Lifshits points for two-dimensional representations

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The existence of Lifshits points at which three phases, two proportional and one nonproportional, are contiguous are shown for two-dimensional irreducible representations which admit a Lifshits gradient invariant. The properties of these points and their immediate neighborhood have been investigated.

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A new type of critical point called a Lifshits point, which divides transition lines of the second kind into proportional and nonproportional phases, was discussed in Ref. 1. This point is defined by the conditions $\alpha=0$, $\delta=0$ for the coefficients in the invariants $\alpha\eta^2+\delta(\Delta\eta)^2$ in a thermodynamic potential for a parameter of order η which transforms like a one-dimensional representation.

In this work we discuss Lifshits points in the framework of the Landau theory for two-dimensional irreducible representations which admit a Lifshits gradient invariant.[2] It is shown that these points differ from those discussed in Ref. 1: the critical fluctuations in their neighborhood have the usual order of magnitude. The properties of the thermodynamic anomalies in the phase transitions have been studied.

We shall discuss the two-dimensional representation E_4, which admits a fourth order anisotropic invariant. Using the convenient polar coordinate system

$$\eta=\rho\cos\phi\,,\quad \xi=\rho\sin\phi \tag{1}$$

for the basis of the representation η,ξ, we write the thermodynamic potential in the

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$$\Phi = \frac{\alpha}{2}\rho^2 + \frac{\beta}{4}\rho^4 + \frac{\beta'}{4}\rho^4\cos 4\phi + \frac{\gamma}{6}\rho^6 - \sigma\rho^2\frac{\partial\phi}{\partial z} + \frac{\delta}{2}\left[\left(\frac{\partial\rho}{\partial z}\right)^2 + \rho^2\left(\frac{\partial\phi}{\partial z}\right)^2\right], \quad (2)$$

where the coefficients β' and σ occur in front of the anisotropic invariant and the Lifshits gradient invariant, respectively. In Eq. (2) it is necessary to assume $\gamma > 0$, $\delta > 0$. As long as the phase diagram in the variables α, β' is symmetrical relative to the axis $\beta' = 0$, we shall assume for definiteness that $\beta' > 0$.

The following solutions and their corresponding phases satisfy the thermodynamic potential [Eq. (2)]. The initial phase 0: $\rho = 0$. The proportional phase 1:

$$\cos 4\phi = -1, \ \rho^2 = [\beta - \beta' + \sqrt{(\beta-\beta')^2 - 4\alpha\gamma}\,]/2\gamma \ . \quad (3)$$

The nonproportional phase $\tilde{1}$:

$$\rho = \rho_0, \ \phi = k_0 z, \text{ where } \rho_0^2 = \frac{\alpha_0 - \alpha}{\beta}, \ k_0 = \frac{|\sigma|}{\delta}, \ \alpha_0 = \frac{\sigma^2}{\delta}. \quad (4)$$

The solution [Eq. (4)] is valid under the condition $(\alpha_0 - \alpha)/\alpha_0 \ll 1$.

If $\beta > 0$ the phase transition $0 \to \tilde{1}$ is of the second kind and occurs along the line $\alpha = \alpha_0$. The phase transition $0 \to 1$ of the second kind $(\beta - \beta' > 0)$ takes place along the line $\alpha = 0$, while that of the first kind $(\beta - \beta' < 0)$ takes place along the line $(\Phi_0 - \Phi_1)$: $\alpha = 3(\beta - \beta')/16\gamma$. The two lines for the phase transition $0 \to \tilde{1}$ and $0 \to 1$ intersect at the point

$$\alpha = \alpha_0, \ \beta' = \beta + \sqrt{16\alpha_0\gamma/3}, \quad (5)$$

further referred to as the Lifshits point.

In the neighborhood of the Lifshits point [Eq. (5)] the phase diagram in the variables α, β' has the form shown in Fig. 1 (the phase diagram in the variables T,P will be similar). Three lines for phase transitions converge at the Lifshits point: $0 \to \tilde{1}$ of the second kind, and $0 \to 1$ and $\tilde{1} \to 1$ of the first kind. The latter two lines have a common tangent at the Lifshits point with a slope $d\beta'/d\alpha = (4\gamma/3\alpha_0)^{1/2}$.

We emphasize that the simplest solution [Eq. (4)] is valid in the neighborhood of the Lifshits point for the nonproportional phase $\tilde{1}$ in the entire region between the $0 \to 1$ and $\tilde{1} \to 1$ phase transitions. This leads to analytical equations for the anomalies of the physical quantities. In particular, we note that the phase transition $\tilde{1} \to 1$ from the nonproportional to the proportional phase in the neighborhood of the Lifshits point is an ordinary transition of the first kind with superheating and supercooling. Along the entire line for the $0 \to \tilde{1}$ phase transition, the value of the wave vector k characterizing the nonproportional phase $\tilde{1}$ is equal to k_0 [Eq. (4)]. Consequently, at the Lifshits point [Eq. (5)] k experiences a jump to k_0 (in contrast to Ref. 1, where k becomes zero at the Lifshits point). In the phase transition of the second kind $0 \to 1$, the heat capacity C increases to $C = \alpha_T^2 T_0/2\beta$, where the transition temperature T_0 is determined from the condition $\alpha = \alpha_T(T_0 - \theta) = \alpha_0$ [Eq. (4)]. In the phase transition $0 \to 1$ of the first kind the heat capacity in the neighborhood of the Lifshits point increases to $C = (\alpha_T^2 T_k/2\beta)(3\beta^2/4\alpha_0\gamma)^{1/2}$, where T_k is determined from the condition

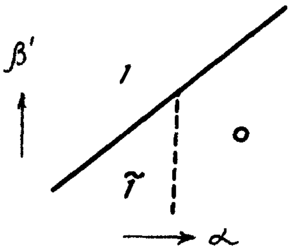

FIG. 1.

$\alpha = 3(\beta - \beta')^2/16\gamma$. Both jumps have a finite value at the Lifshits point. Since normally $\beta^2/\alpha_0\gamma \gg 1$ (actually $\beta^2/\gamma\alpha_T\theta \sim 1$, $\alpha_0 \ll \alpha_T\theta$), the sudden change in the heat capacity for the $0 \to \tilde{1}$ transition is much less than for the subsequent $\tilde{1} \to 1$ transition (in the neighborhood of the Lifshits point).

The E_4 representation was discussed above. Similar results are also obtained for the E_3 representation which admits a third order anisotropic invariant. For two-dimensional representations E_n with any n at $\beta < 0$, Lifshits points may occur at which three lines of phase transitions of the first kind between the same three phases converge. These lines do not have a common tangent at the Lifshits point.

We note that the three phases are also contiguous at the Lifshits point discussed in Ref. 1. However, the self-similar field approximation in all three phase transition lines have a common tangent which coincides with the line for transitions of the second kind, the initial-proportional phase (the aperture angle between the lines for the transitions of the second initial-nonproportional phase kind and of the first nonproportional-proportional phase kind tends to zero as the square of the distance from the Lifshits point). In the neighborhood of the Lifshits point discussed in Ref. 1 the critical fluctuations are anomalously large (because the coefficient δ in the gradient invariant becomes zero). In the neighborhood of the Lifshits point (5) the critical fluctuations have the usual order of magnitude.

Besides the Lifshits points discussed above on the α, β phase diagram, there exists a critical point defined by the conditions $\alpha = 0$, $\beta = 0$. At this point the line for transitions of the second kind crosses the line for transitions of the first kind between the initial and nonproportional phases. The properties of this point are completely similar to those of the usual critical point for phase transitions between the nonproportional phases.[2]

[1]R.M. Hornreich, M. Luban, and S. Strikman, Phys. Rev. Lett. **35**, 1678 (1975).
[2]L.D. Landau and E.M. Lifshits, Statisticheskaya Fizika (Statistical Physics), Nauka, Moscow, Part 1 (1976).