# Oscillations of the magnetic moment and strong diamagnetism in a medium containing superconducting states localized in dislocation loops

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Oscillations of the magnetic moment in a superconducting state localized near a dislocation loop are examined. It is shown that the dependence of the critical temperature on the magnetic field oscillates. When dislocation loops of approximately the same size produce a medium, strong diamagnetic effects occur in it and instabilities appear in a number of cases. The investigation is conducted in the framework of the Ginzburg–Landau equation.

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Oscillations of the magnetic moment occur in the superconducting state near a dislocation loop in an external magnetic field at a temperature exceeding the critical temperature of a homogeneous superconductor. When the dislocation loops form a medium, a strong diamagnetism is developed in the system.

Earlier we showed that a localized state of the order parameter, which in this case has the form of a superconducting filament of diameter $l$ located on the axis of dislocation, is produced near the linear edge dislocation at a higher temperature and field than the critical.[1] Apparently such states were observed in the experiment of Khlyustikov and Khaĭkin.[2]

In addition to the linear dislocations, dislocation loops (DL) are a very common type of defect in crystals. Because the DL are closed, the produced superconducting state is doubly bound and the requirement that the order parameter must be single valued leads to quantum oscillations in the magnetic moment **M** and to the temperature dependence of the superconducting transition $T_0(\mathbf{B})$, where **B** is the magnetic-field induction. This phenomenon is analogous to the Little-Parks effect.[3]

When a large number of the DL of approximately the same area are present in a sample, we are dealing with a medium with an oscillating **M(B)** dependence. The diamagnetic phase transitions analogous to those occurring in the presence of strong diamagnetism in metals due to the de Hass-van Alphen effect may occur in the medium.

For simplicity, we shall examine the case $L \gg l$, where $L$ is the length of the DL. We shall also assume that the condition $\lambda(T) \gg l$ can be satisfied, where $\lambda$ is the penetration depth of the magnetic field.

For a formal solution of the problem, we write the Ginzburg-Landau (GL) functional in the following form

$$G = \int dV \left\{ \frac{1}{2m} \left| \left( -i\hbar \nabla - \frac{2e}{c}\mathbf{A} \right) \psi \right|^2 + (V - E)|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{(\mathbf{B} - \mathbf{B}_e)^2}{8\pi} \right\};$$

$$u_{ii} = b_0 \frac{\sin\phi}{r}; \quad U = a(0) u_{ii}\gamma; \quad \gamma = \partial \ln T_c / \partial \ln V; \quad E = \frac{T - T_c}{T_c} a(0);$$

$$b_0 = \frac{b}{2\pi} \frac{1 - 2\sigma}{1 - \sigma}; \qquad l \approx \xi_0^2 / b_0 \gamma, \tag{1}$$

where $m$ and $e$ are the mass and charge of the electron, $c$ is the velocity of light, $\mathbf{B}_e$ is the external magnetic field, $\psi$ is the order parameter, A is the vector potential, $V$ is the volume, $\alpha(T)$ and $\beta$ are the coefficients in the GL expansion, $T_c$ is the critical temperature of the homogeneous superconductor, $u_{ik}$ is the strain tensor of the DL, **r** is the vector drawn from the nearest point of the DL, **b** is Burgers vector, $\phi$ is the angle between the **r** and **b** vectors, and $\xi_0$ is the coherence length.

The density of the superconducting current and the order parameter of the current state have the form

$$\mathbf{j} = \frac{ie\hbar}{m}(\psi \nabla \psi^* - \psi^* \nabla \psi) - \frac{4e^2}{mc}|\psi|^2 \mathbf{A}; \qquad \psi = q e^{i\phi(\zeta)} \psi_0, \tag{2}$$

where $\vec{\zeta}$ is the vector in the direction of the DL singularities and $\psi_0$ is the eigenfunction of the lower state $E_0$ of the linear part in $\psi$ of the GL equation obtained from Eq. (1) by varying $q^2$. The total current flowing in the superconducting ring has the form

$$\mathbf{J} = \langle \psi_0^2 \rangle \frac{2e}{m} q^2 \left( \hbar \nabla \phi(\zeta) - \frac{2e}{c}\mathbf{A} \right), \qquad \langle \psi_0^2 \rangle \equiv \int \psi_0^2 \, d^2\mathbf{r}. \tag{3}$$

Let us integrate Eq. (3) over $\vec{\zeta}$ along the DL. Taking into account the uniqueness of $\psi$

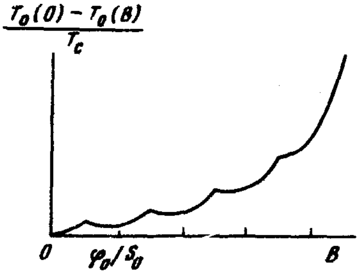


FIG. 1.

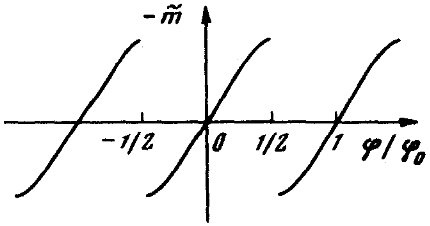


FIG. 2.

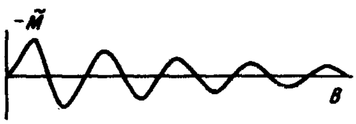


FIG. 3.

we obtain:

$$J = 2hq^2e|n - x|/mL; \qquad x = \Phi/\Phi_0; \tag{4}$$

$$\Phi = \int \mathbf{B}dS; \quad \Phi_0 = hc/2e; \quad n = 0, \pm 1, \pm 2 \ldots \quad .$$

Let us substitute $\psi$ from Eq. (2) in Eq. (1), minimize $G$ with respect to $q^2$, and substitute $J$ according to Eq. (4); as a result, we obtain an equation for $q^2$. Assuming in this equation that $q = 0$, we obtain the coexistence curve $T_0(B)$

$$T_0(0) - T_0(B) = \pi T_c F^2(x) b_0 l/S_0; \qquad F = x - [x - 1/2], \tag{5}$$

where $S_0$ is the area of the DL and [...] is the whole part. It should be taken into account, however, that the true value of $T_0$ is a function of $B$. As we showed earlier, $T_0(B) \sim B^2$.[1]

The coexistence curve with allowance for this fact is shown in Fig. 1.

Determining $q^2$ from the equation, substituting it in the equation for $G$, and differentiating $G$ with respect to $B$, we determine the magnetic moment $\tilde{m}(x)$. (Here the symbol $\sim$ denotes the oscillating part) (Fig. 2).

$$\tilde{m}(x) \approx -\mu_0 L l^2 (T_0(B) - T) F(x)/a^3 T_c, \qquad \mu_0 = e\hbar/2mc. \tag{6}$$

The oscillation period is $\Delta B \sim \Phi_0/S_0$ and $a$ is the interatomic distance.

When the sample has a large number of nonintersecting DL ($N$ loops per unit volume), the magnetic moment of such a medium can be determined by averaging Eq. (6) over the areas of the DL. Let $\phi\,[(S-S_0)/\Delta\,]$ be a rapidly decreasing area-distribution function of the DL, which is normalized to unity.

As a result of averaging, we obtain for the magnetization (Fig. 3):

$$M(B) = N\int \tilde{m}(x)\,\phi((S-S_0)/\Delta)\,dS. \tag{7}$$

It can be easily verified that $\tilde{M}(B)$ is also an oscillating function but the $\tilde{m}$ jumps are smoothed out. The width of the smeared region of the $n$th jump is $\delta B_n \sim \Phi_0 n^2\Delta/S_0^2$.

As it is well known,[4] the system becomes unstable if $\partial\tilde{M}/\partial B > 1/4\pi$. In particular, if the sample has the shape of a thin rod, then first-order phase transitions will occur. A sample in the shape of a plate should have the stratification of domains. It can be easily seen from Eqs. (6) and (7) that the unstable regions of discontinuities $\tilde{m}(BS_0/\Phi_0)$ are those with numbers up to $n_{\max}$,

$$n_{max} \sim (S_0/\Delta)\,NLS_0/\kappa^2, \quad \kappa \sim \lambda(0)/\xi_0. \tag{8}$$

At $n_{\max} \leqslant 1$ there is no instability.

In conclusion, we give some estimates. For $V_3Ge$ at $L \approx 10^{-4}$ cm and $\Delta \approx 10^{-2}\,S_0$ we get:

$$\Delta B \approx 10\ \text{G},\ \delta B_1 \approx 0.1\ \text{G},\quad \tilde{M} \sim 1\ \text{G},\quad (\partial\tilde{M}/\partial B)_{max} \sim 10.$$

These estimates show that experimental observation of the predicted effects is desirable and well within the realm of possibility.

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