Magnetic vortex-topological soliton in a ferromagnet with an easy-axis anisotropy

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A bound, multimagnon state with nonvanishing angular momentum is examined in terms of the Landau-Lifshits equation without damping. It is shown that such a magnetic vortex is associated with a linear topological peculiarity. The characteristics of this vortex are obtained with the help of a computer in a wide range of precessional frequencies of the magnetization vector.

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Lately, localized dynamic states of magnetization of an easy-axis ferromagnet, which were interpreted as bound states of a large number of magnons ("magnon drops"), have been discussed in detail.[1,2] Although magnon drops may have some momentum, it was assumed that they lack angular momentum whose conservation generally does not contradict the symmetry of a magnet. It turned out that magnon drops with a nonvanishing angular momentum are associated with topological singularities that are widely discussed in conjunction with different nonlinear vector fields.[3–6] In the simplest case, the localized inhomogeneous state of a ferromagnet, which has an angular momentum, is equivalent to a magnetic vortex.

A ferromagnet is described by the field of the magnetization vector **M** ($M_0 \sin\theta\cos\phi$, $M_0 \sin\theta\sin\phi$, $M_0 \cos\theta$), where $M_0 = 2\mu_0 s/a^3$, μ_0 is the Bohr magne-

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tron, s is the spin of the atom, a^3 is the atomic volume, and the angle θ is measured from the anisotropy axis $\vec{\xi}$. We assume that magnetization dynamics obey the Landau-Lifshits equations without dissipation.[7] The angular momentum of the magnetization field **K** and the number of spin deviations (magnons) N are

$$\mathbf{K} = -\frac{\hbar M_o}{2\mu_o}\int(1-\cos\theta)[\mathbf{r}\nabla\phi]dV, \qquad N = \frac{M_o}{2\mu_o}\int(1-\cos\theta)\,dV, \quad (1)$$

where the integration is over the volume of the whole magnet.

We examine the axisymmetric solution of the Landau-Lifshits equations for which

$$\theta = \theta(r), \quad \phi = \omega t + pz + \nu\chi + \phi_o, \quad \nu = \pm 1, \pm 2, \dots, \quad (2)$$

where ω is the precession frequency of the magnetic moment, the axis z is in the direction of the vector **K**, (r,χ) are polar coordinates in the perpendicular plane to the vector **K**, and ϕ_0 is an arbitrary constant.

The θ function satisfies the following equation

$$\frac{d^2\theta}{d\rho^2} + \frac{1}{\rho}\frac{d\theta}{d\rho} - \left(1 + \frac{\nu^2}{\rho^2}\right)\sin\theta\cos\theta + \Omega\sin\theta = 0, \quad (3)$$

which is written in dimensionless variables $\rho = r/l_0$, $\Omega = \omega/\omega_0(1 + p^2 l_0^2)$, where l_0 is the exchange length: $l_0 = \sqrt{\alpha/\beta}$, α and β are exchange and anisotropy constants ($\beta > 0$), and $\omega_0 = 2\mu_0 M_0 \beta/\hbar$ is the frequency of the homogeneous resonance.

The boundary conditions for Eq. (3) are homogeneous magnetization at a distance from the preferred axis: $\theta = 0$ when $\rho = \infty$, and boundedness of all the physical values on the axis: $\theta = m\pi$ (m is a natural number) when $\rho = 0$.

The solution of Eq. (2) describes the axisymmetric state of the ferromagnet with a specified circulation of the phase gradient ϕ, i.e., a given circulation of the spin (magnon) flux around the preferred axis. Such a solution is regarded at $p = 0$ as a rotating magnon drop in a two-dimensional magnet or as a linear magnetic vortex in a three-dimensional ferromagnet. In the latter case $p \neq 0$ gives the additional magnon flux along the vortex. It is easy to verify that for this state

$$K = \hbar\nu N = \nu\frac{\Delta M\,\xi}{2\mu_o}. \quad (4)$$

Taking into account the fact that the state along the K axis is homogeneous we calculate the integrals of motion of K and N taken over one atomic layer of thickness α. Moreover, we assume that $m = 0.1$.

It is easy to verify that when $\rho \to 0$ the solution of interest to us (3) behaves like $\theta = (\rho/\rho_0)^{|\nu|}$ for $m = 0$ and like $\theta = \pi - (\rho/\rho_0)^{|\nu|}$ for $m = 1$ ($\rho_0 = \text{const}$), but when $\rho \to \infty$ it vanishes exponentially: $\theta = \theta_0 \exp\{-\rho\sqrt{1-\Omega}\}$. It follows from the behavior of the solution at infinity that the localized inhomogeneous states can exist only when $\Omega < 1$, i.e., when $\omega < \omega_0\{1 + p^2 l_0^2\}$. Since $\omega_0 l_0^2 = 2\mu_0 M_0 \alpha/\hbar$, at $p \neq 0$ the localized-vortex state can also exist in an isotropic ferromagnet ($\beta = 0$) if $\hbar\omega < 2\mu_0 M_0 \alpha\, p^2$.

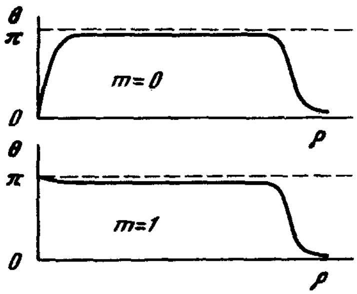

FIG. 1.

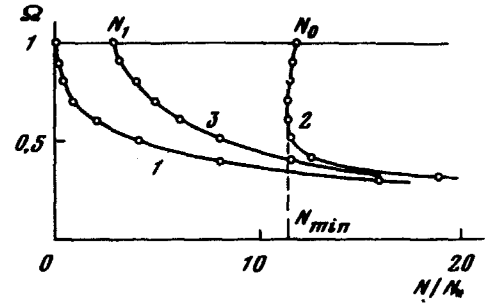

FIG. 2.

We limit ourselves to the case $\nu = 1$, since it follows from Eq. (4) that twice as many magnons are needed to form one vortex with $\nu = 2$ as to form two vortices with $\nu = 1$ and the same total momentum K. The schematic diagrams in Fig. 1 illustrate a magnetic vortex for a large number of bound magnons $N \gg N_*$ ($N_* = 2aM_0 l_0^2 / \mu_0 \sim (l_0/a)^2 \gg 1$). Both solutions give the value $\theta \approx \pi$ in the cylindrical region $r < l_0\sqrt{4N/\pi N_*}$, whose radius is much larger than the transition region l_0 at $N \gg N_*$. These solutions are close to those for the one-dimensional case examined in Ref. 1; the differences arise only near the axis (in a nonrotating drop on the axis $d\theta/d\rho = 0$). For both solutions, $\Omega = \sqrt{\pi}\sqrt{N_*/N}$. The solution with $m = 0$ is topologically equivalent to the homogeneous-magnetization state, and the state with $m = 1$ has a nonvanishing degree of mapping[3] and hence it differs topologically from the homogeneous magnetization. The energy of the vortex with $m = 1$ is lower than the energy of the vortex with $m = 0$.

As N decreases, the solution with $m = 0$ and $\nu = 0$ behaves differently from that with $m = 1$ and $\nu = 1$. For all these cases Eq. (3) was integrated numerically on the computer.[1)] The dependences of the frequency Ω on N, which were constructed by using the obtained solutions, are shown in Fig. 2. Curve 1 corresponds to $m = 1$ and curve 2 corresponds to $m = 0$. For comparison, we give the dependence $\Omega = \Omega(N)$ for the magnetic drop with $\mathbf{K} = 0$ (curve 3).

First, the absence of a static (with $\Omega = 0$) magnetic vortex (which was indicated in Ref. 4) is noteworthy. Second, the solution with $m = 1$ when $N \to 0$ becomes a δ-shaped linear singularity. Here $\Omega(N) \to 1$ and the energy of the vortex tends to $E_0 = a\alpha M_0^2 Q$, where Q is the topological charge ($Q = \nu \int_0^\pi \sin\theta d\theta d\phi$).

The localized solutions such as $m = 0$ exist only at $N > N_{\min} \approx 11.5\ N_*$ and for them $\Omega = 1$ at $N = N_0 \approx 12\ N_*$. The solutions with large precessional frequencies of the magnetization vector, for which $d\Omega / dN > 0$, are apparently unstable.

Finally, we shall discuss briefly the magnon drop without rotation (curve 3 in Fig. 2). The last point on the left-hand side of plot 3 was obtained by us for $\Omega = 0.95$. Extrapolation of the curve for $\Omega = 1$ gives a finite number $N_1 \approx 2.9\ N_*$ of magnons bound in the drop, which coincides with the value obtained earlier by Chiao *et al.*[8] for $\Omega = 1$. Note[1] that in a one-dimensional magnon drop $N \to 0$ as $\Omega \to 1$ and in a three-dimensional magnon drop $N \to \infty$ as $\Omega \to 1$.

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