The annihilation cross section for $e^+e^- \to$hadron near the $N\bar{N}$ threshold

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The behavior of the annihilation cross section for $e^+e^- \to$hadron near the $N\bar{N}$ threshold is discussed for a coupled channel model. It is shown that a strong attraction in the $N\bar{N}$ path leads to the appearance of additional irregularities other than resonances in the energy dependence of the cross section.

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Experimental results recently obtained for e^+e^- collisions apparently indicate a resonance behavior for the annihilation cross section of $e^+e^- \to$hadron in the mass region 1.5–2 GeV.[1] The properties of the resonances which were discovered (proximity of the masses to the $N\bar{N}$ threshold, multipion decay channels) indicate that at least several of them may be vector states of "baryonium",[2] whose existence has already been shown.[3] Along with this, a strong attraction in the $N\bar{N}$ system for continuous spectrum states and the connection with annihilation channels[4] gives a number of additional near-threshold effects (a relatively large value for the annihilation cross section in the $p\bar{p}$ interaction,[5] the growth of an electromagnetic form-factor for the proton when approaching the $N\bar{N}$ threshold from the time-like region,[6] etc.) In this article we show that the presence of a bound state in the $N\bar{N}$ system near the threshold together with a strong attraction between N and $\bar{N}$ in the states of a continuous spectrum leads to a nonmonotonic energy dependence of the cross section for $e^+e^- \to$hadron, in which the position of the maximum in the cross section need not agree with the resonance mass (the energy width of the observed irregularity may also differ from the natural width of the baryonium level).

We shall consider $e^+e^- \to$hadron annihilation near threshold as a coupled channel problem: $e^+e^- \to N\bar{N}$, $N\bar{N} + N\bar{N}$ (channel h), and $N\bar{N} \to$"$\rho\epsilon$" (channel l). A nonrelativistic potential approach is used to describe the annihilation. It should be noted, however, that despite the simplicity and model nature of the discussion, the basic results are a consequence of the general properties of the $N\bar{N}$ interaction. An attractive potential $V_k(r)$ acts between N and $\bar{N}$, leading to the occurrence of quasinuclear resonances in $N\bar{N}$ ("baryonium"). The interaction between the channels h and l takes place through a short-range potential $V_{kl}(r)$, which for simplicity was chosen in separable form: $V_{hl} = \lambda g(r)g(r')$, where $g(r) = 1/(2\pi r_a)^{1/2} \exp(-r/r_a)/r$ ($r_a = 1/2m \approx 0.1$ Fermi is the annihilation radius). We consider the particles in both channels to be zero spin. There is no interaction in the l channel. We selected the masses of the light particles to be close to the masses of real ρ and ϵ mesons.[7]

This process may be represented graphically as follows: The amplitude of the reaction corresponding to diagrams (a) and (b) may be written as

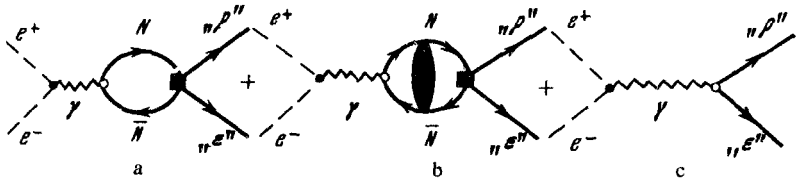


FIG. 1. Feynman diagrams for $e^+e^- \to$"$\rho\epsilon$" annihilation.

$$M = \frac{\langle T_0 | G_h | V_{hl} \rangle}{1 - \lambda^2 \langle g | G_l | g \rangle \langle g | G_h | g \rangle} = \lambda T_0 \frac{\langle 1 | G_h | g \rangle}{1 - \lambda^2 \langle g | G_l | g \rangle \langle g | G_h | g \rangle} g(p) \tag{1}$$

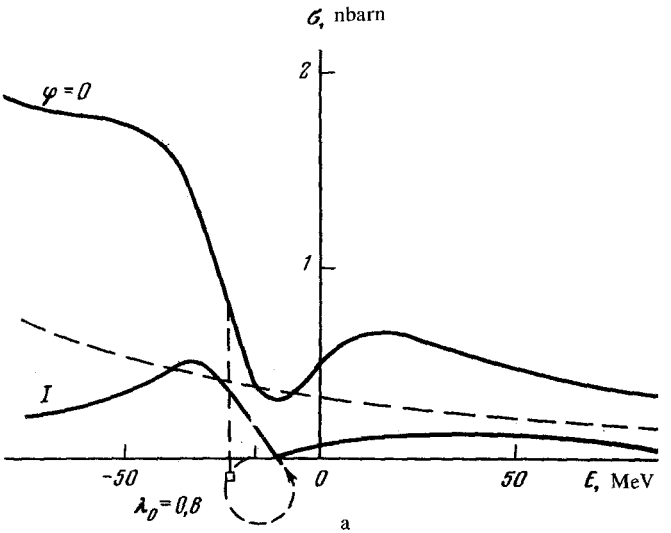
Here G_h and G_l are the Green's functions for the heavy ($N\bar{N}$) and light ("$\rho\epsilon$") particles respectively, T_0 is the $e^+e^- \to N\bar{N}$ annihilation amplitude (without $N\bar{N}$ final state interactions), and p is the light particle momentum in the center of mass system (CMS). In Eq. (1), we made use of the fact that the amplitude T_0 corresponding to the calculation of the annihilation characteristics far from the $N\bar{N}$ threshold may be considered a constant. Component (c) in Fig. 1 takes into account the nonpotential contribution to the $e^+e^- \to$"$\rho\epsilon$" annihilation, and is a background process.

Using the short-range V_{kl} the numerator in Eq. (1) may be rewritten as

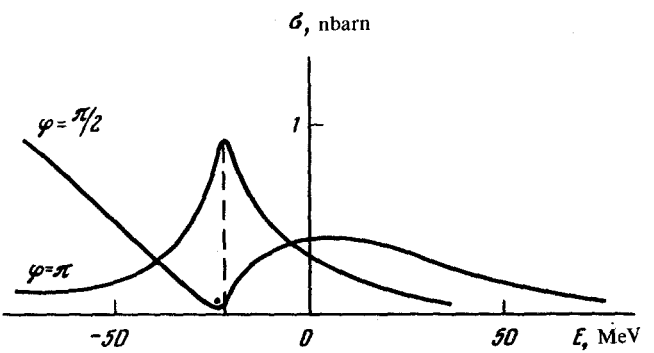
$$\langle 1 | G_h | g \rangle = -\frac{|\phi(0)|^2}{|\epsilon_B| + \epsilon + i\eta} + \int \frac{dk |\phi_k(0)|^2}{k^2/m - \epsilon - i\eta}, \tag{2}$$

where $\phi_k(r)$ and $\phi_0(r)$ are the wave functions for the continuous and discrete spectra of the potential V_h, ϵ_B is the binding energy of the quasinuclear level (we assume for simplicity that there is one bound state in the potential V_h, and $\epsilon = s^{1/2} - 2m$ is the kinetic energy of $N\bar{N}$ in the center of mass system. It is easy to see that the integral over the continuous spectrum in Eq. (2) for $\epsilon < 0$ is positive, while at the same time the contribution of the band corresponding to the bound state changes sign at the point $\epsilon = -|\epsilon_B|$. Therefore, at some point in the range $-|\epsilon_B| < \epsilon \lesssim 0$ a precise compensation of these terms is possible, i.e., the appearance of a zero in the $e^+e^- \to$"$\rho\epsilon$" annihilation amplitude without taking the background into account (the precise position of the zero evidently depends on the relative contribution of the band term and of the integral over the continuous spectrum). Even in the region $\epsilon < -|\epsilon_B|$ an increase in the annihilation cross section may be expected as a result of the constructive interference of both components in Eq. (2). It can be shown that the occurrence of a zero numerator in Eq. (1) is independent of the actual form of the potential $V_k(r)$. Actually, we shall discuss the equation for determining the zeros of the denominator in Eq. (1)

$$1 - \lambda^2 \langle g | G_l | g \rangle \langle g | G_h | g \rangle = 0. \tag{3}$$



a



b

FIG. 2. a—Behavior of the $e^+e^- \to$"$\rho\epsilon$" cross section near the $N\bar{N}$ threshold as a function of energy $\epsilon = s^{1/2} - 2m$. The curves $\phi = 0, \pi/2, \pi$ correspond to different assumptions concerning the phase ϕ of the background amplitude. The solid line shows the behavior of the cross section in the absence of background. The dashed line corresponds to the background cross section. The arrow indicates the position of the quasinuclear level in the complex energy plane ($\epsilon^* = -21 + 14\ i$); b—the same for $\phi = \pi/2$ and π.

It is known that Eq. (3) determines the position of the quasinuclear level in the complex plane ϵ taking into account the annihilation channel. It has been shown that for $\lambda \to 0$ the level with $\Gamma \to 0$, i.e., a pole occurs along the real energy axis for ϵ.[8] It is clear that the value of $\langle g|G_h|g\rangle = (\lambda^2 \langle g|G_l|g\rangle)^{-1}$ should go to zero at this point since $\langle g|G_l|g\rangle \neq 0$. At the same time the position of the zeros for $\langle g|G_h|g\rangle$ and $\langle 1|G_h|g\rangle$ agrees with good accuracy (on the order of $(r_a/R)^2$).

Figure 2a and b shows the behavior of the $e^+e^- \to$"$\rho\epsilon$" annihilation cross section. The potential V_h was chosen so that the undisturbed position of the quasinuclear level would be close to the threshold, and its radius would be large ($\epsilon_B = 12.7$ MeV, $R = 1.2$ Fermi). The dimensionless coupling constant for the annihilation path $\lambda_0 = 0.8$ corresponds to the total annihilation cross section observed in experiments with $\epsilon_{\rm CIS} = 44$ MeV.[8] The proton electromagnetic form factor G_0 corresponding to the calculation of annihilation properties far from the threshold (e.g., in the vector domi-

nance model, to the contribution of ρ, ω, and ϕ mesons) was assumed to be $G_0 = 0.1$. The background amplitude was normalized in the region of the ρ' meson where its contribution is dominant.[7] Curve 1 in Fig. 2a corresponds to the behavior of the cross section in the absence of background (the shift of the quasinuclear level as a function of the coupling constant is shown by a dashed line). The precise calculation evidently confirms the fundamental qualitative relationships which have been discussed earlier. At the same time the irregular behavior of the nonpolar part of the amplitude and interference with the background causes, according to Fig. 2, the complex behavior of the $e^+e^- \to$hadron cross section near the threshold, in particular, the appearance of additional anomalies simulating resonance behavior.

The results of the heuristic model calculations stated earlier indicate that although the origin of irregularities in the energy behavior of the $e^+e^- \to$hadron cross section is itself due to the presence of bound states for $N\bar{N}$, the identification of the observed maxima in the cross section with baryonium levels requires caution. In particular, the position and width of these peaks (and sometimes for the minima as well) in the cross section depend significantly on the inteference of the resonance and "background" terms in the amplitude. Therefore, in order to determine the masses of the sub-threshold resonances in terms of $e^+e^- \to$hadron annihilation processes, a comparative analysis of the data on the energy dependence for the annihilation cross section in different boson channels is necessary.

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