Critical current of Josephson junctions with a semiconductor layer

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The dependence of the critical current of a superconductor–semiconductor–superconductor junction on the free-electron concentration in a semiconductor is determined.

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Much interest has lately been shown in the study of Josephson junctions in which the superconductor is separated by a semiconducting layer.[1,2] The properties of these systems depend on the concentration of electrons in the conduction band of the semiconductor; they vary from ordinary tunnel junctions (at low concentration of electrons) to an *S–N–S* junction (superconductor–normal metal–superconductor). In this

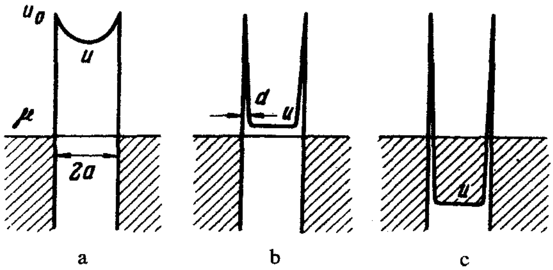

FIG. 1. Band structure of junctions: superconductor–semiconductor–superconductor for different concentrations of free electrons in the semiconductor.

paper we determine the dependence of the critical current of an element near the critical temperature of the superconductor on the concentration of free electrons in the semiconductor.

To determine the critical current we calculated the Green's function of the system $G_\omega(\xi, z, z')$, where z is the coordinate in the perpendicular direction to the plane of the junction, and $\omega = (2n+1)\pi T$ is the Matsubar frequency, $\xi = (p_x^2 + p_y^2)/2m - \mu$ is the energy of the transverse motion counted from the chemical potential μ. Thus, the current through the junction can be determined from the equation[3]

$$j = \frac{ie}{2\pi} T \sum_\omega \int d\xi \left[\left(\frac{\partial}{\partial z'} - \frac{\partial}{\partial z}\right) G_\omega(\xi; z, z')\right]_{z=z'}. \tag{1}$$

The Green's function near T_c can be expressed in terms of the Green's function of the system in the normal state,[4] $G_\omega^n(\xi, z, z')$. To determine the latter function we must know the behavior of the scalar potential $V(z)$.

The potential $V(z)$ is shown in Fig. 1 for different electron concentrations in the conduction band. At low concentration n the chemical potential μ lies below the bottom of the conduction band, so that $u - \mu \gg T$ (Fig. 1a). The thickness d of the barrier layer is large compared with the dimension α, and the $V(z)$ dependence in the semiconductor can be assumed to be quadratic:

$$V(z) = u + Az^2, \qquad A = \frac{2\pi e^2 n}{\epsilon}, \tag{2}$$

where ϵ is the dielectric constant.

Expressing G_ω^n in terms of the amplitude of the D wave transmitted across the barrier, we obtain for the current through the junction

$$j = \frac{2}{\pi} em^3 T\Delta^2 \sin\chi \sum_\omega \int_{-\mu}^{\infty} d\xi \frac{D(k_+)D(k_-)}{k_+k_-(k_+ + k_-)^2} \exp[2i(k_+ + k_-)a], \tag{3}$$

where the pulses $k_\pm = [2m(\pm i\omega - \xi)]^{1/2}$, χ is the difference in the phases of the order parameter in the superconductors, Δ is its modulus, the Cooper constant of attraction between the electrons in the semiconductor is assumed to be zero, the mutual influence of the superconductors is assumed small, and m is the effective mass.

Calculating $D(k)$ for the barrier (2) from Eq. (3), we obtain

$$j_k = \frac{e m^{1/2} \Delta^2 (u_0 - \mu)^{3/2}}{\sqrt{2}\,\pi a \mu T} \exp\left[-4\sqrt{2}\, a m^{1/2} (u_0 - \mu)^{1/2} \left(1 - \frac{a}{3}\right)\right]; \tag{4}$$

$$a = \frac{2\pi e^2 a^2 n}{\epsilon (u_0 - \mu)} << 1.$$

As can be seen, when the concentration of electrons in the semiconductor is small, the superconducting current is determined by electrons tunneling across the barrier, whose height decreases with increasing concentration of the electrons. The concentration at which the thickness of the barrier layer is about the same as that of the junction is determined by $n_0 \sim \epsilon (u_0 - \mu) e^2 \alpha^2$. At large concentrations Eq. (2) cannot be used for the potential. Thus, the transparency of the barrier is small for concentrations up to about n_0 [the exponent in Eq. (4) is $\sim \alpha m^{1/2} (u_0 - \mu)^{1/2}$] and hence the critical current of the junction is exponentially small.

Note that in a certain range of concentrations the chemical potential can be located in the impurity band. In this case jump-like conductivity will become significant in bulk semiconductors[5]; if, however, the thickness of the layer of the semiconductor is smaller than the characteristic length of the jump, then such a charge-transfer mechanism can be disregarded.

At concentrations $n \gg n_0$ the thickness of the barrier layer is small compared to the width of the junction and the wavelength of the de Broglie electrons. In this case the potential in the semiconductor can be assumed to be independent of the z coordinate, and its variation near the boundary can be approximated by a δ function: $V(z) = B\delta(z \pm \alpha)$ (Fig. 1c). For the critical current we obtain the following expression from Eq. (3):

$$j_k = \frac{e\Delta^2 T}{2\pi m^{3/2} a B^4} \sum_\omega \frac{[(u - \mu)^2 + \omega^2] \exp[-4 a m^{1/2} (u - \mu + [(u-\mu)^2 + \omega^2]^{1/2})^{1/2}]}{[(\mu^2 + \omega^2)^{1/2} - \mu](u - \mu + [(u-\mu)^2 + \omega^2]^{1/2})^{1/2}}. \tag{5}$$

The subsequent expressions depend greatly on the location of the chemical potential relative to the bottom of the conduction band in the semiconductor. If $u - \mu \gg T$ (nondegenerate semiconductor), then the exponent in Eq. (5) can be assumed to be independent of ω and the sum may be calculated by

$$j_k = \frac{e\Delta^2 \mu (u - \mu)^{3/2}}{4\sqrt{2}\,\pi a m^{3/2} B^4} \exp[-4\sqrt{2}\, a m^{1/2} (u - \mu)^{1/2}], \tag{6}$$

where the dependence of $u - \mu$ on the concentration n in the nondegenerate semiconductor can be determined from the standard formula: $u - \mu = T \ln (n/N_c)$, where $N_c = (mT)^{3/2}/\sqrt{2}\pi^{3/2}$ is the effective density of states in the conduction band. The critical current in this case is determined, as usual, by the tunneling of electrons across the barrier, although its height is greatly reduced.

At $u - \mu \ll T (n \sim N_c)$ the chemical potential is near the bottom of the conduction band (Fig. 1b). To determine the critical current according to Eq. (5), we confine ourselves to the first term in the sum over ω. As a result, we obtain

$$j_k = \frac{e\Delta^2 \mu T^{1/2}}{\pi^{3/2} a m^{3/2} B^4} \exp(-4\sqrt{\pi}\, a m^{1/2} T^{1/2}). \quad (7)$$

It follows from Eq. (7) that when the barrier is only at the boundaries of the semiconductor with the superconductor the critical current decreases exponentially as a result of increase of the thickness of the semiconducting layer. This has a simple physical explanation. The electrons from the superconductor, after passing through the semiconductor, will lose their coherence at distances of the order of the size ξ of the pair, and at thicknesses $\alpha \gg \xi$ can no longer generate a superconducting current. The quantity ξ is determined by the velocity v of the electrons with energies of order T measured from the bottom of the band: $\xi \sim v/T \sim (mT)^{-1/2}$. This distance is much shorter than the usual size ξ_0 of the pair, since the velocity v is low compared with the Fermi velocity in the metal. Nonetheless, the characteristic size of the junction, which can handle a noticeable superconducting current: $\alpha \sim (1/p_F)(\mu/T)^{1/2}$, greatly exceeds the thickness of the atomic layer.

In the degenerate semiconductor: $\mu - u \gg T (n \gg N_c)$ (Fig. 1c) and for the critical current from Eq. (5) we obtain

$$j_k = \frac{\sqrt{2}\, e\Delta^2 \mu (\mu - u)^{5/2}}{\pi^4 a m^{3/2} T^2 B^4} \exp[-2\sqrt{2}\,\pi a m^{1/2} (\mu - u)^{-1/2} T], \quad (8)$$

where $\mu - u = (3\pi^2)^{2/3}\, n^{2/3}/2m$. The exponent decreases in this region of concentration due to an increase of the Fermi velocity and hence the size of the pair. Note that at $n \gg N_c$ the mean free path of the electrons l in the semiconductor can be shorter than the size ξ of the pair. In this case the critical current is proportional to $\exp(-\alpha/\xi)$, but in the impure case $\xi_d = (\xi_c l)^{1/2}$.[6,7]

In the experiment we used a wide range of electron concentration in the conduction band; to accomplish this, we introduce impurities[2] and used a light-sensitive semiconductor.[1] The experimental dependence of the critical current on the electron concentration in the conduction band is not available at this time; however, the limitations we have set on the thickness of the junctions are consistent with the available experimental data.

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