3 we have

$$\delta\Omega=-T\ln\frac{\Pi_{k\sigma}(1+e^{-\beta E_k})-\Pi_{k\sigma}(1-e^{-\beta E_k})}{\Pi_{k\sigma}(1+e^{-\beta E_k})+\Pi_{k\sigma}(1-e^{-\beta E_k})}=-T\ln\frac{1-f(T)}{1+f(T)}, \tag{3}$$

where

$$f(T)=\Pi_{k\sigma}\tanh(\beta E_k/2). \tag{4}$$

The energy in the spectrum of elementary excitations is

$$E_k=\sqrt{\xi^2+\Delta_k^2}. \tag{5}$$

In a normal metal we would have $\Delta_k=0$, in an ordinary superconductor we would have $\Delta_k=\Delta$, and in a superconductor like the A phase of ^{3}He, for which the gap in the spectrum vanishes at isolated points on the Fermi surface, we would have $\Delta_k^2=(3/2)\Delta^2(\hat{k}_x^2+\hat{k}_y^2)$. In a superconductor in whose spectrum the gap vanishes on an isolated line we choose $\Delta_k^2=3\Delta^2\hat{k}_z^2$, as in a polar phase.

A calculation of the low-temperature behavior $f(T)$ yields, respectively,

$$f_N(T)=\exp(-\pi^2N_0VT), \tag{6}$$

$$f_S(T)=\left(\tanh\frac{\Delta}{2T}\right)^{N_{\text{eff}}}, \tag{7}$$

$$f_A(T)\approx\exp\left(-\frac{\pi^4}{18}N_0V\frac{T^3}{\Delta^2}\right), \tag{8}$$

$$f_P(T)=\exp\left(-CN_0V\frac{T^2}{\Delta}\right). \tag{9}$$

Here N_0 is the density of states, V is the volume of the island,

$$N_{\rm eff}=4N_0V\int_\Delta^\infty \frac{EdE}{\sqrt{E^2-\Delta^2}}e^{-\frac{B-\Delta}{T}}\approx 2\sqrt{2}N_0V\sqrt{\Delta T}, \tag{10}$$

and C is a constant on the order of one.

For ordinary superconductors of sufficiently small volume ($N_0V\approx 4\times 10^3$ K^{-1} under the experimental conditions of Ref. 1) we would have

$$\delta\Omega=\Delta-T\ \ln N_{\rm eff}. \tag{11}$$

Consequently, if the conditions

$$\Delta>e^2/2C_\Sigma>T \tag{12}$$

hold, then under the further condition

$$T<T^*=\Delta/\ \ln N_{\rm eff} \tag{13}$$

the current through the electrometer in states with an odd number of electrons is weakened.

For a superconductor whose spectrum has zeros at isolated points, at sufficiently low temperatures,

$$T\ll T^*=\Delta^{2/3}(36/\pi^4N_0V)^{1/3}, \tag{14}$$

$$\delta\Omega=T\ln\frac{36\Delta^2}{\pi^4N_0VT^3}, \tag{15}$$

the conditions for a weakening of the current in states with an odd number of electrons are considerably more stringent:

$$T\ln\frac{36\Delta^2}{\pi^4N_0VT^3}>\frac{e^2}{2C_\Sigma}>T. \tag{16}$$

There is also another limitation on the possibility of observing a $2e$ -periodicity of the current through a superconducting electrometer with a Coulomb blockade made from a superconductor whose spectrum of elementary excitations has zeros. The superconducting state in such a superconductor is probably suppressed in a surface layer with a thickness $\sim\xi_0$. The density of states in this layer is close to the density of states of a normal metal. Consequently, the difference $\Omega_{\rm odd}-\Omega_{\rm even}$ may remain exponentially small. A corresponding analysis is valid for superconductors whose spectra have zeros on isolated lines on the Fermi surface.

In summary, a doubling of the period of the current through a superconducting electrometer with a Coulomb blockade could be realized in practice only in superconductors whose spectrum of excitations has no zeros. Nontrivial superconducting phases may also be free of zeros in their spectra of elementary excitations.[8] In general, a doubling of the current period would not make it possible to distinguish ordinary and unusual phases without zeros in the spectrum.

This study had partial financial support from the International Science Foundation (ISF; Grant MG1000).

[1] M. T. Tuominen *et al.*, Phys. Rev. Lett. **69**, 1997 (1992).
[2] P. Lafarge *et al.*, Phys. Rev. Lett. **70**, 994 (1993).
[3] M. T. Tuominen *et al.*, Phys. Rev. B **47**, 11599 (1993).
[4] P. Joyez *et al.*, Phys. Rev. Lett. **72**, 2458 (1994).
[5] A. Amar *et al.*, Phys. Rev. Lett. **72**, 3234 (1994).
[6] R. P. Feynman *et al.*, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, Mass., 1963), Vol. 3.
[7] K. A. Matveev *et al.*, Phys. Rev. Lett. **70**, 2940 (1993).
[8] G. E. Volovik and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **88**, 1412 (1985) [Sov. Phys. JETP **61**, 843 (1985)].

Translated by D. Parsons