The special role of above-the-barrier states in the channeling of electrons in crystals

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It is shown that an appreciable number of relativistic electrons propagate through a crystal as a result of channeling. These electrons in the low-lying, above-the-barrier states experience anomalously weak inelastic interaction with the nuclei, multiple scattering and a small energy loss.

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1. The transition from channeling of positively charged particles in a crystal to channeling of electrons is characterized by a "reversal" of the entire physical effect, rather than by a simple change of the sign of the charge image (see Fig. 1). This is illustrated particularly clearly by the above-the-barrier (AB) states.

In fact, a fast transit above the wells near the crystal nuclei and a long "residence" in the regions between the crystallographic planes are characteristic for electrons whose energy of transverse motion lies directly above the potential barrier (Fig. 1b). It is easy to see that, because of this, the probabilities of all the processes of interaction with the atoms, which require small impact parameters (in the case of positively charged particles, the opposite picture occurs—the residence above the nuclei and the fast transit in the regions between them increase the corresponding cross sections—see Ref. 1), decrease sharply (compared to the situation in the amorphous medium). It is important that in the case of electrons the population of the AB states, is large even at much smaller angles of incidence θ than the critical angle θ_c, as follows from the geometry of the charge image.

The potential wells near the crystal nuclei inevitably predetermine the bound levels for the transverse motion of a particle [more precisely, the narrow below-the-barrier (BB) zones] whose number is actually determined by the relativistic electron mass. Localization of the corresponding wave functions near the crystal nuclei appreciably increases the cross sections of all the processes requiring "close" collisions at least in the case of the *s* states (see, for example, Ref. 2 and the references cited therein). Thus, we have a situation which is opposite in every respect to that occurring during channeling of positively charged particles.

After passing through a limited thickness of the crystal L_{coh}, in which the nondiagonal elements of the density matrix are dampened (see Refs. 3 and 4 for a detailed description), the electron beam is broken down into two parts-BB and AB beams, and the resulting interaction with the crystal at $\theta < \theta_c$ is a superposition of two completely different interaction patterns that are characteristic for BB and AB particles. In the AB beam, the large-angle scattering, excitation of the atomic *K* shells, etc., as well as all inelastic processes leading to multiple scattering and energy loss, are suppressed. In fact, for this fraction, channeling is realized in the original meaning of this word, and

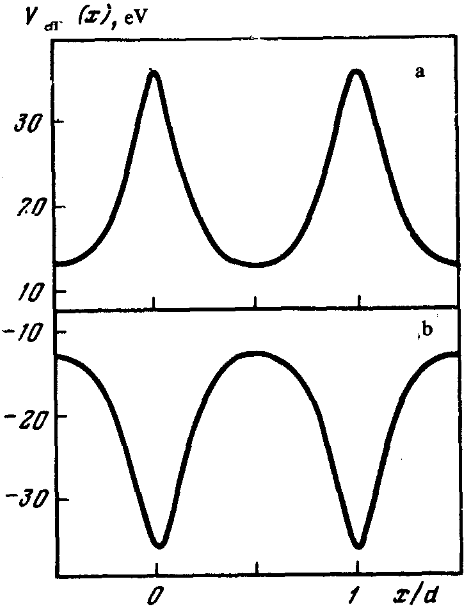

FIG. 1. Effective periodic potential (in Moliére approximation) for the $\{110\}$ planar channeling in silicon at room temperature: (a) positron case; (b) electron case.

becomes more pronounced with increasing electron energy. On the other hand, the BB beam is characterized by an enhanced intensity of the inelastic process, and, in this sense, these particles experience "antichanneling."

2. To illustrate the ideas developed above, we analyzed the planar channeling of electrons with an energy $E = 5$ MeV in the Kronig-Penney[5] model with parameters characteristic for $\{110\}$-planar channel in silicon: the potential depth is $V_0 = 23$ eV and the constant is $d = 1.92$ Å. The width $a = 0.55$ Å of the Kronig-Penney potential well was selected in such a way that the location of the second BB zone (see Fig. 2a) would coincide with that of the corresponding zone in the Molière potential of the $\{110\}$ silicon planes.

Figure 2b illustrates the behavior of $|\psi_{qn}(x)|^2$ in the first five zones and Fig. 2c shows the population probabilities $|a_{qn}|^2$ of these zones as a function of the angle θ between the momentum $\mathbf{p}_0$ of the incident electrons and the channel plane ($q = p_0 \sin\theta$—for details, see Ref. 1). We can see in these figures that at $\theta < \theta_c = (2EV_0)^{1/2}/cp_0$ the wave function near the crystal nuclei is formed largely by the BB zones, whereas the AB zones contribute predominantly to the particle density in the space between the nuclei. The total relative number of particles in the AB state for $\theta = 0$ is 0.554.

Figure 3 shows the θ dependence of the yield, integrated over the thickness, of the inelastic processes with a small impact parameter in the crystal nuclei (see Ref. 1):

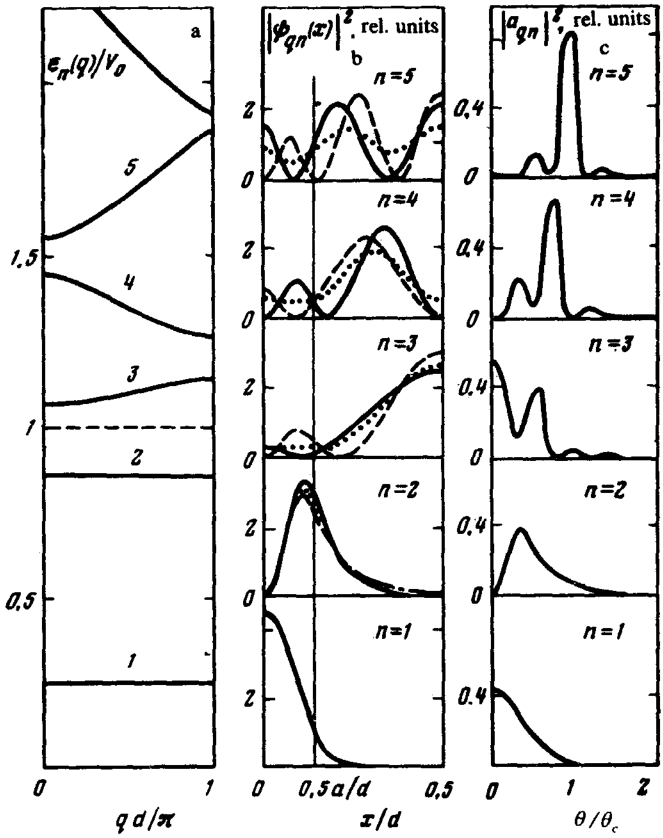

FIG. 2. Planar channeling of electrons in the Kronig-Penney model with the parameters indicated in the text: (a) structure of energy bands of transverse motion, (b) dependence of $|\psi_{qn}(x)|^2$ on x in different bands for different values of the quasi momentum q(———— $q = 0$, ... $q = \pi/2d$, --- $q = \pi/d$; the arrows denote the average thermal values $\langle|\psi_{qn}(x_{\text{nucl}})|^2\rangle_T$ in the crystal nuclei), (c) population of the bands as a function of the angle of incidence θ.

$$I/I_{amorph} = \sum_n |a_{qn}|^2 < |\psi_{qn}(x_{nucl})|^2 >_T ,$$

where $\langle ... \rangle_T$ represents averaging over the thermal vibrations [in our calculations $(\overline{u_1^2})^{1/2} = 0.077$Å]; the summing is done over all the zones of transverse motion. The given results are valid up to noticeable thicknesses $L > L_{\text{coh}}$, which are bounded above by the condition of particle knockout from the BB states due to inelastic scattering.

Our calculations fully confirm the aforementioned ideas about channeling of electrons in crystals.

3. The compatibility of populations of the AB and BB states at small incidence angles allowed us to demonstrate experimentally the dramatic differences in the behavior of the electrons in both parts of the beam in the region of thicknesses $L > L_{\text{coh}}$. In the angular distribution of particles behind the crystal we can, therefore, observe

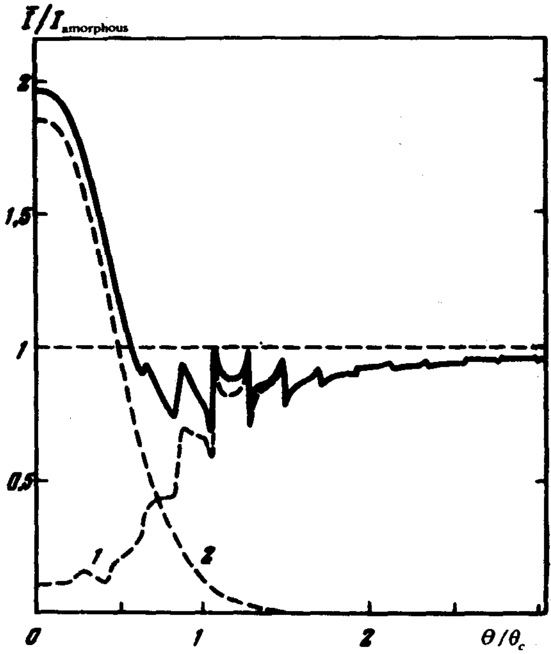

FIG. 3. Yield of the inelastic processes integrated over the thickness with a small impact parameter in the crystal nuclei due to planar channeling of electrons; 1, contribution of above-barrier states; 2, contribution of subbarrier states. The channeling model is the same as in Fig. 2.

simultaneously an increased scattering efficiency at large angles (BB part) and an anomalously large number of electrons, which move in a narrow range of angles $\Delta\theta \sim \theta_c$ near the incident direction and experience multiple, weak scattering (AB part). In addition, the low-lying AB states must be characterized by much lower energy losses than those in the amorphous medium, whereas these losses in the BB states are noticeably larger. In the case of axial channeling, all these effects manifest themselves even more dramatically.

Some aspects of this unexpected picture of anomalous transmission of the AB electrons through crystals were apparently experimentally observed in Refs. 6–9. On the whole, however, this effect requires a detailed investigation. In this connection, the experimental studies of the energy spectra of electrons under channeling conditions would be of particular interest.

In conclusion, we would like to say the obtained results show that the systematic BB levels cannot effectively compete with the low-lying AB states in forming the weakly interacting fraction of electrons (see Ref. 8). In fact, if there is a weakening of the interaction with the crystal nuclei for the particles in these levels, it is very small the thermal vibrations of atoms play an important role in this case [see Fig. 2b in which the $\langle|\psi_{qn}(x_{\text{nucl}})|^2\rangle_T$ values are shown]; however, the interaction with the electron subsystem of a crystal in such levels increases due to an increasing probability of finding the particles in the regions of localization of the atomic shells.

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