Upper critical magnetic field in quasi-one-dimensional superconductors

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The effect of magnetic field on superconducting ordering in quasi-one-dimensional superconductors is examined. It is suggested that the three-dimensional, superconducting order parameter is established due to electron transfer between the chains. The orbital effect in this case leads to the result that the perpendicular critical field with respect to the chains $H_{c2\perp}\to\infty$ as $T\to 0$, and T_c turns out to be a periodic function of the field that is parallel to the chains.

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In view of the experimental studies of the superconducting properties of the polymeric compound $(SN)_x$ [1,2] and organic crystals $(TMTSF)_2PF_6$,[3] it would be of interest to analyze the theory of the upper critical magnetic field in quasi-one-dimensional superconductors. The indicated compounds contain $(SN)_x$ conducting chains and a stack of TMTSF molecules, respectively. The superconducting properties of quasi-one-dimensional systems depend substantially on the relationship between the resonance integrals J describing the electron motion between the conducting chains and the energy gap Δ that characterizes the ground state within the chain.[4,5] In this paper we examine the case when $J\ll\Delta$, and the phase transition in a system of chains corresponds to the appearance of a long-range, three-dimensional order for a singlet superconducting pairing. This situation will occur if the Cooper instability within the chain is stronger than the Peierls instability, and the electron interaction in different chains is small compared with the electron motion between the chains.[8] The electron transfer between the chains in this case allows a phase transition to the superconducting state with a critical temperature T_c, where $T_c\to 0$ as $J\to 0$. We shall investigate the dependence of T_c on the magnetic field H for $J\ll\Delta$. Such an investigation was performed by Manneville[6] and Turkevich and Klemm[7] in the context of the Ginzburg-Landau (GL) theory. Below we shall show that an allowance for one-dimensional fluctuations changes substantially the results of the GL theory.

To calculate the dependence of T_c on H of H_{c2} on T, we shall use the self-consistent field approximation with respect to interaction between the chains (SFI).[9] At $H=0$ the temperature T_c is given by

$$W(q_\perp) - \chi^{-1}(q_z, T) = 0, \tag{1}$$

where $W(q_\perp)$ is a Fourier transform of the parameter $W_{\mathbf{n},\mathbf{n}'}$ for the interaction $W_{\mathbf{n},\mathbf{n}'}\psi^*_{\mathbf{n}}(z)\,\psi_{\mathbf{n}'}(z)$ of the superconducting parameters of order $\psi_{\mathbf{n}}(z)$ and $\psi_{\mathbf{n}'}(z)$ of the chains $\mathbf{n}$ and $\mathbf{n}'$ and $\chi(q_z T)$ is a function of the superconducting response of one chain for the perturbations with a wave vector q_z directed along the chain. In the limit $J\ll\Delta$ the value $W_{\mathbf{n},\mathbf{n}'}$ is determined by the Josephson interaction of the nearest chains. For a square lattice of the chains $\mathbf{n} = (n_1, n_2)$, where $n_{1,2}$ are integers, we have $W(\mathbf{q}_\perp) = (W\cos q_x + \cos q_y)$ and $W = J^2/\epsilon_F$. In terms of renormalized group theory[8] we obtain for $\chi(q_z, T)$ the expression

$$\chi(q_z, T) = \frac{\epsilon_F}{T_1^2}\left(\frac{T_1^2}{T^2 + v^2 q_z^2}\right)^{y/2}, \tag{2}$$

where $y > 0$ is a numerical parameter of the order of unity, v is of the order of v_F and T_1 coincides with Δ with an accuracy to the numerical factor. It follows from Eqs. (1) and (2) that T_c reaches the maximum value at $\mathbf{q} = 0$, and $T_c = J^{2/y} T_1^{1-2/y}$. To allow for the magnetic field, we substitute $\partial/\partial\mathbf{r} - 2e\,\mathbf{A}/\hbar c$ for $\mathbf{q}$, where $\mathbf{A}$ is a vector potential. Turning to the equation for the superconducting order parameter $\psi_{\mathbf{n}}(z)$, we obtain an eigenvalue problem for a differential-difference equation. Let us examine the case $y = 2$, which allows us to reduce the problem to solving the Mathieu equation. Note that the qualitative behavior of $H_{c2}(T)$ is independent of the value of $y > 0$.

Let us assume that the magnetic field is directed perpendicularly to the chains at an angle ϕ with respect to the x axis. Thus, we obtain from Eqs. (1) and (2) the differential equation

$$\frac{2v^2}{T_c^2}\frac{d^2\psi_{\mathbf{n}}}{dz^2} + \left(\cos\frac{2\,eHaz\cos\phi}{c\hbar} + \cos\frac{2\,eHaz\sin\phi}{c\hbar} - \frac{2T^2}{T_c^2}\right)\psi_{\mathbf{n}} = 0, \tag{3}$$

where a is the distance between the chains. At $\phi = \pi/4$ we obtain from Eq. (3) the Mathieu equation, and

$$H_{c2} = \frac{2\Phi_0}{\pi a \xi}\frac{T_c - T}{T_c}, \qquad T_c - T \ll T_c,$$
$$H_{c2} = \frac{\Phi_0}{2\pi a\xi}\frac{T_c}{T}, \qquad T \ll T_c, \tag{4}$$

where ϕ_0 is a quantum of flux and $\xi = v/T_c$. We can see from Eq. (4) that $H_{c2} \to \infty$ as $T \to 0$, since the interaction between the chains is "shut off" under the influence of the field, and the temperature of the three-dimensional ordering decreases with decreasing effective interaction. Note that at large H the paramagnetic effect, which gives the finite values of H_{c2} for $T = 0$ (for singlet pairing), must be taken into account. In the context of the GL theory[6,7] the magnetic field also decrease T_c, but this decrease is negligible, and $T_c(H) \to T_c(0) - 2W$ as $H \to \infty$ [we recall that $W < T_c(0)$].

We obtain in a field applied parallel to the chains the difference equation

$$\psi_{n_1,n_2}\cos\frac{2eHa^2 n_2}{c\hbar} + \frac{1}{2}\psi_{n_1,n_2+1} + \frac{1}{2}\psi_{n_1,n_2-1} - \frac{T^2}{T_c^2}\psi_{n_1,n_2} = 0. \tag{5}$$

The eigenvalue problem for a difference equation was analyzed by Turkevich and Klemm,[7] and we shall use the results of the investigation. First, $T(H)$ is a periodic H function with a period Φ_0/a^2 and $T_c(H)$ oscillates between $T_c(0)$ and $0.8T_c(0)$. Near $T_c(0)$ we have $H_{c2} = 2\sqrt{2}\Phi_0(T_c - T)/\pi a^2 T_c$. Apparently, the $T_c(H)$ oscillations actually cannot be observed, since the paramagnetic effect changes substantially the behavior of $T_c(H)$ in the fields of order Φ_0/a^2. The GL theory also gives $T_c(H)$ oscillations, but T_c oscillates in a narrow temperature range between $T_c(0)$ and $T_c(0) - 2W$.

According to Refs. 1–3, however, the $H_{c2\perp}$ field in a $(SN)_x$ polymer and in an organic superconductor $(TMTSF)_2PF_6$ approaches a finite value which is appreciably smaller than the paramagnetic limit. Thus, the situation with $J \gtrsim \Delta$ is apparently realized in these superconductors: at $J \gg \Delta$ the behavior of $H_{c2}(T)$ can be described by a model of an ordinary, three-dimensional, anisotropic superconductor, and the $(SN)_x$ polymer undoubtedly is such a superconductor. The behavior of $H_{c2}(T)$ at $J \approx \Delta$ requires further investigation.

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