Transverse hadron moments in quark and gluon jets

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It is shown that for large Q^2 the hadron distributions in jets depend on the transverse momentum k through the parameter $\kappa^2 = \mathbf{k}^2/z^2Q^2f$, where f is independent of $\mathbf{k}$.

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We consider a jet of hadrons produced by a parton p for a partonometer resolution corresponding to a momentum transfer Q. Let $D_p^h(z,\mathbf{k};z_0,\mathbf{k}_0;t)$ be an inclusive function of the hadron distribution h in terms of the transverse momentum k and the fraction of the jet z carried away by a hadron, in a reference system in which the jet carries the fraction z_0 of the interaction momentum. In quantum chromodynamics (QCD) the change in D_p^h with increasing Q^2 is determined by the development of a cascade of gluons—quark-antiquark pairs—with increasing "depth"

$$t(Q^2) = (1/b)\ln[1 + (\alpha_0 b/2\pi)\ln Q^2/Q_0^2], \tag{1}$$

where $b = (33 - 2n_f)/6$, n_f is the number of quark flavors, Q_0 is the transmitted momentum for which $D_p^h(z,k;z_0,k_0;0)$ is given, the initial hadron distribution in the jet, and α_0 is the constant for the color interaction of $\alpha(Q^2)$ for $Q^2 = Q_0^2$.

The relationship between $D_p^h(z,\mathbf{k};z_0,\mathbf{k}_0;t)$ and $D_p^h(z,\mathbf{k};z_0,\mathbf{k}_0;0)$ in QCD is easily obtained using group renormalization equations.[1] Following Refs. 1 and 2, we have

$$D_p^h(z,\mathbf{k};z_0,\mathbf{k}_0;t) = \sum_{p'} \int \frac{dy}{y} d\mathbf{k}' D_{p'}^h(z,\mathbf{k};y,\mathbf{k}';0)\, G_p^{p'}(y,\mathbf{k}';z_0\mathbf{k}_0;t), \tag{2}$$

where $G_p^{p'}(y,\mathbf{k}';z_0,\mathbf{k}_0;t)$ is the inclusive distribution of a parton of type p' in a parton of type p at a resolution Q^2.

Strictly speaking, the group renormalization equations are used for the direct calculation of the parton momentum distribution producing jets in which there is a hadron in the final state with a given momentum, i.e., D_p^h in the reference system where $z = 1$, $\mathbf{k} = 0$ (these are the distribution functions which occur in the inclusive hadron production cross sections). The distribution functions in the stream reference system where $z_0 = 1$ and $\mathbf{k}_0 = 0$ are related to them by the equations

$$D_p^h(z, z\mathbf{k}_0; 1, 0; t) = D_p^h(1, 0; 1/z, \mathbf{k}_0; t). \tag{3}$$

We shall first determine the nature of the dependence of $D_p^h(1,0;z_0,\mathbf{k}_0;t)$ on k_0. Multiplying Eq. (2) by $(\mathbf{k}_0^2)^l$ and integrating in terms of k_0, we obtain the recurrence equations relating the l-th order moments $K_p^h(z_0,l,t) = \int(\mathbf{k}_0^2)^l D_p^h(1,0;z_0,\mathbf{k}_0;t)d\,\mathbf{k}_0$ with lower order momenta. Then, by the successive application of the recurrence relations we write $K_p^h(z_0,l,t)$ as a sum of terms $A_p^h(l,n,z_0,t)$ containing integrals of the moments of the parton distributions in the partons $K_p^{p'}(z,l,t)$ of degrees Q^2 and $K_p^h(z_0,0,0)$:

$$K_p^h(z_0, l, t) = \sum_{n=0}^{l-1} A_p^h(l, n, z_0, t). \tag{4}$$

A simple calculation shows that for sufficiently large Q^2 where $\exp bt \gg a_0 b/2\pi$

$$A_p^h(l, n, z_0, t) = [a(Q^2)]^{n+1} Q^{2l} a_p^h(l, n, z_0, t) \tag{5}$$

and, consequently,

$$K_p^h(z_0, l, t) = a(Q^2) Q^{2l} \sum_{n=0}^{l-1} a_p^h(l, n, z_0, t)[a(Q^2)]^n. \tag{6}$$

It follows from Eq. (6) that, as in the case of the parton distribution in the hadrons $G_p^h(x,Q^2,t)$, for sufficiently large Q^2 the dependence of $D_p^h(1,0;z_0,\mathbf{k}_0;t)$ on k_0 is determined by a parton cascade through the moments $K_p^{p'}(z,l,t)$ and is concentrated in the parameter $\boldsymbol{\kappa}_0^2 = \mathbf{k}_0^2\,[Q^2 f_p^h(z_0,\alpha(Q^2),t)]$, where f_p^h is a slowly varying function of Q. Correspondingly, the transverse momentum distributions for hadrons in jets will depend on $\boldsymbol{\kappa}^2 = \mathbf{k}^2/z^2 Q^2 f_p^h(1/z,\alpha(Q^2),t)$, i.e., the distribution of hadrons in terms of $\mathbf{k}^2$ is z^2 times narrower.

Thus, in QCD the hadron distribution in terms of k in jets formed by partons possesses the same approximate scale invariance as the parton distribution in hadrons. As a result, the Gribov-Lipatov equations are satisfied for D_p^h and G_h^p.[4]

The nature of the dependence of f_p^h on z and t is successfully determined only for the limiting cases $z \ll 1$ and $1 - z = \delta \ll 1$. For $\delta \ll 1$ we have $f_p^h \approx \delta C_p^h(\alpha(Q^2))$ and for $z \ll 1$ $f_p^h \approx \bar{C}_p^h(\alpha(Q)^2)$, so that the hadron distribution in a jet in terms of k^2 first broadens with increasing z, becoming broadest for medium z $(0.1 < z < 0.9)$, and then again becomes narrower.

We shall now consider the rms transverse momentum for hadrons in jets $\langle k^2 \rangle_p^h$. For definiteness we restrict ourselves to the so-called primary pion distributions.[3] We have

$$\langle \mathbf{k}^2 \rangle_p^\pi = K_p^\pi(z, 1, t) / K_p^\pi(z, 0, t). \tag{7}$$

We shall first consider the properties of the $K_p^\pi(z,0,t)$ distribution in terms of z for pions in jets formed by quarks and gluons. For sufficiently large fixed Q^2 at $\delta \ll 1$ the number of pions in jets formed by quarks and antiquarks is about five times larger than in streams formed by gluons, and for $\delta \to 0$ they decrease in exponential fashion $K_q^\pi(1-\delta,0,t) \sim \delta^{8t/3}$. For small z the numbers of pions in jets produced by quarks, antiquarks, and gluons is of the same order. The pion distribution in terms of z is similar to the particle distribution of a quark-antiquark sea: the number of pions increases sharply with decreasing z, while with increasing Q^2 the number of pions with a given z decreases—the particles "migrate out" in the region of smaller and smaller z (the shoaling analog in a quark-antiquark sea.)[5]

The rms transverse pion momenta in jets produced by quarks (antiquarks) and gluons have the form

$$\langle \mathbf{k}^2 \rangle_p^\pi = \langle \mathbf{k}^2 \rangle_{0,p}^\pi + \beta_p^\pi \, \alpha(Q^2)\, Q^2 \delta / \pi \quad (8)$$

for $\delta \ll 1$ and

$$\langle \mathbf{k}^2 \rangle_p^\pi = \langle \mathbf{k}^2 \rangle_{0,p}^\pi + 21\, \alpha(Q^2)\, Q^2 z^2 / 10\pi \quad (9)$$

for $z \ll 1$. Here $\langle k^2 \rangle_{0,p}^\pi$ is the initial pion transverse momentum in the jets, $\beta_q^\pi = \frac{4}{3}$, and $\beta_g^\pi = 3$.

From Eq. (8) we see that as $z \to 1$ the rms transverse pion momenta in the jets produced by quarks are somewhat less than in jets produced by gluons, and for $z \ll 1$ $\langle \mathbf{k}^2 \rangle_q^\pi \approx \langle \mathbf{k}^2 \rangle_g^\pi$. As Q^2 increases $\langle \mathbf{k}^2 \rangle_p^\pi$ gets larger; however, this change only becomes important for very large $Q^2 \sim 10^3$–10^4 GeV. The rms transverse pion momentum integrated over z in the jet varies by $\langle \mathbf{k}^2 \rangle - \langle \mathbf{k}^2 \rangle_0 \sim \alpha(Q^2) Q^2 \langle z^2 \rangle / 2\pi n$, where $\langle z^2 \rangle = \int z^2 K_p^\pi(z,1,t)\, dz$ and n is the average multiplicity of pions in the jet. For present energies, $\langle \mathbf{k}^2 \rangle^\pi - \langle \mathbf{k}^2 \rangle_0^\pi \lesssim \langle \mathbf{k}^2 \rangle_0^\pi$ is in good agreement with experiment.[6]

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