Inversion of a wave front associated with a four-wave interaction in media with nonlocal nonlinearity

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(Submitted 18 April 1979)
Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 1, 6–11 (5 July 1979)

We identify and study the phenomenon of generation of complex-conjugate wave fronts in the case of a degenerate four-wave interaction in ferroelectric $LiNbO_3$ and $LiTaO_3$ crystals with nonlocal nonlinear response.

PACS numbers: 77.80. – e, 78.20. – e

Frequency degenerate mutually-collinear four-wave interaction of light (FIL) in media with a cubic nonlinearity has attracted attention in view of the possibility of practically inertialess amplification and generation of complex-conjugate wave fronts,[1–3] an effect which may be used for compensation of laser radiation wave fronts.[4]

Heretofore, FIL has been studied in nonlinear liquids[2] and metallic vapors,[3] i.e., media with a local nonlinear response, where a change in the refractive index $\Delta\kappa$ is proportional to the light intensity $I = |E|^2$ at each coordinate point r. In this work, we identify and investigate the generation of complex-conjugate wave fronts in the case of FIL in lithium niobate and lithium tantalate crystals in which nonlocal response of the form $\Delta\kappa \sim (d/dr) \ln I(r)$ is achieved (so called "diffusion nonlinearity"[5,6]).

Figure 1 shows a FIL scheme. A helium-cadmium laser beam ($\lambda = 0.44$ μm, $P = 10$ mW) is split into two beams with complex amplitudes C_1 and C_3 ($E_e = 0.5 \times [C_e \exp i(wt - k_e r) +$ complex conjugate]), which were combined in the specimen volume. Having traveled through the crystal, beam C_1 is reflected backward by the mirror thus forming the second pumping wave C_2. In certain cases wave C_2 is formed as a result of Fresnel reflection from the back edge of the specimen. As a result of FIL,

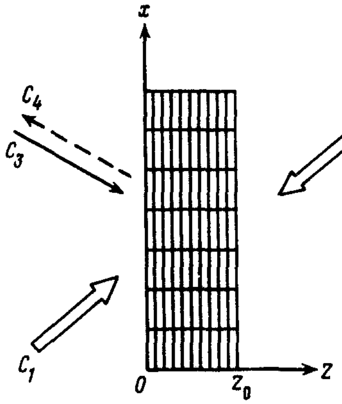

FIG. 1. FIL scheme and isophase surfaces of light-induced gratings of the reflecting and transmitting type.

wave C_4 was generated and amplified, and was directed toward the signal wave C_3. Figure 1 also shows the isophase surfaces of light-induced gratings of the refractive index (for simplicity, only two gratings are shown: reflective, formed when waves C_2 and C_3 interfere, and transmitting, due to waves C_1 and C_3). In the language of light-inducing holographic grids, the occurrence of wave C_4 is determined by the diffraction of wave C_2 at the transmitting grating and wave C_1 at the reflecting grating; and changes in wave C_3 are caused by diffraction of C_1 at the transmitting grating and of C_2, at the reflecting grating.

Figure 2 shows characteristic examples of measurements of intensity of the waves C_3 and C_4 in the case of interaction in nominally pure reduced $LiTaO_3$ crystals for two orientations of the C-axis: along and against the direction of a bisectrix of an angle between the beams C_2 and C_3. Clearly, in contrast to the known data on the FIL in media with a local response,[2,3] in this case a strong anisotropy is seen, which is expressed in terms of intensity changes of wave C_4 and the sign of wave C_3.

The intensity of the resultant wave C_4 within a interval $C_3^2 \lesssim 0.1 C_1^2$ varied linearly with the decrease in the initial value of intensity of wave C_3. A strong dependence of intensity of the fourth wave on a degree of collinearity of the pumping waves was observed: a 15″ detuning of wave C_2 with respect to C_1 resulted in the attenuation of the stationary value of $|C_4|^2$ to half its value.

Experiments conducted to verify the foregoing have confirmed that the wave C_4 is complex-conjugate with respect to the signal wave C_3. When a negative lens was inserted into wave C_3, wave C_4 became a converging spherical wave which, at a certain distance, could be focused to a point. Introduction of transparencies into wave C_3 had resulted in the reconstruction of their real images in wave C_4, examples of which are shown in Fig. 3. The resolution obtained, ~ 20 mm^{-1}, is not critical and is determined by the aperture of the interaction region (~ 1 mm) and distance to a transparency (~ 10 cm). Using the inhomogeneous phase plate method,[7] we established that the portion of energy coupled solely to the complex-conjugate wave is not less than 70% of the total radiation that propagates toward the signal beam.

The theory of this effect is based on the simultaneous solution of Maxwell's equations and the equations of electrotransport-diffusion for an electrooptic crystal.[6] The

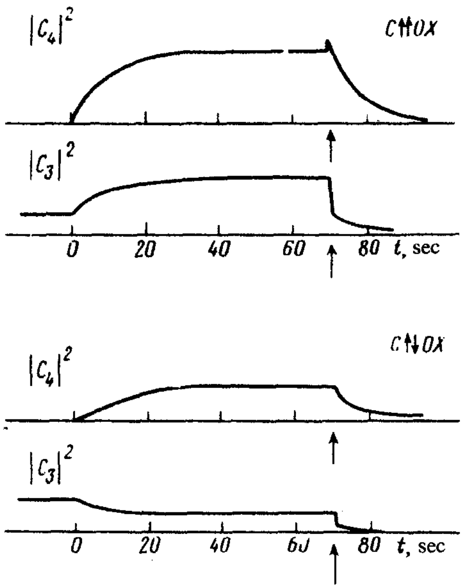

FIG. 2. Time dependence of intensities of signal C_3^2 and generated C_4^2 waves for different C-axis orientations in $LiTaO_3$ crystal. $C_1(0)^2 \approx C_2(z_0)^2$; $C_3(0)^2$ $0.15C_1(0)^2$. The arrow indicates the moment at which the signal beam is turned off $C_3(0) = 0$.

equations for the dimensionless intensities $v_e = I_e/I_0$ (I_0 is a sum of intensities of light waves incident on a crystal) and phases ϕ_e ($c_e = \sqrt{I_e}\exp i\phi_e$), for the C-axis of a crystal which is parallel to the OX axis (Fig. 1), are as follows:

$$\frac{\partial v_1}{\partial z'} = -\frac{\partial v_3}{\partial z'} = v_1 v_3 + \sqrt{v_1 v_2 v_3 v_4}\ \cos\phi,$$

$$\frac{\partial v_2}{\partial z'} = -\frac{\partial v_4}{\partial z'} = v_2 v_4 + \sqrt{v_1 v_2 v_3 v_4}\ \cos\phi, \tag{1}$$

$$\frac{\partial\phi}{\partial z} = \frac{1}{2}\left[(v_2 - v_3)\sqrt{\frac{v_1 v_4}{v_2 v_3}} + (v_1 - v_4)\sqrt{\frac{v_2 v_3}{v_1 v_4}}\right]\sin\phi,$$

where

$$\phi \equiv \phi_3 - \phi_1 + \phi_2 - \phi_4;\ z' = 4z k_0^2 r_{33}\kappa^4 T k_{1x}\left[k_{1z}\left(1 - \frac{\sigma_T}{\sigma_{ph}}\right)\right]^{-1},$$

$z_0' \equiv z'(0)$ is the dimensionless thickness of a crystal, $k_{1x} = k_0\kappa\sin(\theta/2)$, $k_{1z} = k_0\kappa\cos(\theta/2)$, $k_0 = 2\pi/\lambda$, θ is the angle of convergence of the C_1 and C_3 beams in a medium, σ_T/σ_{ph} is ratio of dark conductivity to photoconductivity of a crystal, and r_{33} is the electrooptic coefficient.

It follows from Eq. (1) that for $v_4(z_0) = 0$ there exists for ϕ a integral $\phi = n\pi$,

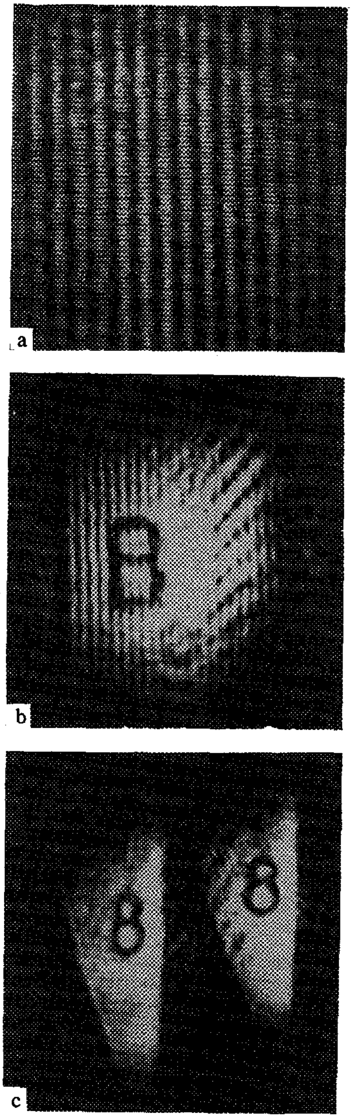

FIG. 3. Real images of transparencies introduced into a signal wave, reconstructed in the complex-conjugate wave C_3: a—grating with 20 mm^{-1} spatial frequency; b—examples of reconstruction of complex signals.

$n = 0, \pm 1, \ldots$ which determines the linear relationship between the phases of generated and original waves

$$\phi_4 = -\phi_3 + \phi_2 + \phi_1 + n\pi, \tag{2}$$

Expressions for the intensities of interacting waves in the $v_1 v_2 v_3^{-1} \gg v_2 \gg v_4$ approximation are

$$\begin{gathered} v_2 = v_2(z_0);\quad v_1 + v_3 = v_1(0) + v_3(0) \equiv v_{13}, \\ v_3(z_0') = v_{13} v_3(0)\,[v_2(0) + v_1(z_0)\exp(z_0' v_{13})]^{-1}. \end{gathered} \tag{3}$$

If, in this case, $z_0'(v_{13} - v_2) \ll 1$ and $v_{30} < v_{10} \exp(v_{13} z')$, the expression for v_4 takes on a simple form

$$v_4(0) = v_2 \frac{v_3(0)}{v_1(0)} [1 - \exp(-v_{13} z_o')]^2. \quad (4)$$

Equations (3) and (4) indicate that depending on the C-axis orientation (z_0' sign change), v_4 has a different value and v_3 may be greater or smaller than $v_3(0)$ in accordance with the experimental data. Equation (4) also indicates the experimentally observed linear dependence of intensity of a complex-conjugate wave on the initial intensity of the signal wave $v_3(0)$. The experimental value of gain $\Gamma = v_{13}(z_0'/z_0) \approx 5$ cm^{-1} in $LiTaO_3$ at the covergence angle $\theta = 22°$ is half the theoretical value.

Thus, media with nonlocal nonlinearity may also be used to generate and amplify the complex-conjugate wave fronts. The important advantage of FIL in these materials is compliance with the law of conservation of phases of the type of Eq. (2) for any ratio of intensities of the interacting waves.[1)] The consequence of this law is independence of the phase of wave C_4 from the initial intensities of interacting waves and the linear relationship among the phases of all beams. This determines the practical independence of the wave front of a reconstructible complex-conjugate wave C_4 from the degree of inhomogeneity of a pumping wave with respect to intensity on the one hand, and the possibility of conducting the processing and comparison of the phase transparencies, on the other.

The authors thank B.Ya. Zel'dovich for a stimulating critique of this work.

[1)]For media with a local response in the approximation of a given pumping wave field, Eq. (2) holds under the condition of equal pumping wave intensities.

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