Spiral cyclotron waves in metals with an open Fermi surface

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A new type of electromagnetic excitations in gallium—spiral cyclotron waves—were investigated theoretically and experimentally. They propagate in the open directions and are polarized in the same way as helicons in uncompensated metals. The spiral waves can be effectively excited by longitudinal ultrasound.

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1. The spectrum of electromagnetic excitations in metals with an open Fermi surface has several unique characteristics compared to metals whose Fermi surface is closed.[1] Let us consider, for example, a metal with a Fermi surface modeled by a round cylinder with an axis along O_x $[\epsilon_{\mathbf{p}} = (p_y^2 + p_z^2)/2m]$. In the magnetic field **H** for a wave propagating in the "open direction" O_x, the hf conductivity tensor $\hat{\sigma}$ in the coordinate system $\{xyz\}$ associated with the crystal axes has the form:

$$\hat{\sigma} = \sigma_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{\gamma^2}{1+\gamma^2} & -\dfrac{\gamma}{1+\gamma^2} \\ 0 & \dfrac{\gamma}{1+\gamma^2} & \dfrac{\gamma^2}{1+\gamma^2} \end{pmatrix}, \qquad \sigma_0 = \frac{ne^2}{|m|(\nu - i\omega)}, \qquad \gamma = (\nu - i\omega)/\Omega. \tag{1}$$

Here e is the charge, m is the effective mass, $\mathbf{v}$ is the velocity, ν is the relaxation frequency, n is the concentration of the conduction electrons, ω is the wave frequency,

$$\Omega = \frac{eH}{mc}\sin\phi \tag{2}$$

is the cyclotron frequency, and the vector **H** is in the xOz plane at an angle ϕ to Oz.

It follows from the Maxwell equations and Eq. (1) that a circularly polarized electromagnetic wave

$$E_{\pm}(x,t) = E_y \pm iE_z = E_{\pm}(0)e^{i(qx-\omega r)} \tag{3}$$

can propagate in metal, with the coupling between the wave vector q and ω determined by the dispersion relation

$$q^2c^2 - 4\pi i\omega\sigma_{\pm} = 0, \qquad \sigma_{\pm} \equiv \sigma_{yy} \pm i\sigma_{zy} = \sigma_0 \frac{\gamma}{\gamma \mp i} \ . \tag{4}$$

This relation shows that for a cylindrical electronic Fermi surface ($m > 0$) the wave is negatively polarized and for the hole surface it positively polarized, with

$$\omega(q) = \frac{|\Omega| - i\nu}{1 + (\omega_p/qc)^2}, \qquad \omega_p^2 = \frac{4\pi ne^2}{|m|} \ . \tag{5}$$

For small $q \ll \omega_p/c$ the $\omega(q)$ dependence is quadratic. As the wave accelerates the spectrum approaches the cyclotron frequency (2). The relative attenuation of the wave is small: $\mathrm{Im}\omega/\omega = -\nu/|\boldsymbol{\Omega}|$. Thus, the spiral cyclotron wave (SCW) at small q is an ordinary helicon which is converted to a cyclotron mode as q increases.

2. The SCW might also be called a wave near the diamagnetic resonance. In fact, if $\mathbf{q}\|Ox$ for the cylindrical Fermi surface $\mathbf{qv}\equiv\mathbf{0}$ the field of the wave for such electrons will be uniform, and resonance between the electrons and the wave will occur only at the fundamental frequency of the cyclotron resonance (2). However, in a general configuration (a small angle between $\mathbf{q}$ and Ox) spatial dispersion will appear and resonance peculiarities in $\hat{\sigma}$ will occur at frequencies which are multiples of Ω. These resonances are responsible for multiple SCW's whose properties are analogous to those examined above.

The presence of other groups of carriers in the metal will of course change the spectrum and the attenuation of the SCW. Because of the resonance behavior of the conductivity $\hat{\sigma}$ in Eq. (1) at $\omega \approx |\boldsymbol{\Omega}|$ the spectrum and polarization of the excitation nontheless are described, as usual, by relations (5) and (3) in the immediate vicinity of the cyclotron frequency. To obtain the $\omega(q)$ dependence in the region in which the SCW dispersion is large, we must take into account the contribution to $\hat{\sigma}$ of other "nonresonance" electrons.

3. SCW's should be manifested by their resonance interaction with ultrasound. At small angles of deflection ϕ an SCW can be effectively excited only by longitudinal sound oscillations. To verify this, we need only examine the expression for the induced field

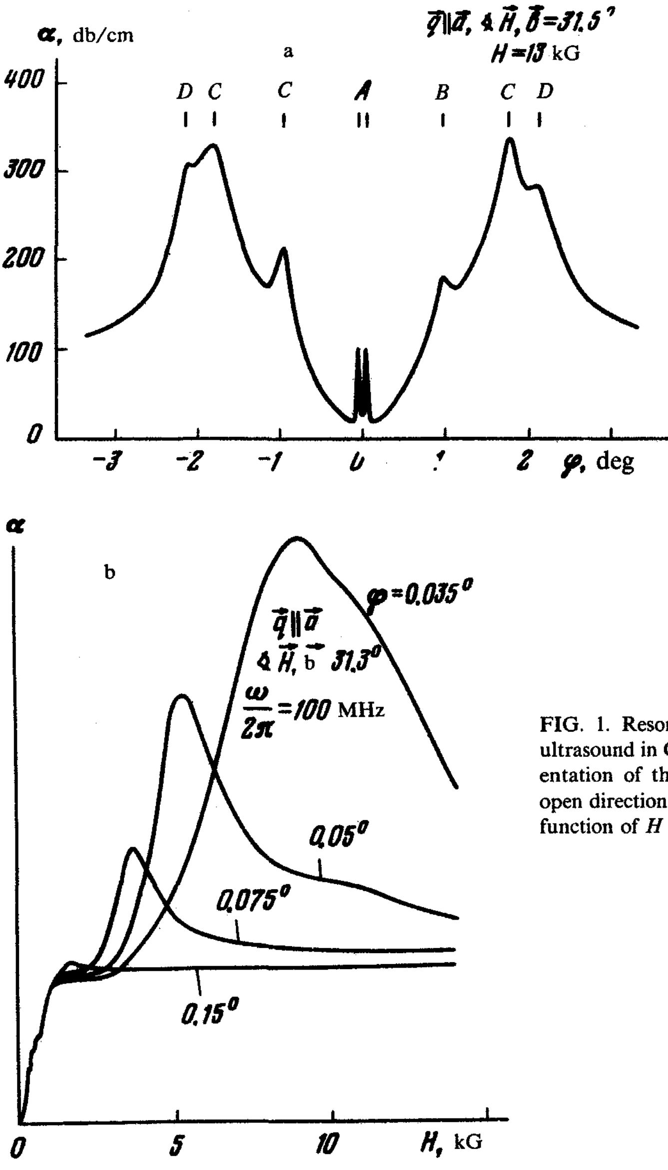

FIG. 1. Resonance absorption of longitudinal ultrasound in Ga: a—due to variation of the orientation of the magnetic field relative to the open direction, with H and ω constant; b—as a function of H for fixed ω and ϕ.

$$\mathbf{E}' = -\frac{i\omega}{c}[\mathbf{u}\mathbf{H}],$$

which builds up electromagnetic waves in the metal. For transverse sound (the displacement vector $\mathbf{u}\perp Ox$) the projection of $\mathbf{E}'$ on the yOz plane, in which the SCW is polarized, is proportional to $\sin\phi$ and for longitudinal sound ($\mathbf{u}\|Ox$) it is proportional to $\cos\phi$. Therefore, at $\phi\ll 1$ the SCW can couple only with the longitudinal lattice vibrations.

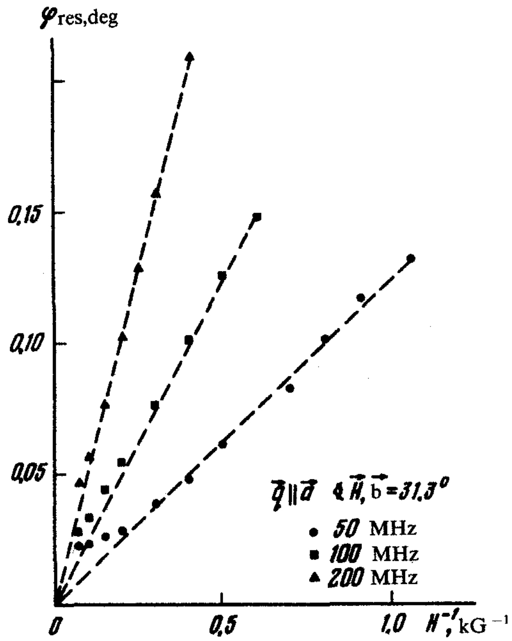

FIG. 2. Experimental dependence of the angle of deflection ϕ on H in the resonance absorption peak. The dashed line represents the theoretical line (6).

The interaction of the SCW with the sound reaches a maximum when the external parameters are such that the length of the SCW, $2\pi/q$, is comparable to that of the sound wave, $2\pi s/\omega$ (s is the velocity of the longitudinal sound oscillations). Using Eq. (5) we obtain the condition for resonance excitation of the SCW:

$$\sin\phi = \frac{\omega}{H}\,\frac{|m|c}{e}\left[1+\left(\frac{\omega_p}{\omega}\,\frac{s}{c}\right)^2\right] \tag{6}$$

and the width of the resonance line

$$\delta(H\sin\phi) \sim \nu\frac{|m|c}{e} \tag{7}$$

4. An excitation of SCW's by sound was observed experimentally by us in gallium. We measured the absorption α of longitudinal sound oscillations in pure Ga single crystals at $T = 1.7$ K and frequencies $\omega/2\pi = 50$, 100, and 200 MHz (maximun value $\omega/\nu \approx 8$). The $\alpha(\phi)$ dependence (see Fig. 1a) was automatically recorded at specified ω and H. In addition to the maxima for the usual tilt effect (B, C, and D with $\phi \approx 1$–2° in Fig. 1a), we observed at very small angles ($\phi \approx 0.02$–0.2°) sharp absorption peaks A which were distributed symmetrically along Φ with respect to the open directions. A transition via the absorption peak α can be accomplished not only by varying ϕ at $H =$ const but also by varying H in a specified experiment geometry (Fig. 1b) Peaks such as A were observed only in the longitudinal sound if the vector **q** was parallel to the open directions (**a** and **c** axes, [100] and [001], respectively) and *in all cases* when the orientation of **H** ensured strongly prolate electron orbits at the Fermi

surface. Estimates show that the "resonance" orbits penetrate about 100 unit cells of the reciprocal lattice.

The distance between the A peaks in Fig. 1a, equal to $2\phi_{res}$, varies with the magnetic field and frequency. These dependences are illustrated in Fig. 2 for one experiment geometry. It can be seen that, except for the region of the strongest fields, the experimental dependence of ϕ_{res} on ω and H obeys the law $\phi_{res} \propto \omega/H$. This is consistent with Eq. (6) in which the parameter $(\omega_p s/\omega c)^2$ is small. We obtain from this an upper limit on the concentration of resonance electrons:

$$n \lesssim \frac{|m|}{4\pi}\left(\frac{\omega c}{es}\right)^2 \sim 10^{17}\ \text{cm}^{-3}, \qquad s = \begin{cases} 4.28 \cdot 10^5\ \text{cm/sec} & (\mathbf{q} \parallel \mathbf{a}); \\ 4.9 \cdot 10^5\ \text{cm/sec} & (\mathbf{q} \parallel \mathbf{c}); \end{cases}$$

and from the slope of the lines (6) in Fig. 2 we can determine the effective mass:

$$|m| = \begin{cases} 0.12 \times 10^{-27}\ \text{g} & \text{at} \quad \measuredangle\, \mathbf{H}, \mathbf{b} = 31.3^\circ; \\ 0.05 \times 10^{-27}\ \text{g} & \text{at} \quad \mathbf{H} \perp \mathbf{a}. \end{cases}$$

The relaxation frequency ν can be estimated by using Eq. (7) and Fig. 1b. We obtain from them $\nu \sim (2\text{–}5) \times 10^8\ \text{sec}^{-1}$. This value coincides with ν, which was measured from the shape of the tilt-effect line and from the dispersion of the sound velocity.

5. We have observed a divergence from the $\phi_{res} \propto \omega/H$ law in strong magnetic fields (see Fig. 2). This effect can be explained by the influence of other nonresonance carriers on the SCW spectrum, rather than by the model of the cylindrical Fermi surface. This problem and the curious effects occurring as a result of divergence of q from the open directions will be discussed in a separate paper.

[1]E.A. Kaner and V.G. Skobov, Adv. in Phys. **17**, 605 (1968).