

# Focusing of conduction electrons by an inhomogeneous magnetic field

Yu. A. Kolesnichenko and V. G. Peschanskii

*Physicotechnical Institute of Low Temperatures, Ukrainian Academy of Sciences*

*A. M. Gor'ki Khar'kov State University*

(Submitted 6 July 1979)

Pis'ma Zh. Eksp. Teor. Fiz. **30**, No. 4, 237–240 (20 August 1979)

It is shown that transverse focusing of electrons by an inhomogeneous magnetic field gives detailed information on their interaction with the surface of the conductor. In particular, the angular dependence of the image parameters  $q_i(\phi)$  can be fully restored.

PACS numbers: 72.10.Di, 72.15.Gd

Electron focusing in metals by a constant uniform magnetic field<sup>[1-3]</sup> is an effective method of studying the electronic energy spectrum and reflection of charge carriers that are incident almost normally to the surface of the sample.

Investigation of the interaction of charges with the conductor boundary can be greatly expanded by using an inhomogeneous magnetic field for transverse focusing. For example, electron focusing in a magnetic field  $H_y = \alpha + 2\beta z/d$  in thin metal plates of thickness  $d \ll l$  ( $l$  is the mean free path of the electrons) gives information on the scattering of conduction electrons by the surface of the sample at different angles of incidence.

A nonmonotonic dependence of the potential difference on the parameters  $\alpha$  and  $\beta$  results from electron drift in the orthogonal direction to the magnetic field and its gradient. The displacement  $\Delta R$  of electrons along the line of contacts during the time between two successive collisions with the upper surface, if the electrons are ejected from the emitter at angles  $\phi$  close to  $\pi$ , is always different from zero (Fig. 1), i.e., in a certain region of angles  $0 \leq \phi \leq \phi_0$  the function  $\Delta R(\phi)$  is unique [ $\Delta R(\phi_0) = \Delta R(\pi)$ ] and in the remaining interval it has two values. Therefore, the difference  $\phi_p(\alpha, \beta; L)$  in potentials between the potential contact and the peripheral point of the sample, a quantity proportional to the number of nonequilibrium electrons transmitted from the emitter to the collector without scattering, will, as a function of the distance  $L$  between the contacts, have a singularity at  $L = L_0 = \Delta R(\phi_0)$ , because when  $L > L_0$  electrons ejected from the emitter at angles  $\phi_0 \leq \phi \leq \pi$  will begin to reach the potential contact

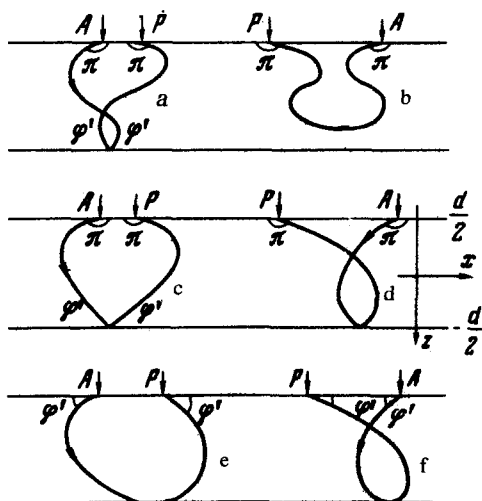


FIG. 1. Trajectories of the effective electrons produced as a result of their motion from the current junction  $A$  to the potential  $P$  in a non-uniform magnetic field.

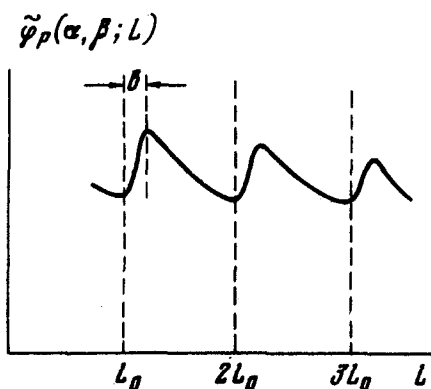


FIG. 2. Schematic behavior of the nonmonotonic part of the potential  $\tilde{\phi}_p$  at the point  $P$  on the distance to the far edge of the current junction  $L$ .

point. As a result, the variation of the potential is greater by a factor  $r_{\text{eff}}/b$  at a distance of the order of the characteristic size  $b$  of the contact than in a section of the same size at some distance from the point  $L_0$  ( $r_{\text{eff}} \sim r_\beta = c p_0 / |e| \beta \gg b$ ;  $c$  is the velocity of light,  $e$  is the electron charge, and  $p_0$  is the Fermi momentum).

If the reflection of the charges by the plate is not purely diffusive, then the increase of the potential described above will periodically repeat as the collector moves along the surface of the plate; moreover the ratio of the amplitudes of the two adjacent focusing lines is proportional to the probability of the mirror reflection of the charge carriers by the boundaries and the angle of incidence depends on the parameters of the nonuniform magnetic field.

The distribution of the electric potential  $\phi_p(L)$  at the surface of the sample can be easily determined by using the equation for the electrical neutrality and the kinetic Boltzmann equation with boundary conditions which take into account the reflection of the charges by the plates.

In the approximation of the image parameter and of the isotropic dispersion law for the charge carriers we have the following cases:

1)  $\gamma = |\alpha/\beta| < 1$ , i.e., the magnetic field is oppositely directed on the surfaces  $z_1 = d/2$  and  $z_2 = -d/2$ . For definiteness, we assume throughout that  $\beta > 0$  and that the nonuniform magnetic field is weak when  $d/r_\beta = \alpha^2 < 1$  and  $\alpha_\pm = \alpha|1 \pm \gamma| < 1$ .

Analysis shows that if  $\alpha > 0$  the potential lines can be observed only when the scattering of the charges by the lower surface is nondiffusive. The location of the line can be effectively determined by the electrons that collide with the upper boundary at the angle  $\phi_1 = \pi$  and with the lower boundary at the angle  $\phi_2 = \phi' = \pi - \arccos(1 - 4\alpha^2\gamma)$  (trajectory *a* in Fig. 1). The amplitude of the *n*th line is

$$\tilde{\phi}_p^{(n)} = q_2(\phi') [q_1(0)q_2(\phi')]^{n-1} J_e R_k \left(\frac{b}{r_\beta}\right)^2 \frac{1+\gamma}{a [K(a_+) - F(\chi, a_+)]}, \quad (1)$$

and the parameters  $\alpha$  and  $\beta$  at which it should be observed are determined by

$$n\Delta R = \frac{nd}{2a} [K(a_+) - F(\chi, a_+) - 2E(a_+) + 2E(\chi, a_+)] = L. \quad (2)$$

Here  $J_e$  is the total current transmitted through the emitter,  $R_k$  is its resistance,  $K(k)$ ,  $F(\chi, k)$  and  $E(k)$ ,  $E(\chi, k)$  are complete and incomplete elliptic integrals of the first and second kind, and  $\chi = \arccos \alpha_-/\alpha_+$ . We assume that the current through the measuring contact is negligible,  $|L| \ll l$ , and  $q_1(\phi) \neq 1$ .

When  $\alpha < 0$  the focusing lines exist independently of the reflection by the lower face. The charges which produce the nonmonotonic region  $\phi_p$  and collide with the upper surface at small angles do not interact with the lower boundary (trajectory *b* in Fig. 1). The expression for the amplitude of the *n*th peak of the potential has the following form:

$$\tilde{\phi}_p^{(n)} = q_1^{n-1}(0) J_e R_k \left(\frac{b}{r_\beta}\right)^2 \frac{1-\gamma}{aK(a_-)}, \quad (3)$$

and the characteristic magnetic field, which determines the location of the lines, satisfies the condition

$$n\Delta R = \frac{nd}{2a} [K(a_-) - 2E(a_-)] = L. \quad (4)$$

2)  $\gamma > 1$ , i.e., the magnetic field in the plate has a constant sign. In this case two types of focusing lines exist. One of them is associated with electrons incident on the upper wall at angles close to zero and  $\pi$  and on the lower wall at angles close to  $\phi'$  and  $\pi - \phi'$  (trajectories *c* and *d* in Fig. 1). The amplitude of the *n*th line is described by the equations

$$\tilde{\phi}_p^{(n)} = q_2(\phi') [q_1(0)q_2(\phi')]^{n-1} J_e R_k \left(\frac{b}{r_\beta}\right)^2 \Phi_\pm(\alpha, \beta), \quad (5)$$

$$\Phi_+ = \frac{1 + \gamma}{\alpha F(\chi, \alpha_+)} ; \quad \Phi_- = \frac{1 - \gamma}{\alpha [K(k_-) - F(\psi, k_-)]} , \quad (6)$$

where  $k_- = (1 + \alpha_-^2)^{-1/2}$  and  $\psi = \arcsin 2\alpha\sqrt{\gamma}$ .

The second type of line, which exists for any reflection by the lower boundary, is produced by charges whose orbit height is equal to the thickness of the plate. The trajectories of these electrons form together with the lower surface the angle 0 or  $\pi$  and with the upper surface the angle  $\phi'$  or  $\pi - \phi'$  (trajectories *e* and *f* in Fig. 1).

The nonuniform magnetic field  $H_y = \alpha + 2\beta z/d$  can be produced in the plate in a weak uniform magnetic field  $\mathbf{H} = (0, \alpha, 0)$ ; through the end contacts of this plate a current *j* is passed perpendicular to  $\mathbf{H}$ , whose magnetic field is described with sufficient accuracy by the formula  $H_y = 4\pi jz/c$ . To obtain electron focusing in 0.01-cm diameter plates with 0.1-cm spacing between the contacts, we must have a current density  $j = 10^4$  A/cm<sup>2</sup> and an external field  $\mathbf{H} = 10$  G, which can be achieved experimentally.

Since the charge displacement  $\Delta R(\phi)$  depends not only on  $\alpha$  and  $\beta$  but also on the dispersion law for the conduction electrons, the position of the focusing line contains information on the dynamic properties of the charge carriers, which can be used to control the reliability of the theoretical models of the electronic energy spectrum.

<sup>1</sup>Yu.V. Sharvin, Zh. Eksp. Teor. Fiz. **48**, 984 (1965) [Sov. Phys. JETP **21**, 655 (1965)].

<sup>2</sup>Yu.V. Sharvin and L.M. Fisher, Pis'ma Zh. Eksp. Teor. Fiz. **1**, 54 (1965) [JETP Lett. **1**, 152 (1965)].

<sup>3</sup>V.S. Tsoi, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 114 (1974) [JETP Lett. **19**, 70 (1974)].