Potential-free formulation of the inverse scattering problem

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A new technique is proposed for formulating the partial wave functions of the *N–N* system directly in terms of *N–N* scattering phase shifts in explicit analytic form. A comparison is made with results obtained within the framework of the usual potential approach.

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A traditional problem of nuclear physics, the correct description of *N–N* interaction down to the region of small internucleon distances, has attracted progressively more attention in recent years. There are a number of experimental data (see, e.g., Refs. 1 and 2) which do not yield to a natural explanation within the framework of the usual and modified potential and quasipotential models.[3,4] We may assert rather definitely that we have reached the limits of applicability for the concept describing the *N–N* interaction in terms of local and quasi-local potentials.

In this regard, it is of great interest to study the possibility of a potential-free recreation of the wave functions of the *N–N* system directly in terms of scattering data. In this work we announce a technique for solving this problem. The idea for the suggested technique goes back to Ref. 5 and subsequent work.[6-9]

It is clear that a complete rejection of the potential concept requires some additional information providing for the existence and uniqueness of the solution for the recreation of the wave functions in terms of scattering data. With this aim in view, we may use the condition for the equivalence of the dispersion and Shroedinger descriptions of the deuteron electro-disintegration process. Then, using Watson's theorem on the final state interaction and assuming the validity of the Mandelstam representation for the invariant amplitudes of the deuteron electro-disintegration process,[7] we obtain the following explicit representation for the *N–N* system wave functions:

$$\Psi_{lsj}(k, r) = a_{lsj}(k, r) - \frac{1}{\pi f_{lsj}(k)} \int_{-\infty}^{\infty} \frac{\mathrm{Im} f_{lsj}(q) a_{lsj}(q, r)}{q - k - i0} dq , \tag{1}$$

where

$$a_{lsj}(k, r) = u_l(kr) + b_{lsj}(k, r) \tag{2}$$

u_l is the Riccati-Bessel function, f_{lsj} is the Jost function,[10] and the function b_{lsj} takes into account the contribution of the so-called nonphysical cross section due to the dynamics of the interaction and admits the following general integral representation:

$$b_{lsj}(k, r) = \mathrm{Re} \int_m^{\infty} dx \int_{m/2}^{x/2} dy \frac{e^{-ikr - xr} - e^{yr - xr}}{k - iy} c_{lsj}(x, y), \tag{3}$$

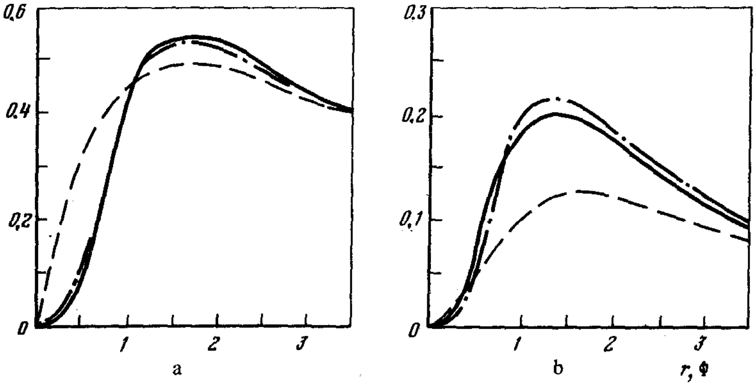

FIG. 1. S-wave (a) and D-wave functions for a deuteron. Our results are shown by a solid line. For comparison we have plotted the corresponding results obtained for the Reid potential with a soft core (dot-dash line) and for the Hulthén-Sugavar potential with a soft core (dashed line).

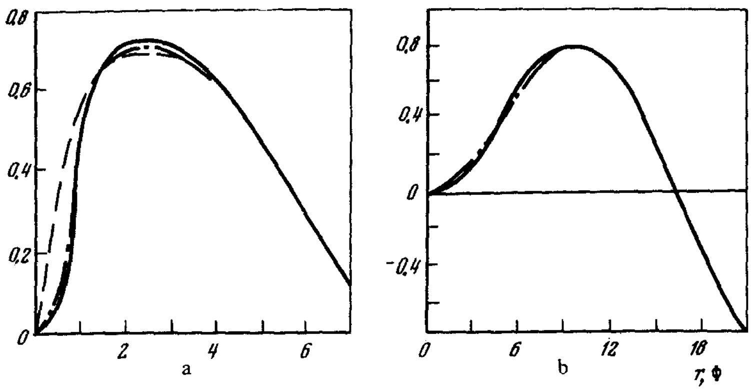

FIG. 2. The wave function for the S_0 path (a) and the $^3P_0$ path (b) for n–p scattering at a relative energy of motion for nucleons in the center of mass system of $E = 3$ MeV. Symbols are the same as in Fig. 1.

determined by the spectral function c_{lsj} for the contribution of nonphysical cross sections. Here the mass of the π meson is denoted by m.

Equations (1)–(3) are valid for the singlet scattering channel as well as for the related channels in terms of quantum numbers. In the subsequent case u_l should be understood as the diagonal matrix in terms of angular momentum, f_{lsj} as a transformed Jost matrix, and a_{lsj}, b_{lsj}, and c_{lsj} as general matrices.[9] The deuteron bound state wave function is determined naturally in terms of the corresponding scattering wave function by taking the remainder at the bound state point.

The problem of formulating the Jost function in terms of scattering data has been discussed extensively in the literature (see, e.g., the references given in Ref. 9). In order to find the spectral function c_{lsj} it is proposed here by analogy with the bootstrap concept to use the normalization condition for the wave scattering functions, considered as a quadratic nonlinear integral equation relative to the function c_{lsj}. In the linear approximation this integral equation belongs to a class of incorrect problems according to Tikhov, and requires that regularization be introduced. In our case regularization was brought about by reducing the problem considered to a problem in compactibility. For c_{lsj} we used an expansion in a finite sum of an orthogonal system of Laguerre polynomials:

$$c_{lsj}(x, y) = e^{-\frac{\epsilon(x - m)}{m}} \sum_{i,k=0}^{N} L_i\left(\frac{2y}{m} - 1\right) L_k\left(\frac{x - 2y}{m}\right) g_{lsj}^{i,k} , \tag{4}$$

where the values of the parameter ϵ determines the degree of suppression of the nonphysical cross sections from the long distance contribution. Thus, the problem of determining the function is reduced to solving a system of quadratic nonlinear algebraic equations for the coefficients $g^{i,k}$ by standard numerical analysis techniques. We also note that by taking into account the series in Eq. (4) for the function Ψ_{lsj} it is possible to obtain an explicit expression in elementary and special functions (which we will not carry out here because of its unwieldy nature).

The realistic nature of the approach stated is confirmed graphically by the results given in Figs. 1 and 2. In comparison with the potential description of the *N–N* interaction the proposed method, besides its structure and simplicity, has the fundamentally important advantage that it is completely lacking in phenomenologically-introduced parameters which would have been determined from the condition of agreement between theory and experiment. It should be noted that, within the framework of the approach which was developed and also using results from Ref. 11, it is possible in a simple and constructional level to put these nonrelativistic wave functions into relativistic form. The corresponding results are unquestionably of interest for wide number of applications. In particular, they permit a realistic calculation of the contribution of three forces (the so-called meson exchange currents) in electromagnetic processes with the participation of nucleonic systems.

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