# Theory of superdiamagnets

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The phenomenological $\Psi$ theory of superconductivity, which may be of interest in describing superdiamagnets—a class of materials with strong diamagnetism but differing from conventional superconductors—is generalized. The proposed generalization, moreover, also involves superconductors.

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As is well known, under certain conditions superconductors can be considered ideal diamagnets (magnetic susceptibility $\chi_{id} = -1/4\pi$) and, moreover, can have a resistance equal to zero. It is quite natural, however, to raise the question[1] about the possibility of the existence of solids having only one of these properties, specifically, superdiamagnetism (susceptibility $\chi = \chi_{id} = -1/4\pi$ or $\chi \sim \chi_{id}$ and $\mu = 1 + 4\pi\chi \geqslant 0$) in the absence of metallic conductivity. Recently, this question became relevant in connection with the examination of models of materials with spontaneous currents[2-5] which undergo a phase transition from the dielectric (semiconducting) state to the superdiamagnetic state.

A key problem here is selection of the order parameter. Volkov *et al.*[3] used the spontaneous current density **j**, which was introduced by Landau[6] as a relevant parameter. But above the transition temperature $T_c$ the material in this case should behave like a London superconductor rather than a dielectric. Moreover, because of the field equations $\mathbf{j} = (c/4\pi)\,\mathrm{rot}\,\mathbf{H}$, and this as a minimum complicates the selection of the density **j** as the order parameter. Volkov *et al.*[5] considered a certain value of the momentum density $p$ as the order parameter, but the macroscopic measurement of this parameter and the general form of the free energy remain insufficiently clear. Therefore, it is appropriate to focus attention on the large "reserves" available in the description of the superconducting and superdiamagnetic systems by using the order parameter—the complex macroscopic wave function $\Psi$.

Let us write that expression for the density of the free energy which we shall use here

$$F = F_0 + \frac{\hbar^2}{2m^*}\left|\left(-i\hbar\nabla - i\frac{e^*}{c}\mathbf{A}\right)\Psi\right|^2 + a|\Psi|^2 + \frac{b}{2}|\Psi|^4$$
$$+ \frac{d}{4}\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right|^4 + \frac{f}{2}\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)^2\Psi\right|^2 \qquad (1)$$
$$+ g|\Psi|^2\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right|^2 + \frac{B^2}{8\pi},$$

where the magnetic field strength (identified with the induction) is $\mathbf{B} = \mathrm{rot}\,\mathbf{A}$ and all the coefficients generally may depend on the temperature $T$.

If we assume that $d = f = g = 0$ and $m^* = \mathrm{const}$, $b = \mathrm{const}$, and $a = \alpha(T - T_c)$, then we come to the $\Psi$ theory of superconductivity, where the phase transition occurs at $T = T_c$ (see Refs. 7 and 8, Sec. 45). For ordinary superconductors $e^* = 2e$ ($e$ is the electron charge) and it is convenient to assume that $m^* = 2m$ ($m$ is the electron mass; the choice of the constant $m^*$ is determined solely by normalization of $\Psi$). At $e^* = 0$ we come to the $\Psi$ theory of superfluiditiy[9,10] and its generalizations.

The new possibilities, which we shall examine here, are associated primarily with the temperature dependence of the coefficient $m^*$. For some phase transitions the coefficient $1/m^*$ with the gradient term is known to vanish (the so-called Lifshitz point). Bearing in mind the establishment of conformity with the results of Volkov *et al.*[4] and Volkov *et al.*,[5] we assume that

$$m^* = \mu(T - T_c), \quad a = \alpha(T - T_c); \; b = \mathrm{const} > 0, \qquad (2)$$

i.e., we assume that $m^*$, rather than $1/m^*$, vanishes.

Of course, we can also develop a more general scheme in which the $m^*$ and $\alpha$ coefficients vanish at different temperature. Under conditions (2) and assuming that the coefficients $d$, $f$, and $g$ have no singularities (say, they are constant, positive, and not anomalously large), at $T > T_c$ the equilibrium value of $\Psi = 0$. However, near $T_c$ the fluctuations of $\Psi$ increase, which produces an anomalous diamagnetism. Repeating this deduction for ordinary superconductors (see Ref. 8, Sec. 49) gives the susceptibility

$$\chi = -\frac{(e^*)^2 k_B T_c}{24\sqrt{2}\pi\hbar c(\mu a)^{1/2}(T - T_c)} . \tag{3}$$

This result is in agreement with that of Volkov *et al.*,[4] whereas in the superconductors $\chi \sim (T - T_c)^{-1/2}$.

As usual, the equations for $\Psi$ and $\mathbf{A}$ are obtained by varying the free energy $\tilde{F} = \int F dV$. For simplicity, we limit ourselves here to the special case when $F = g = 0$. thus, we have equations (we use the gauge div $\mathbf{A} = 0$):

$$\frac{1}{2m^*}\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)^2\Psi + \frac{d}{2}\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\left\{\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right|^2\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right\}$$

$$+ a\Psi + b|\Psi|^2\Psi = 0, \tag{4}$$

$$\Delta\mathbf{A} = -\frac{4\pi}{c}\mathbf{j}_s, \ \mathbf{j}_s = \frac{1}{2}\left[\frac{1}{m^*} + d\left|\left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A}\right)\Psi\right|^2\right]$$
$$\times\left\{-i\hbar e^*(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{2(e^*)^2}{c}|\Psi|^2\mathbf{A}\right\}. \tag{5}$$

Below the transition point $T_c$ (i.e., in the ordered phase), where $m^* < 0$ we cannot assume that $d = 0$ and the solutions differ substantially from those corresponding to ordinary superconductors. For orientation, we start with the superfluid system ($e^* = 0$) for which Eq. (4) has the form

$$-\frac{\hbar^2}{2m^*}\Delta\Psi - \frac{d\hbar^4}{2}\nabla\left\{|\nabla\Psi|^2\nabla\Psi\right\} + a\Psi + b|\Psi|^2\Psi = 0. \tag{6}$$

From Eq. (6) and analogous to the equation for $\Psi^*$ we obtain in the usual way the equation of continuity for the density of the flowing liquid $\tilde{\mathbf{j}}_s$

$$\text{div}\,\tilde{\mathbf{j}}_s = 0, \ \tilde{\mathbf{j}}_s = \text{const}\left[\frac{1}{m^*} + d\hbar^2|\nabla\Psi|^2\right](\Psi^*\nabla\Psi - \Psi\nabla\Psi^*). \tag{7}$$

Of course, this expression for $\tilde{\mathbf{j}}_s$ is consistent with Eq. (5). We seek a solution in the form

$$\Psi = \eta e^{i\mathbf{q}\mathbf{r}}, \quad \eta = \text{const}, \quad \mathbf{q} = \text{const}. \tag{8}$$

Substituting Eq. (8) in the expression for the free energy [i.e., in Eq. (1) with $e^* = 0$] and minimizing in $\eta$ and $\mathbf{q}$, we obtain (at $\eta \neq 0$ and $\mathbf{q} \neq 0$)

$$\frac{\hbar^2}{2m^*} q^2 + \frac{d\hbar^4}{2} q^4 \eta^2 + a + b\eta^2 = 0, \tag{9}$$

$$1/m^* + d\hbar^2 q^2 \eta^2 = 0. \tag{10}$$

Equation (9) follows from both Eqs. (6) and (8), but for the solution of Eq. (8) div $\mathbf{j}_s \equiv 0$ and Eq. (10) must also be obtained in the prescribed manner. From Eqs. (9) and (10) we determine [using also Eq. (2)]:

$$\eta^2 = -\frac{a}{b} = \frac{a(T_c - T)}{b}, \quad q^2 = -\frac{1}{\hbar^2 dm^* \eta^2} = \frac{b}{\hbar^2 d\mu a(T_c - T)^2}. \tag{11}$$

Note that for the solution of Eqs. (8) and (11) the flow density $\mathbf{j}_s = 0$ [see Eqs. (7) and (8)]. Evidently as $T \to T_c$ the solutions of Eqs. (8) and (11) are unsuitable and we probably cannot assume that $f = g = 0$ in Eq. (1). For a superdiamagnet (i.e., at $e^* \neq 0$) the solution of Eqs. (8) and (9) satisfies Eqs. (4) and (5) at $\mathbf{A} = 0$. Such a solution can probably be used in a simply connected sample in the absence of the external magnetic field. Substituting in Eq. (5) the solution of Eqs. (8) and (11) as the zeroth approximation for $\Psi$, we can see that superdiamagnetism generally holds. In the simplest case when the $\mathbf{q}$ and $\mathbf{A}$ vectors are collinear and linearization of Eq. (5) over $\mathbf{A}$ is possible, the current $\mathbf{j}_s$ in (5) has the form

$$\mathbf{j}_s = -2\hbar^2 d \frac{(e^*)^2}{c} \eta^4 q^2 \mathbf{A} = -2 \frac{(e^*)^2 \eta^2}{|m^*| c} \mathbf{A}.$$

Therefore, according to Eq. (5), we get an exponential attenuation of the field deep into the material with a penetration depth $\delta = [|m^*|c^2/4\pi(e^*)^2 n_s]^{1/2}$, where in order to conform with conventional superconductors we assumed that $\eta^2 = |\Psi|^2 = n_s/2$.

It is difficult to doubt that the use of expression (1) is justifiable in the extension of the usual theory of superconductivity to the case when the coefficient $m^*$ depends on temperature, where $m^*$ or $1/m^*$ passes through zero [one more order parameter, for example, magnetization $\mathbf{M}$ may have to be introduced in Eq. (1), if the superconductors in question are ferromagnetic; see for example, Refs. 11 and 12]. However, the question of whether superdiamagnetic materials different from superconductors exist cannot be answered on the phenomenological level, since the response of the system to the electric field in Eq. (1) cannot be predetermined. One of the systems of interest in this regard is a dielectric which goes directly to the superconducting state, rather than simply to the metallic state. As to the materials examined in Refs. 2, 4, and 5, it is not clear whether they can be described in the way indicated above. In general, the problem of superdiamagnetism, which cannot be reduced to the usual superconductivity, remains open. But, this situation, in our opinion, justifies a many-sided approach to this problem, in particular, on the basis of the analysis of the phenomenological possibilities.

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