Instability of unrestricted cumulation

E. I. Zebakhin

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It is shown that all the phenomena of an unrestricted cumulation are unstable, i.e., among the arbitrary small deviations of the initial conditions from the ideal there are always those for which focusing (the appearance of an unrestricted energy density) breaks down.

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Known unrestricted cumulation phenomena, such as converging spherical shock wave, collapsing bubbles in a liquid, pinch effect in a pulsed discharge and others, could give an unrestricted energy connection in the ideal case. This is of considerable interest since it is accompanied by new physical phenomena, such as a large radiant heat conductivity, nuclear reactions, etc.

The natural restriction of cumulation due to atomism (the difference between real and continuous media) is always very slight. Energy dissipation due to viscosity and heat conductivity usually also does not eliminate it. An unrestricted cumulation would be technically attainable if it were stable, i.e., if it also occurred when the test conditions deviate from the ideal. If, however, it is unstable, then its probability is equal to zero, the maximum energy density is finite and its magnitude is determined by the degree to which the initial conditions are not ideal.

Let us note that the unrestricted cumulation is not disturbed by every change in the initial conditions (for example, all the states, through which the system passes after the ideal initial conditions, are applicable); also, it is not necessarily disturbed with an increase in small perturbations (for example, a spherical wave can be converted into toroidal, also be amplified without limit, etc.)

For an unrestricted cumulation the volume density of the energy at some point becomes infinite, and its reciprocal value α becomes zero. It is non-negative, for a restricted cumulation $\alpha_{min} > 0$ and for unrestricted $\alpha_{min} = 0$. The corresponding state is called focusing.

The initial state of the system is characterized by the spatial distribution of the independent parameters $\alpha(r)$ as well as $\beta_0(r)$, $\gamma_0(r)$... (density, temperature, velocity, magnetic field, etc.). After dividing the region of space that we need into small cells and writing the values of all the parameters in each of them, we describe the initial state by a set of n quantities or by a point in n-dimensional space. The different initial states correspond to an n-dimensional region in this space.

In all the possible states of the unrestricted cumulation one of the n quantities is fixed—$\alpha_{min} = 0$ (we stipulate that the smallest of the α values is written in each cell). Thus, the set of focused conditions for any n occupies a region of measurements one fewer in number. For stability there must be a one-to-one correspondence between the

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points of the n- and $(n-1)$-dimensional regions, and this correspondence is established by the equations of the process. This still does not predetermine instabilities since, in principle, such a correspondence is possible between regions having different numbers of measurements, for example, the points of the unit segment $0, a_1\, b_1\, a_2\, b_2 \ldots$ correspond one-to-one to the points of the unit square $(0, a_1\, a_2 \ldots; b_1\, b_2 \ldots)$. However, such a correspondence cannot be continuous or piecewise continuous and it does not correspond to any physical processes. In fact, if the set of n variables $x_1 \ldots x_n$ is continuously and on a one-to-one basis transformed into $y_1 \ldots y_{n-1}$, then

$$x_1 = f_1(y_1 \ldots y_{n-1})$$

$$x_2 = f_2(y_1 \ldots, y_{n-1})$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x_n = f_n(y_1 \ldots y_{n-1}),$$

where f_i are continuous functions. From the last $n-1$ equations we obtain

$$y_1 = g_1(x_2 \ldots, x_n)$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$y_{n-1} = g_{n-1}(x_2 \ldots x_n).$$

Thus, $y_1 \ldots y_{n-1}$ do not depend on x_1 or points far from one another in the first space (differing in terms of x_1) correspond to one point in the second space, i.e., the requirements of continuity and single-valuedness are not satisfied together.

Thus, there can be no continuous one-to-one correspondence between the points of regions with a different number of measurement, i.e., focusing is not preserved when there are arbitrary variations in the initial conditions. This also means that any unrestricted cumulation is unstable or its probability is equal to zero and the sometimes assumed property of self-focusing does not exist. (Let us note that the probability of realizing any other $\alpha_{\min} > 0$ is also equal to zero as the probability of landing at a given point on line). The instability of not only an instantaneous cumulation at a point, but also its other versions (on a line, on a surface, at a running point, etc.,) follows from these same considerations.

This statement has already been set forth in the form of an assumption in Refs. 1 and 2 and contain nothing that is fundamentally new compared with the conclusion of Lifshits, Sudakov, and Khalatnikov[3] on the improbability of an unrestricted compression of the universe because of the chaotic nature of the initial state, but it differs in the fact that it relates not only to gravitational phenomena but also to all cases of unrestricted cumulation, including attempts to realize it in practice.

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