Ground state energy shift in the proton-antiproton atom

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We have obtained limits on the isosinglet and isotriplet scattering lengths for $N\bar{N}$ based on energy shift data for the ground state of the proton-antiproton atom. A technique is indicated for finding the nuclear level binding energy in the $N\bar{N}$ system based on the shift of the atomic $1S$ state and one of the $N\bar{N}$ scattering lengths.

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Results have recently been published for the first measurements[1,2] of radiative transitions in the $\bar{p}p$ atom. Transitions to the $2P$[1] and $1S$[2] levels (L and K lines) were observed. Measurement of the K-line energies showed that the shift in the energy of the $1S$-level due to nuclear $N\bar{N}$ interaction is 3.04 ± 0.06 keV, in which the level shifts toward smaller binding energies (the unperturbed binding energy of the $1S$ level is 12.5 keV.

The fact that the nuclear interaction leads to an effective pushing out of the $1S$ level leads to the conclusion[3] that, in addition to the atomic levels in the $N\bar{N}$ system, there is a lower quasinuclear S level. The possible existence of a near-threshold S state (having a binding energy on the order of 1 MeV) in the $N\bar{N}$ system was first indicated in Ref. 4 before the appearance of experimental results concerning the shift of the $1S$-

level based on the relative $\bar{p}$ annihilation cross section from the atomic S- and P-states. An attempt was made[5] to determine the position of the near-threshold quasinuclear state using experimental data for the $1S$-level shift. The authors proposed a model-free technique for solving the problem based on the smallness of the nuclear force radius in comparison with the Bohr radius of the $\bar{p}p$ atom. According to Ref. 5, the quasinuclear S-state binding energy is 930 keV, and the $\bar{p}p$ scattering length is 6.66 Fermi. However, Ref. 5 did not take into account the interaction between the $\bar{p}p$ and $\bar{n}n$ channels arising from the exchange of isovector mesons. It is shown below that, because of the interaction with the $\bar{n}n$ channel in the problem of determining the position of the quasinuclear level from the shift of the atomic $1S$ state, a free parameter arises which must be found from an independent experiment.

Following Ref. 5, we will solve the problem by joining the logarithmic derivative of the wave function of the $\bar{p}p$ system in the outer and inner regions. Thus, the existence of a radius r_0 is assumed such that for $r > r_0$ the nuclear interaction may be neglected, while for $r < r_0$ the Coulomb interaction may be negelcted (according to Ref. 5 $r_0 \approx 3$ Fermi). The mass difference $\Delta = 2(m_n - m_p)$ between the $\bar{n}n$ and $\bar{p}p$ paths will be considered as a kinetic effect which manifests itself in the condition $k_1 \neq k_2$, where k_1 and k_2 are the momenta in the $\bar{p}p$ and $\bar{n}n$ channels, respectively.[1)] The wave function in the outer region is expressed by the Whittaker function $W_{y,1/2}(2r/y)$, where $E = -1/2y_2$ is the energy of the level in atomic units. The continuity condition obtained in Ref. 5 by expanding the logarithmic derivative of the function $W_{y,1/2}(2r/y)$ in terms of the small parameters r_0/a_B ($a_B = 57.6$ Fermi is the Bohr radius of the $\bar{p}p$ atom) and neglecting terms of order r_0/a_B has the form

$$1/a = F(y), \tag{1}$$

$$F(y) = 2\left[\psi(1-y) + \frac{1}{2y} - (1+4r_o)\ln y + (1-4r_o)\ln r_o + C_o\right]. \tag{2}$$

Here $\psi(2) = \Gamma'(2)/\Gamma(2)$, and $C_0 = 1.848$. The quantity a is the scattering length for the nuclear potential. It is known[6] that for the multi-channel case the scattering length determined by the equation

$$1/a = k_1 \operatorname{ctg}\delta_1 \;, \tag{3}$$

is a function of energy even for the case of zero-radius force (formally the dependence of a on energy is expressed as a function of the momenta k_i, $i \neq 1$). Thus, in the "zero-radius" approximation but taking into account the $\bar{n}n$ channel, Eq. (1) should be rewritten as

$$\frac{1}{S}\left(\frac{z-S}{z-\gamma S}\right) = F(y) \,. \tag{4}$$

Here $2S = -(a_0 + a_1)$, where a_0 and a_1 are the isosinglet and isotriplet $N\bar{N}$ scattering lengths, respectively, $z = (y^{-2} + \Delta)^{-1/2}$, and $\gamma = a_0 a_1 (a_0 + a_1)^{-2}$.

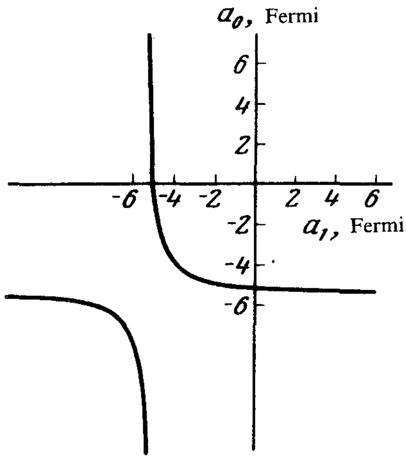

FIG. 1. Values for the scattering lengths a_0 and a_1 compatible with experimental results.[2]

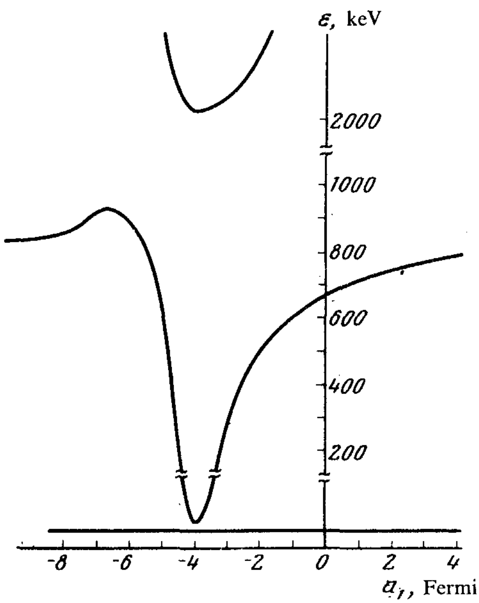

FIG. 2. Dependence of the quasinuclear S-level binding energy on scattering length a_1 (the horizontal line in the lower part of the figure corresponds to the observed position of the atomic $1S$ level).

Solving Eq. (4) with respect to a_0 with $y = y_0 = 1.148$, where y_0 corresponds to the experimental value for the energy of the $1S$ level,[2] we find the relation between the scattering lengths a_0 and a_1. The dependence of a_0 on a_1 is shown in Fig. 1. Therefore, it is possible to determine only the allowable values for the scattering lengths a_0 and a_1, in terms of the shift of the $1S$ level of the $\bar{p}p$ atom, and not the values of the lengths themselves. In order to determine a_0 and a_1 it is necessary to measure one of them (or a nonlinear combination of the two) in an independent experiment. For example, the scattering length a_1 may be obtained from experiments based on the scattering of slow $\bar{n}$ by p (the possibility of generating a beam of slow $\bar{n}$ has been discussed in the literature[7]).

If the value of a_1 (or a_0) is known, based on the known shift of the $1S$ level, Eq. (4) leads to the calculation of the quasinuclear S state which was mentioned earlier. To do this it is necessary to substitute a_1 into Eq. (4) with $y = y_0$, find a_0, and then look for solutions of Eq. (4) in the region $y < y_0$, i.e., for large binding energy values $\mathscr{E} = 1/2y^2$. The curve for the dependence of the binding energy $\mathscr{E}$ on a_1 is shown in Fig. 2. This curve shows that the value for the binding energy of the quasinuclear S state combined with the measured energy shift of the atomic $1S$ level, lie in a broad energy range near the $N\bar{N}$ threshold.[2)]

Despite the uncertainty in the prediction of the binding energy for the quasinuclear S state, the very fact of the existence of the near-threshold S level in the $N\bar{N}$ system necessarily follows from the positive sign and the large value of the atomic level energy shift. This conclusion also agrees with other experimental data for the $\bar{p}p$ atom.[4] Thus, the search for the near-threshold state under consideration is of great interest. A possible technique for its observation was suggested by I.S. Shapiro, and consists of measuring the radiative transitions to this level from the atomic $2P$-state.

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[1)] In this connection see Ref. 6; the rejection of this assumption would lead to the necessity of introducing one more parameter characterizing the breakdown of isotopic invariance.

[2)] We should also keep in mind the error due to the fact that for solving the problem the simplest approximation of "zero radius" was used.

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