# Geometrical resonance in an intermediate state of superconductors

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We consider the absorption of ultrasound (geometrical resonance) in an intermediate state with $ka_N \gg 1$. We obtain an amplitude-modulated oscillatory dependence of the absorption as a consequence of Andreev reflections of electron excitations from the boundaries of normal and superconducting layers. It can be shown that this function explains the experiments in Refs. 1 and 2.

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Interest in study of the absorption of ultrasonic waves (UW) in the intermediate state (IS) is based on the fact that this technique is in essence unique, leading to experimental information concerning IS structure produced by an external magnetic field within a superconductor of the first kind. Electron excitation of the normal layers, whose distintive characteristic as first shown by Andreev[3] is a mechanism for reflection from the boundaries with a superconductor, is a fundamental contribution to the interaction with UW in the temperature domain $T < T_k$ in IS. Proceeding from the Andreev laws for the dynamics of electron excitations in the IS, several cases for the passage of UW waves through the IS were studied theoretically.[4-9] An absorption mechanism was found in Ref. 4 which is related to the vibration of the interphase boundaries, and has subsequently been studied in a region of IS filamentary structure.[5] In 1967 Andreev was the first to predict oscillatory absorption of UW[6]; to calculate it, he solved the kinetic equation in the Fourier components in terms of the inverse value of the double thickness of the normal layer. For $ka_N \sim 1$ and $l, D > a_N$ the periodicity of the absorption was found as a function of $a_N$. The influence of magnetic quantization on the absorption has also been discussed,[7,8] and the properties of UW propagation in the case of their oblique impact on an IS structure.[9]

Below in the framework of a simple model (a closed Fermi surface with two revolution points in the extremal cross section) the propagation of a longitudinal UW through an IS is analyzed at $T \ll T_c$ assuming the following sequence of inequalities is satisfied, $l > Da_N \gg \lambda$, where $l$ is the electronic excitation mean free path length, $D$ is their orbit diameter in the critical magnetic field $H_c$, $a_N$ is the thickness of the normal phase layer, and $\lambda$ is the UW wavelength. The UW with wave vector **k** is propagated transverse to the system of alternating normal and superconducting layers in the direction of the $X$-axis. A magnetic field equal to $H_c$ in the normal layer is oriented along the $Z$-axis. The boundary dividing the normal and superconducting phases (the NS-boundary) lies in the $YZ$-plane.

The kinetic equation in the field of the UW deformation $u_{ik} = u_{ik,0}\exp(i\mathbf{k}\mathbf{r} - i\omega t)$ in the normal layer may be written as (see Refs. 10 and 11)

$$(i\mathbf{k}\mathbf{v} - i\omega + \nu)\psi + \partial\psi/\partial t_1 = \Lambda_{ik} u_{ik}, \quad (1)$$

  

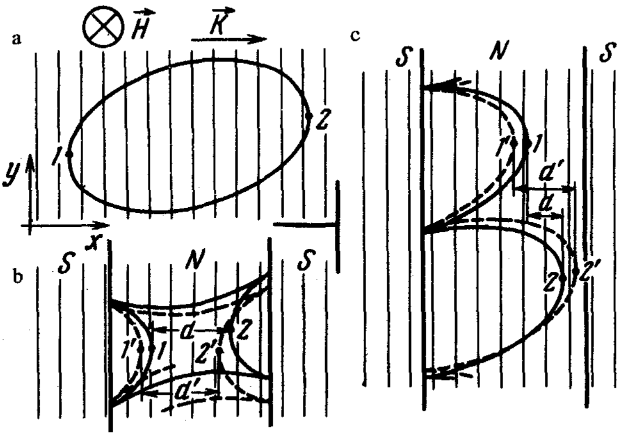


FIG. 1. Electron trajectory in a magnetic field: *a*-normal metal; *b*-intermediate state (two reflections between the revolution points 1 and 2; the distance between 1 and 2 along the *X*-axis is equal to *d* and is independent of the position of point 1); *c*-intermediate state (one reflection, *d* depends on the position of revolution point 1, $d \neq d'$). The thin vertical lines are planes of equal phase of the UW.

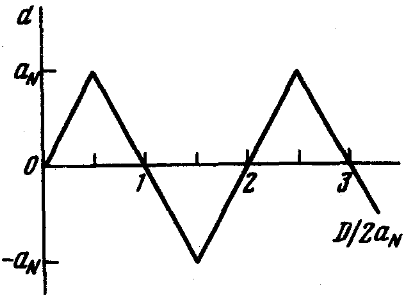


FIG. 2. The distance *d* between revolution points in the direction of the *X*-axis for an even number of reflections as a function of the ratio $D/2a_N$.

where $\psi$ is found from $f = f_o - (\partial f_o/\partial\epsilon)\,\psi$ ($f_o$ and $f$ are the equilibrium and nonequilibrium electron distribution functions), the notation is analogous to that in Ref. 11. In view of the small value for the speed of sound in comparison with the electron velocity **v**, we neglect the derivative of $\psi$ which is proportional to $\omega$ in Eq. (1). We shall seek a solution in the normal layer, as usual, in the form

$$\psi(t_1) = \int_{-\infty}^{t_1} \Lambda_{ik}(t_2)\,\dot{u}_{ik}(t_2)\exp\{\int_{t_1}^{t_2}(i\mathbf{k}\mathbf{v}(t_3) + \nu)\,dt_3\}\,dt_2 \,. \tag{2}$$

We note that in the coordinate system which is related to the electron excitation $\dot{u}_{ik} \sim (\mathbf{kv} - \omega)$ and, since $\mathbf{kv} \gg \omega$, then on the NS-boundary for a change in the sign of the velocity $\mathbf{v} \rightarrow -\mathbf{v}$ we shall have $\psi(\mathbf{v}) + \psi(-\mathbf{v}) = 0$, which is needed to satisfy the Andreev boundary condition.

Taking into account $\mathbf{k} \perp \mathbf{H}$ and the periodicity in the selected geometry $\Lambda_{ik}\dot{u}_{ik}$ in terms of the period of revolution of the electron excitation in the magnetic field, the equation for the absorption coefficient, which is formally analogous for the normal metal, may be written in the form

$$\Gamma(X_{(1)}) = \frac{2}{(2\pi\hbar)^3} A \int \frac{\Omega}{\mu} \{ |\gamma_{(1)}|^2 + |\gamma_{(2)}|^2 + \gamma^*_{(1)}\gamma_{(2)} \sin(\int_{t_{(1)}}^{t_{(2)}} \mathbf{kv}\, dt) \} dp_z, \quad (3)$$

where $A$ is a coefficient depending on the properties of the medium and the frequency of the UW,[10,11] and $\gamma(l) = A_{ik(l)}\dot{u}_{ik}/|\mathbf{kv}'_{(l)}|^{1/2}$, where the lower index in parenthesis indicates at which of the two revolution points for which $\mathbf{kv}_{(1),(2)} = 0$ the values of the corresponding quantities should be taken. In the IS $\Gamma(X_{(1)})$ turns out to depend on the position of the electron orbit on the $X$-axis, i.e., on the coordinate of the revolution point $X_{(1)}$.

We shall expand the integral which is the argument of the sine in the equation for the oscillating part of the absorption into a sum of integrals, where for the limits we select the successively fixed times of collision of excitation with the NS-boundaries, i.e., $\int_{t_{(1)}}^{t_{(2)}}... = \int_{t_{(1)}}^{\tau_1}... + \int_{\tau_1}^{\tau_2}...4...4 \int_{\tau_n}^{t_{(2)}}...$ . Taking into account that $\int_{t_{(1)}}^{t_{(2)}}|\mathbf{kv}|dt = kD$, we calculate the entire integral, writing it as $\int_{t_{(1)}}^{t_{(2)}}\mathbf{kv}dt = kd$, where $d$ is a certain effective diameter of the excitation trajectory, introduced in analogy with the normal metal (Fig. 1). For the case of an even number of reflections from the NS-boundaries between the revolution points, $d$ is

$$d = (-1)^l \{ (l+1)\, 2a_N - D \}, \quad (4)$$

where $l = [(D - a_N)/2a_N]$, $[\cdots]$ is the integral part of the expression in brackets ($[x] = -1$ if $-1 \leqslant x < 0$). The curve for $d = f(D/2a_N)$ is given in Fig. 2. For the case of an odd number of reflections $d$ will be a piecewise linear function of $X_{(1)}$ with limits varying from $-a_N$ to $a_N$ whose explicit form in this case is not significant. Averaging the oscillating part of the absorption $\tilde{\Gamma}(X_{(1)}$ for $ka_N \gg 1$ over the thickness of the layer leads to the value $\langle\tilde{\Gamma}\rangle = \int_0^{a_N}\tilde{\Gamma}(X_{(1)})dX_{(1)}/\int_0^{a_N}dX_{(1)}$. Since the number of reflections is a function of $X_{(1)}$ for given $a_N$ and $D$, the integral in the numerator is divided into two integrals--for orbits with an even and the odd number of reflections. Thus, $\tilde{\Gamma}$ (even) is independent of $X_{(1)}$ and the problem is reduced to the calculation of $\int_0^{a_N}dX_{(1)}$ (even), which is equal to $a_N - |d|$. $\tilde{\Gamma}$ (odd) is a rapidly oscillating sign-variable function of $X_{(1)}$, and its integral tends to zero. Thus, $\langle\tilde{\Gamma}\rangle$ turns out to be a modulated function equal to $a_N - |d|/a_N\tilde{\Gamma}$ (even). We shall write the equation for $|d|$:

$$|d| = (-1)^n (D - \{ m + \frac{1}{2}(1 - (-1)^n) \} 2a_N, \quad (5)$$

where $m = [D/2a_N]$, $n = [D/a_N]$.

We obtain the oscillating part of the absorption in the IS in the form

$$\tilde{\Gamma} \sim \Delta\tilde{\Gamma}_{\text{norm.}} (H_K)\, \eta\, \frac{a_N - |d|}{a_N} \sin(kd \pm \frac{\pi}{4}), \quad (6)$$

where $\Delta\tilde{\Gamma}_{\text{norm.}}(H_k)$ is the amplitude of the oscillations of the geometrical resonance in the normal metal in the field $H_c$, and $\eta$ is the normal phase density.

From Eq. (6) we see that the number of oscillations of the absorption in one modulation period is equal to twice the thickness of the normal layer $2a_N$ expressed in units of the wavelength of the UW $\lambda$, i.e., $2a_N = n\lambda$, where $n$ is the number of oscillations in one modulation period. For the case $D \ll a_N$ the results go over to the equation for the normal metal, i.e., $d \to D$, $[(a_N - |d|)/a_N] \to 1$, and only the density variation of the oscillation amplitudes remains in Eq. (6) which may be used to evaluate the normal phase density in the IS.

A comparison made with the experimental results[1,2] shows that Eq. (6) qualitatively describes the complex periodicity of the observed relationships. A quantitative comparison with experimental data is proposed to be carried out in a subsequent publication. Here we shall only indicate that the thickness of the layer of normal phase in Ref. 1 and 2 in the calculation based on Eq. (6) is varied from $10^{-2}$ to $10^{-3}$ cm and depends on the external magnetic field.

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