Transition to turbulence in Couette flow

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It is shown that the transition to turbulence in Couette flow combines the features of Landau's picture and the stochastic-attractor concept. An increase of the Reynolds number causes excitation of new modes and a smooth broadening of the spectral peaks that were produced earlier.

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1. There are two points of view on the transition from the laminar state to the turbulent state. According to Landau,[1] this transition comprises a series of bifurcations that excite new types of motion with incommensurate frequencies. As a result, sharp peaks begin to appear in the power spectrum, which becomes continuous as the number of peaks increases. In terms of this theory, the stochastization of the motion entails the excitation of a large number of degrees of freedom. According to the other point of view, which is based on the concept of a stochastic (strange) attractor,[2,3] randomization of motion is possible if the number of interacting modes is finite.

In this paper we give some results on the transition to the turbulent mode in circular Couette flow. Our interpretation of the onset of turbulence combines the features of Landau's picture and the idea of the stochastic attractor. An increase in the Reynolds number will both excite new modes and smoothly broaden the spectral lines produced earlier. These conclusions are inconsistent with the results of Gollub and Swinney[4]; in their experiments the transition to a continuous spectrum occurs abruptly. This discrepancy is attributable primarily to the fact that our experiments were conducted in a system with a wider gap between the cylinders ($R_2/R_1 = 1.57$ and $R_1 = 17.5$ mm; in Ref. 4, $R_2/R_1 = 1.14$).

By maintaining a quite high accuracy of the parameters of the system (the temperature and the rotation frequency of the inner cylinder Ω were maintained with an accuracy of 0.02° and 10^{-4}, respectively, and the radial wobbling of the cylinders was 5 μm), we were able to obtain a higher resolution in both frequency and Reynolds number. The velocity of the fluid (water) was recorded by a laser Doppler anemometer. In contrast to Ref. 4, we measured the azimuthal projection of the velocity $V_\phi(t)$ at a point close to the center of the gap. The position of the measuring volume could be varied $\pm$ 16 mm in the vertical direction. The results of the measurements were fed to a computer and analyzed.

2. In our geometry the laminar flow becomes unstable at $\mathrm{Re} = \Omega R_1(R_2 - R_1)/\nu = 76 \pm 1$ (ν is the kinematic viscosity). The number of vortices varies (from 22 to 36) in the 300-mm-high cylinders, depending on the acceleration mode. The results corresponding to the 30-vortex mode are given below.

At $\mathrm{Re} = \mathrm{Re}_1 = 995 \pm 1$, bending vibrations of the vortices occur and a sharp

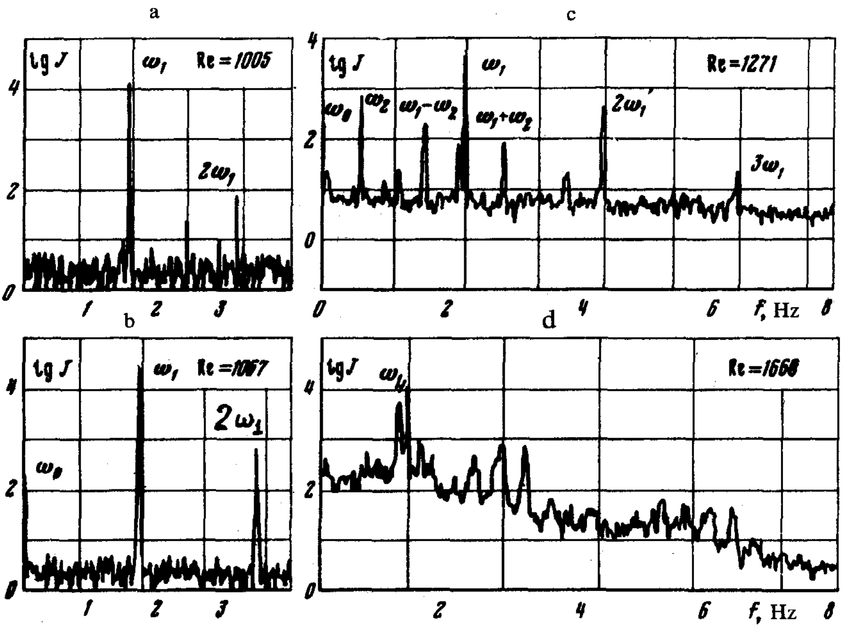

FIG. 1. Vertical lines correspond to the frequencies $n\Omega$.

peak, whose relative width at the 10^{-3} level is 10^{-3}, is produced in the power spectrum $J_\omega = \langle |V_\phi(\omega)|^2 \rangle$ at the frequency $\omega_1 = 1.93\ \Omega$ (Fig. 1a). The second harmonic in the spectrum becomes important with increasing Re. A bifurcation, which slightly broadens the line $\omega_1(\Delta\omega/\omega_1 \sim 10^{-2}$ at $\epsilon = 0.05)$, occurs under conditions of supercriticality $\epsilon = (\mathrm{Re} - \mathrm{Re}_1)/\mathrm{Re}_1 = 0.025$. This produces a low-frequency motion with a characteristic variation time $1/\Delta\omega$ (Fig. 1b). In the $1200 < \mathrm{Re} < 1300$ range the combination harmonics and motion at the frequencies $\omega_2 = 0.58\ \Omega$, $\omega_3 = 0.36\ \Omega$, and $\omega_4 = 0.95\ \Omega$ occur sequentially (Fig. 1c). The peaks broaden with increasing number

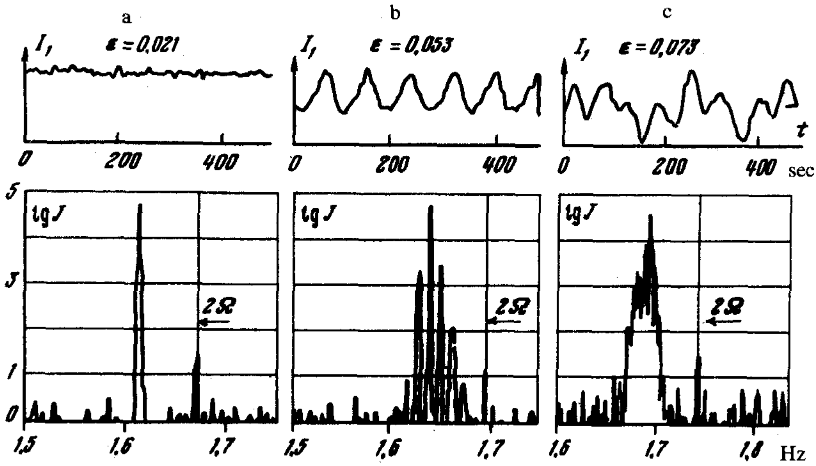

FIG. 2.

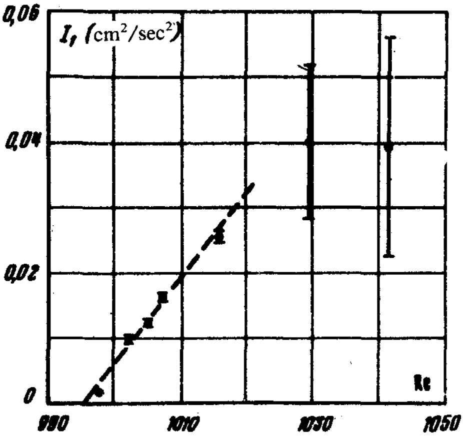

FIG. 3.

of revolutions and their widths gradually become comparable to the distance between them (Fig. 1d). With further increase of Re (Re > 2600), sharp peaks appear in the background of the continuous spectrum. This system of secondary peaks again collapses in the same way as the primary system.

Thus a sequence of bifurcations which excite new modes can be observed in a definite range of supercriticality. This is clearly illustrated by the evolution of the spectra. Moreover, the original $V(t)$ dependences contain much more detailed information on the phase space of the system. If the spectrum consists of several (N) sharp peaks, and the noise component can be neglected, then $V(t)$ can be written in the form

$$V(t) = \sum_{n=1}^{N} \left(A_n(t) e^{-i\omega_n t} + A_n^*(t) e^{i\omega_n t} \right) .$$

The $\{A_n, A_n^*\}$ values determined in this way form a 2 N-dimensional subspace S of phase space. The $A_n(t)$ dependence can be obtained by filtration of $V(t)$ in the frequency window, which is larger than $\Delta\omega_n$ but smaller than the distance between the peaks. The $A_n(t)$ trajectory in S is an attractor for the established modes.

At low supercriticality $\epsilon < 0.01$ the spectrum has only one peak and S is equal to 2. The experiment shows that $|A_1|^2 = \text{const}$ (Fig. 2a) and hence the attractor is the limiting cycle. Thus, $I = |A_1|^2 \sim \epsilon$ (Fig. 3), fully consistent with Landau's law. At $\epsilon > 0.025$ this limiting cycle breaks down. First, this gives rise to periodic modulations of $I(t)$ and to formation of satellite lines in the fine structure (Fig. 2b); then, at $\epsilon \approx 0.08$ an irregular time dependence of $I(t)$ with a modulation depth of order unity can be observed. In this case, the line structure has a noise component (Fig. 2c). It is curious that the power dispersion $D_\omega = (\langle |V_\omega|^4 \rangle / J_\omega^2 - 1)^{1/2}$, which is equal to unity for the Gaussian statistics and to zero for a coherent signal, at frequency ω_1 and for $\epsilon < 0.025$ is equal to 0.01–0.02, while for $\epsilon \approx 0.08$ such motion is attributable to the strange attractor. In this case the dimensionality of the effective phase space S is 5. When Re is large the dimensions of S increase, the structure of the trajectory in it becomes more complicated, and D_ω is equal to ≈ 1 in the entire frequency range.

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